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Uniqueness Polynomials for Holomorphic Curves into the Complex Projective Space

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Abstract. In this paper, by making use of uniqueness polynomials for meromorphic functions, we obtain a class of uniqueness polynomials for holomorphic curves from the complex plane into complex projective space. The related uniqueness problems are also considered.

1. Introduction and Results

We first recall the definitions of sharing values and sets which play an important role in the development of uniqueness theory of meromorphic functions. Let f and g be two non-constant meromorphic functions in the complex plane $\mathbf{C}$ and let a be a finite complex number. We say that *f and g share the value a CM (counting multiplicities)* , provided that $f-a$ and $g-a$ have the same zeros with the same multiplicities. Similarly, we say that *f and g share the value a IM (ignoring multiplicities)*, provided that $f-a$ and $g-a$ have the same set of zeros, where the multiplicities are not taken into account. In addition we say that *f and g share ∞ CM (IM)*, if $1/f$ and $1/g$ share 0 CM (IM).

Let S be a set of distinct elements of $\mathbf{C}\cup\{\infty\}$ and

$$E_f(S)=\bigcup_{a\in S}\{z;\ f(z)-a=0\},$$

where each zero is counted according to its multiplicity. If multiplicities are not counted, then the set is denoted by $\overline{E}_f(S)$. If $E_f(S)=E_g(S)$ we say that *f and g share the set S CM.* On the other hand, if $\overline{E}_f(S)=\overline{E}_g(S)$, we say that *$f$ and g share the set S IM.*

In [1], F. Gross proposed the following problem (known as "Gross-problem ") which has a significant influence on uniqueness theory of meromorphic functions: *Whether there exist two (even one) finite sets S_j $(j=1,2)$ such that $E_f(S_j)=E_g(S_j)$ $(j=1,2)$ can imply $f\equiv g$ for any pair of nonconstant entire functions f and g ?* Since then many authors have found such two finite sets (called unique range sets) with as small cardinalities as possible. See [2–4, 6].

P. Li and C. C. Yang [13] seem to have been the first to draw a connection between unique range sets and zeros of polynomials.

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A polynomial $P \in \mathbf{C}[t]$ is called a *uniqueness polynomials for meromorphic functions* (UPM) if

$$P(f) = P(g) \Rightarrow f = g$$

for all nonconstant meromorphic functions f and g on $\mathbf{C}$.

In the last years, much attention has been given to find uniqueness polynomials for meromorphic functions. For instance, Yi [7], Yang and Hua [8] proved

Theorem 1.1. *[7, 8] For $m, n \in \mathbf{N}^*$, let $P(z) = z^n - az^{n-m} + b,\ a, b \in \mathbf{C}^*$. Then $P(z)$ is a UPM if $(m, n) = 1, n > m + 1, m \geq 2$.*

Recall that the $N-$dimensional complex projective space

$$\mathbf{P}^N(\mathbf{C}) = \mathbf{C}^{N+1} - \{0\}/\sim,$$

where

$$(a_0, \ldots, a_N) \sim (b_0, \ldots, b_N) \text{ if and only if } (a_0, \ldots, a_N) = \lambda(b_0, \ldots, b_N)$$

for some $\lambda \in \mathbf{C}$. We denote by $[a_0 : \cdots : a_N]$ the equivalence class of $(a_0, \ldots, a_N)$. Throughout this paper, we fix homogeneous coordinates $[x_0 : \cdots : x_N]$ on $\mathbf{P}^N(\mathbf{C})$. Let H be a hypersurface of degree d in $\mathbf{P}^N(\mathbf{C})$ defined by the equation

$$\sum_{I \in \mathcal{T}_d} a_I X^I = 0$$

where $\mathcal{T}_d = \{(i_0, \ldots, i_N) \in \mathbf{N}^{N+1};\ i_0 + \cdots + i_N = d\}$, $X^I = x_0^{i_0} \cdots x_N^{i_N}$ for $I = (i_0, \ldots, i_N)$. Sometimes, we identify the hypersurface H with its defining polynomial, i. e. , we will write

$$H(x_0, \ldots, x_N) = \sum_{I \in \mathcal{T}_d} a_I X^I.$$

Since a meromorphic function on $\mathbf{C}$ is also a holomorphic curve from $\mathbf{C}$ into the complex projective with dimension 1, it is natural to generalize the results about UPM to the case of holomorphic curves from $\mathbf{C}$ into $\mathbf{P}^N(\mathbf{C})$. Now we recall the following definition

Definition 1.2. *A homogeneous polynomial P of variables $x_0, \ldots, x_N$ is called a uniqueness polynomials for holomorphic curves (UPC) if*

$$P(\tilde{f}) = P(\tilde{g}) \Rightarrow f = g$$

for all algebraically nondegenerate holomorphic curves f and g from $\mathbf{C}$ into $\mathbf{P}^N(\mathbf{C})$.

In 1997, Shirosaki [9] proved the homogeneous polynomial

$$H(x_0, x_1) = x_0^n + x_0^m x_1^{n-m} + x_1^n$$

is a uniqueness polynomial for holomorphic curves from $\mathbf{C}$ into $\mathbf{P}^1(\mathbf{C})$ if $(m, n) = 1, n > 2m + 8, m \geq 2$. Afterwards, he constructed inductively uniqueness polynomials for algebraically nondegenerate holomorphic curves into $\mathbf{P}^N(\mathbf{C})$. In 2005, T. V. Tan [10] improved Shirosaki's result to more general cases and hence obtained a larger class of UPCs.

In 2011, V. H. An and T. D. Duc [11] obtained a UPC related to Theorem 1.1.

Theorem 1.3. *[11] Suppose that $m, n \in \mathbf{N}^*$ with $(m, n) = 1, m \geq 2, n \geq 2m + 9$. Let*

$$P_i(x_i, x_N) = x_i^n - a_i x_i^{n-m} x_N^m + b_i x_N^n,\ (0 \leq i \leq N - 1),$$

where $a_i, b_i \in \mathbf{C}^*, 0 \leq i \leq N - 1$ and $b_i^{2d} \neq b_j^d b_l^d$ with $i \neq j, i \neq l$. Then $P_{N,d} := \sum_{i=0}^{N-1} P_i^d(x_i, x_N)$ is a UPC if $d \geq (2N - 1)^2$.

Note that the homogeneous polynomial $P_i(x_i, x_N) = x_i^n - a_i x_i^{n-m} x_N^m + b_i x_N^n$ is the homogeneous equation of the polynomial $\widetilde{P}_i(x) = x^n - a_i x^{n-m} + b_i$ as in Theorem 1.1, that is $P_i(x_i, x_N) = x_N^n \widetilde{P}_i(\frac{x_i}{x_N})$. Inspired by this heuristic, we present, in this article, a connection between the UPCs and the UPMs, which provides a class of uniqueness polynomials for holomorphic curves from $\mathbf{C}$ into complex projective space.

Theorem 1.4. *(Main Result) Suppose that $m, n, d \in \mathbf{N}^*$ with $n \geq 2m + 9, d \geq (2N-1)^2$. Let*

$$\widetilde{P}_i(x) = \sum_{\mu=0}^{m} a_{n-\mu}^i x^{n-\mu} + b_i$$

be a UPM, where $a_\mu^i, b_i \in \mathbf{C}, 0 \leq \mu \leq m, a_n^i \neq 0, b_i \neq 0, a_{n-\mu_0}^i \neq 0$ for some $\mu_0 \in \{1, \ldots, m\}$ $(0 \leq i \leq N-1)$. Set

$$P_i(x_i, x_N) = \sum_{\mu=0}^{m} a_{n-\mu}^i x_i^{n-\mu} x_N^{\mu} + b_i x_N^n, \ (0 \leq i \leq N-1).$$

If $b_i^{2d} \neq b_j^d b_k^d$ with $i \neq j, i \neq k$, then $P_{N,d} := \sum_{i=0}^{N-1} P_i^d(x_i, x_N)$ is a UPC.

In particular, Theorem 1.4 generalizes Theorem 1.3 in the case of $\widetilde{P}_i(x) = x^n - a_i x^{n-m} + b_i$ $(0 \leq i \leq N-1)$. In addition, Theorem 1.4 can yield some new UPCs. For example, as a corollary of the result of G. Frank and M. Reinders [12], we have the polynomial

$$P(x) = \frac{(n-1)(n-2)}{2} x^n - (n-1)(n-2) x^{n-1} + \frac{n(n-1)}{2} x^{n-2} - c$$

is a UPM, where $n(\geq 11)$ is a positive integer and $c(\neq 0, 1)$ is a constant. Thus, Theorem 1.4 implies the following

Corollary 1.5. *Suppose that $n \in \mathbf{N}^*$ with $n \geq 11$. For $0 \leq i \leq N-1$, let*

$$P_i(x_i, x_N) = \frac{(n-1)(n-2)}{2} x_i^n - (n-1)(n-2) x_i^{n-1} x_N + \frac{n(n-1)}{2} x_i^{n-2} x_N^2 + b_i x_N^n,$$

where $b_i^{2d} \neq b_j^d b_k^d$ with $i \neq j, i \neq k$. Then $P_{N,d} := \sum_{i=0}^{N-1} P_i^d(x_i, x_N)$ is a UPC if $d \geq (2N-1)^2$.

Let $P_{N,d}$ be the homogeneous polynomial defined in Theorem 1.4. Now consider the hypersurface S in $\mathbf{P}^N(\mathbf{C})$, which is defined by the equation $P_{N,d}(x_0, \ldots, x_N) = 0$. For a holomorphic curve $f : \mathbf{C} \to \mathbf{P}^N(\mathbf{C})$, we denote by f^*S the pull-back of the divisor S in $\mathbf{C}$ by f. By Theorem 1.4, we have the following uniqueness theorem.

Corollary 1.6. *Suppose that $m, n, d \in \mathbf{N}^*$ with $n \geq 2m + 9, d \geq (2N-1)^2$. Let f and g be two algebraically nondegenerate holomorphic curves from $\mathbf{C}$ into $\mathbf{P}^N(\mathbf{C})$. Let S be the hypersurface defined as above. Assume that $b_i^{2d} \neq b_j^d b_k^d$ with $i \neq j, i \neq k$. If $f^*S = g^*S$, then $f = g$.*

2. Preliminaries

We start with relevant notions and definitions. For details see [13–15]. Let D be a domain in $\mathbf{C}$, $f : D \to \mathbf{P}^N(\mathbf{C})$ be a holomorphic curve and U be an open set in D. Any holomorphic curve $\tilde{f} : U \to \mathbb{C}^{N+1}$ such that $\mathbf{P}(\tilde{f}(z)) \equiv f(z)$ in U is called a *representation* of f on U, where $\mathbf{P} : \mathbf{C}^{N+1} - \{0\} \to \mathbf{P}^N(\mathbf{C})$ is the standard projective map.

Definition 2.1. *For an open subset U of D we call a representation $\tilde{f} = (f_0, \ldots, f_N)$ a reduced representation of f on U if $f_0, \ldots, f_N$ are holomorphic functions on U without common zeros.*

Remark 2.2. *As is easily seen, if both* $\tilde{f}_j : U_j \to \mathbf{C}^{N+1}$ *are reduced representations of* f *for* $j = 1, 2$ *with* $U_1 \cap U_2 \neq \phi$ *then there is a holomorphic function* $h(\neq 0) : U_1 \cap U_2 \to \mathbf{C}$ *such that* $\tilde{f}_2 = h\tilde{f}_1$ *on* $U_1 \cap U_2$.

Remark 2.3. *Every holomorphic curve* $f : \mathbf{C} \to \mathbf{P}^N(\mathbf{C})$ *has a reduced representation on the totality of* $\mathbf{C}$. *See [16].*

Definition 2.4. *Let* $f : \mathbf{C} \to \mathbf{P}^N(\mathbf{C})$ *be a holomorphic curve with a representation* $\tilde{f}$. *If there exists no nonzero homogeneous polynomial* $H(x_0, \ldots, x_N)$ *such that* $H(\tilde{f}) \equiv 0$, *then it is said that* f *is algebraically nondegenerate.*

Obviously, for holomorphic curves from $\mathbf{C}$ into $\mathbf{P}^1(\mathbf{C})$, i.e., meromorphic functions, algebraically nondegeneracy coincides with nonconstantness.

In order to prove our main result, we need the following lemmas.

Lemma 2.5. *[15] Let* $F_j \not\equiv 0, 0 \leq j \leq N$ *be holomorphic functions on* $\mathbf{C}$, *and let* $d \in \mathbf{N}^*$. *Assume that*

$$F_0^d + \cdots + F_N^d = 0.$$

If $d > (N+1)(N-1)$, *there is a partition of indices,* $\{0, 1, \ldots, N\} = \cup I_\alpha$ *such that*
(i) the cardinality $|I_\alpha| \geq 2$ *for every* I_α,
(ii) $F_i/F_j = c_{ij} \in \mathbf{C}$ *for all* $i, j \in I_\alpha$,
(iii) $\sum_{i \in I_\alpha} F_i^d = 0$.

Lemma 2.6. *[17] Let* $g_j(x_0, \ldots, x_N)$ *be homogeneous polynomial of degree* δ_j *for* $0 \leq j \leq N$. *Suppose there exists a holomorphic curve* $f : \mathbf{C} \to \mathbf{P}^N(\mathbf{C})$ *so that its images lies in*

$$\sum_{j=0}^{N} x_j^{d-\delta_j} g_j(x_0, \ldots, x_N) = 0.$$

and $d > (N+1)(N-1) + \sum_{j=0}^{N} \delta_j$. *Then there is a nontrivial linear relation among* $x_1^{d-\delta_j} g_1(x_0, \ldots, x_N), \ldots, x_N^{d-\delta_j} g_N(x_0, \ldots, x_N)$ *on the image of* f.

3. Proofs

3.1. Proof of Theorem 1.4

Proof. Suppose that f and g be two holomorphic curves from $\mathbf{C}$ into $\mathbf{P}^N(\mathbf{C})$ with reduced representations $\tilde{f} = (f_0, \ldots, f_N)$, $\tilde{g} = (g_0, \ldots, g_N)$, respectively, such that $P_{N,d}(\tilde{f}) = P_{N,d}(\tilde{g})$. Then we get

$$P_0^d(f_0, f_N) + \cdots + P_{N-1}^d(f_{N-1}, f_N) - P_0^d(g_0, g_N) - \cdots - P_{N-1}^d(g_{N-1}, g_N) = 0. \tag{3.1}$$

Since $d \geq (2N-1)^2$, f and g are algebraically nondegenerate holomorphic curves, from Lemma 2.5 it follows that there exists some permutation, says σ, $\sigma : \{0, 1, \cdots, N-1\} \to \{0, 1, \cdots, N-1\}$ such that

$$P_i(f_i, f_N) = A_i P_{\sigma(i)}(g_{\sigma(i)}, g_N), \tag{3.2}$$

where $A_i^d = 1,\ 0 \leq i \leq N-1$. Fix B_i such that $B_i^n = A_i,\ 0 \leq i \leq N-1$. Then

$$\tilde{\hat{g}} = (\hat{g}_0, \ldots, \hat{g}_N) := (B_i g_0, \ldots, B_i g_N)$$

is also a reduced representation of g and

$$P_i(f_i, f_N) = P_{\sigma(i)}(\hat{g}_{\sigma(i)}, \hat{g}_N), \tag{3.3}$$

for $0 \leq i \leq N-1$.

Claim 1 $b_i f_N^n = b_{\sigma(i)} \hat{g}_N^n$ *for* $0 \leq i \leq N-1$.

We have from (3.3) that

$$\hat{g}_{\sigma(i)}^{n-m}\Big(\sum_{\mu=0}^{m} a_{n-\mu}^{\sigma(i)}\hat{g}_{\sigma(i)}^{m-\mu}\hat{g}_N^{\mu}\Big) - f_i^{n-m}\Big(\sum_{\mu=0}^{m} a_{n-\mu}^{i} f_i^{m-\mu} f_N^{\mu}\Big) - b_i f_N^n + b_{\sigma(i)}\hat{g}_N^n = 0. \tag{3.4}$$

for $0 \le i \le N-1$. We now define the holomorphic curve F_1 from $\mathbf{C}$ into $\mathbf{P}^3(\mathbf{C})$ induced by the mapping $\tilde{F}_1(z) = (\hat{g}_{\sigma(i)}, f_i, f_N, \hat{g}_N)$. By (3.4), we see that the images of F lies in

$$x_0^{n-m}\Big(\sum_{\mu=0}^{m} a_{n-\mu}^{\sigma(i)} x_0^{m-\mu} x_3^{\mu}\Big) - x_1^{n-m}\Big(\sum_{\mu=0}^{m} a_{n-\mu}^{i} x_1^{m-\mu} x_2^{\mu}\Big) - b_i x_2^n + b_{\sigma(i)} x_3^n = 0.$$

Since $n > 2m + 8$, it follows from Lemma 2.6 that the homogeneous polynomials

$$x_1^{n-m}\Big(\sum_{\mu=0}^{m} a_{n-\mu}^{i} x_1^{m-\mu} x_2^{\mu}\Big), b_i x_2^n, b_{\sigma(i)} x_3^n$$

are linearly dependent on the image of F_1. Hence, there exist constants C_1, C_2, C_3 with $(C_1, C_2, C_3) \neq (0, 0, 0)$, such that

$$C_1 b_{\sigma(i)}\hat{g}_N^n + C_2 b_i f_N^n + C_3 f_i^{n-m}\Big(\sum_{\mu=0}^{m} a_{n-\mu}^{i} f_i^{m-\mu} f_N^{\mu}\Big) = 0. \tag{3.5}$$

Note that the holomorphic curve f is algebraically nondegenerate, we then have $C_1 \neq 0$. If $C_1, C_2, C_3 \neq 0$, we can define the holomorphic curve F_2 from $\mathbf{C}$ into $\mathbf{P}^2(\mathbf{C})$ induced by the mapping $\tilde{F}_2(z) = (\hat{g}_N, f_N, f_i)$. Similarly, by (3.5) and Lemma 2.6, we obtain

$$D_1 b_i f_N^n + D_2 f_i^{n-m}\Big(\sum_{\mu=0}^{m} a_{n-\mu}^{i} f_i^{m-\mu} f_N^{\mu}\Big) = 0$$

for some constants D_1, D_2 with $(D_1, D_2) \neq 0$. Which is a contradiction to the assumption that f is algebraically nondegenerate. Therefore, we have $C_1 \neq 0$ and one of C_2, C_3 is 0. We next consider the following two possible cases.

If $C_2 = 0$, then $C_3 \neq 0$. By the assumption of the theorem that $a_{n-\mu_0}^{i} \neq 0$ for some $\mu_0 \in \{1, \dots, m\}$, we can rewrite (3.5) as the following

$$C_1 b_{\sigma(i)}\hat{g}_N^n + C_3 a_{n-\mu_0}^{i} f_i^{n-\mu_0} f_N^{\mu_0} + C_3 f_i^{n-m}\Big(\sum_{\mu\in\{0,\dots,m\},\mu\neq\mu_0} a_{n-\mu}^{i} f_i^{m-\mu} f_N^{\mu}\Big) = 0.$$

In the exactly same way, we obtain f is algebraically degenerate by Lemma 2.6. Again, we get a contradiction.

If $C_3 = 0$, then $C_2 \neq 0$. Thus, we deduce by (3.5) that

$$b_{\sigma(i)}\hat{g}_N^n = -\frac{C_2}{C_1} b_i f_N^n. \tag{3.6}$$

Then $\hat{g}_N = c f_N$ holds for some constant $c \neq 0$. Combing this with (3.4) and (3.6) yields that

$$\hat{g}_{\sigma(i)}^{n-m}\Big(\sum_{\mu=0}^{m} a_{n-\mu}^{\sigma(i)} c^{\mu}\hat{g}_{\sigma(i)}^{m-\mu} f_N^{\mu}\Big) - f_i^{n-m}\Big(\sum_{\mu=0}^{m} a_{n-\mu}^{i} f_i^{m-\mu} f_N^{\mu}\Big) - b_i\Big(1 + \frac{C_2}{C_1}\Big) f_N^n = 0.$$

Suppose that $1 + \frac{C_2}{C_1} \neq 0$. By the similar arguments above for the holomorphic curve F_3 from $\mathbf{C}$ into $\mathbf{P}^2(\mathbf{C})$ induced by the mapping $\tilde{F}_3(z) = (\hat{g}_{\sigma(i)}, f_N, f_i)$ we obtain a contradiction. Hence, $1 + \frac{C_2}{C_1} = 0$ and Claim 1 holds.

Claim 2 *The map σ is an identity, that is $\sigma(i) = i$ for $0 \leq i \leq N-1$.*

Suppose that there exists $i_0 \in \{0, 1, \ldots, N-1\}$ such that $\sigma(i_0) \neq i_0$. We will arrive at a contradiction below. By Claim 1, we have $b_i f_N^n = A_i b_{\sigma(i)} g_N^n$ for $0 \leq i \leq N-1$. Recall that $A_i^d = 1$, we deduce $b_i^d f_N^{nd} = b_{\sigma(i)}^d g_N^{nd}$ for $0 \leq i \leq N-1$. We thus obtain

$$\frac{{b_{i_0}}^d}{(b_{\sigma(i_0)})^d} = \frac{g_N^{nd}}{f_N^{nd}} = \frac{(b_{\sigma^{-1}(i_0)})^d}{{b_{i_0}}^d}.$$

However, this contradicts the assumption that for $i \neq j, i \neq k, b_i^{2d} \neq b_j^d b_k^d$. And hence, The map σ is an identity.

We are now ready to get back to our original task of showing that $f = g$. Claims 1,2 imply that $f_N^n = \hat{g}_N^n$. This clearly implies, together with (3.3), that

$$P_i\Big(\frac{f_i}{f_N}, 1\Big) = P_i\Big(\frac{\hat{g}_i}{\hat{g}_N}, 1\Big),$$

for $0 \leq i \leq N-1$. Note the definition of $\widetilde{P}_i(z)$, we then have

$$\widetilde{P}_i\Big(\frac{f_i}{f_N}\Big) = \widetilde{P}_i\Big(\frac{\hat{g}_i}{\hat{g}_N}\Big) = \widetilde{P}_i\Big(\frac{g_i}{g_N}\Big),$$

for $0 \leq i \leq N-1$. Since $\widetilde{P}_i(z),\ 0 \leq i \leq N-1$, are UPMs, we have $\frac{f_i}{f_N} = \frac{g_i}{g_N}$ holds for $0 \leq i \leq N-1$. Thus, $f = g$. This completes the proof. □

3.2. Proof of Corollary 1.6

Proof. Suppose that f and g be two holomorphic curves from $\mathbf{C}$ into $\mathbf{P}^N(\mathbf{C})$ with reduced representations $\tilde{f} = (f_0, \ldots, f_N)$, $\tilde{g} = (g_0, \ldots, g_N)$, respectively. Since $f^*S = g^*S$, $\frac{P_{N,d}(\tilde{f})}{P_{N,d}(\tilde{g})}$ is an entire function without zeros, denote by $h(z)$. Thus $P_{N,d}(\tilde{f}) = P_{N,d}(h\tilde{g})$, where $h\tilde{g} = (hg_0, \ldots, hg_N)$ is also a reduced representation of g. By the definition of $P_{N,d}$ and Theorem 1.4, $f = g$. □

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References

[1] F. Gross, Factorization of meromorphic functions and some open problems, In: Complex Analysis. Lecture Notes in Mathematics, vol.599, pp. 51-67, Springer, Berlin, 1977.
[2] F. Gross and C. C. Yang, On preimage and range sets of meromorphic functions, Proc. Japan Acad. 58 (1982), 17–20.
[3] M. L. Fang and W. S. Xu, On the Uniqueness of Entire functions, Bull. of Malaysian Math Soc. 19(1996),29–37.
[4] H. X. Yi, On a question of Gross concerning uniqueness of entire functions, Bull Austral Math. Soc. 57(1998), 343–349.
[13] P. Li and C. C. Yang, Some further results on the unique range sets of meromorphic functions, Kodai Math. J. 18(1995), 437–450.
[6] H. Fujimoto, On uniqueness of meromorphic functions sharing finite sets, Am. J. Math. 122(6)(2000), 1175–1203.
[7] H. X. Yi, Unicity theorems for meromorphic or entire functions III, Bull. Austral. Math. Soc. 53(1996), 71–82.
[8] C. C. Yang and X. H. Hua,Unique polynomials of entire and meromorphic functions, Mat. Fiz. Anal. Geom. 4(3)(1997), 391–398.
[9] M. Shirosaki, On polynomials which determine holomorphic mappings, J. Math. Soc. Japan 49(2)(1997), 289–298.
[10] T. V. Tan, Uniqueness polynomials for entire curves into complex projective space, Analysis 25 (2005), 297–314.
[11] V. H. An and T. D. Duc, Uniqueness theorems and uniqueness polynomials for holomorphic curves, Compl. Var. Ellipt. Equat. 56 (2011), 253–262.
[12] G. Frank and M. Reinders, A unique range set for meromorphic functions with 11 elements, Complex Var. Theory Appl. 37(1998), 185–193.
[13] L. Yang, Value distribution theory, Springer-Verlag, Berlin, 1993.
[14] M. Ru, Nevanlinna theory and its relation to Diophantine approximation, World Scientific, 2001.
[15] J. Noguchi and J. Winkelmann,Nevanlinna Theory in Several Complex Variables and Diophantine Approximation, Springer, Tokyo, 2014.
[16] L. Yang, C. Y. Fang and X. C. Pang, Normal families of holomorphic mappings into complex projective space concerning shared hyperplanes, Pacfic J. Math 272(2014), 245–256.
[17] Y. T. Siu and S. K. Yeung, Defects for ample divisors of Abelian varieties, Schwarz lemma, and hyperbolic hypersurfaces of low degrees, Amer. J. Math. 119 (1997), 1139–1172.