# Some Methods to Reduction on Electrical Transmission Lines by Using Rough Concepts 

A. A. Nasef ${ }^{\text {a }}$, M. Shokry ${ }^{\text {b }}$, S. Mukhtar ${ }^{\text {b }}$<br>${ }^{a}$ Department of Physics and Engineering Mathematics, Faculty of Engineering, Kafrelsheikh University, Egypt.<br>${ }^{b}$ Department of Physics and Engineering Mathematics, Faculty of Engineering, Tanta University, Egypt.


#### Abstract

This paper discuss some concepts of rough set, nano topology, relations, discernibility matrix, discernibility function and new method of reductions, by modeling electrical application, converting to data table, comparing electrical results with mathematical results, indicating the state of conflicting results and trying to solve them. We got a new way to delete unnecessary components of any circuit by turning it into an information system.


## 1. Introduction

In this section, we will recall some fundamental concepts needed in this paper, which can also be found in [1], [3], [4], [5], [8], [10], [11], [12] and [13].

### 1.1 Databases

Databases are a set of logical data elements that are linked to each other in a mathematical relationship. We can represent databases using information systems that divide data into one or more types of classes. Such that classification of any number of students, according to age, color and length. It is characterized by rapid access and retrieval of data to ease of the customer, such as his number or his name. Methods of reduction can decrease the vast space occupied by the archive offices. Data integration where we can link the banking system with the civil register, to be customer-related data to create the account, and therefore the account will be linked to the national registry, to get data from a reliable source. In this, sense the importance of intermittent mathematics and its multiple classifications, such as rough sets, soft sets, and relationships, where these mathematical treatments deal with the ambiguity affecting information systems. For example, the rough sets deal with these data by substituting each group that is not defined by a pair of defined groups. In this paper, we will review the efforts of previous scientists in the analysis and classification of a set of data in different ways and access to the decision making and the reduction of some data if possible and indicate the compatibility between these applications and some of them. In addition we try to develop a new mathematical method through an analysis of the engineering model [1, 4, 7].

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### 1.2 The Electrical Power System

The power system is based on three basic process [2]. Generation system, transportation system and finally the electrical distribution system. Figure (1) shows a general outline of all stages of the electrical system where we note that it starts at the generation stage, then the stage of transport and then the distribution stage. There are a variety of energy sources generated by electricity, such as steam stations, diesel stations, solar power plants, or wind power plants. In this research, we will highlight the second stage, the stage of transport, where the scientists of electricity class transport to different sections according to the length of the distance of transport. In this research we will review the second type of this classification and take the case, two that called Nominal- $\pi$ representation and try to extract some information and to convert them into data tables. We try to study these data through the characteristics of the various combinatorial groups and compare their results with the electrical results obtained in advance by the specialized electrical scientists.

## 2. High Voltage Transmission Line.

In this section we describe different types of high voltage transmission lines can be formed by simple diagrams as shown in Figure (1). Show the stages of generating electricity starting from the steam generation station and then moving to the high voltage transformer [3], to the high voltage electric transmission lines and then to the electric power transformers that ends in the final distribution stage.


Figure 1: A general form of electrical power system.

The line models are classified by their length as following:

- Short line approximation for lines that are less than 80 km long.
- Medium line approximation for lines whose lengths are between 80 km to 250 km .
- Long line model for lines that are longer than 250 km .


### 2.1 Nominal $\pi$-Representation.

Nominal $\pi$-representation in this application the lumped series impedance is placed in the middle while the shunt admittance is divided into two equal parts and placed at the two ends [6]. Nominal $\pi$-representation is shown in Figure (2). This representation is used for load flow studies, as we shall see later. Also, a long transmission line can be modeled as an equivalent $\pi$-network for load flow studies. The lumped series impedance is placed at the middle of the circuit whereas the shunt admittance are at the ends. As we can see from the diagram of Nominal $\pi$-representation below, the total lumped shunt admittance is divided into two equal halves, and each half with value is placed at both the sending and the receiving end while the entire circuit impedance is between the two. The shape of the circuit so formed resembles that of a symbol Nominal $\pi$-representation, and for this reason, it is known as Nominal $\pi$-representation application of a medium transmission line. It is mainly used for determining the general circuit parameters and performing load flow analysis.


Figure 2: Nominal $\pi$-representation.
As we can see here $e(t)$, and $U 2$ is the supply receiving end voltages, $i 1$ is the current flow through the supply end, $i 2$, and $i 3$ are the values of currents flowing through the admittances, respectively, where $L_{t}+L_{g}$ : the total induction impedance of generator and transformer, $R+x_{l}$ : the total impedance of the line, and $C / 2$ : is sending and receiving lumped shunt admittance.

### 2.2 Simplification Electrical Circuit.

Suppose that total induction impedance of the generator and transformer: $L_{t}+L_{g}=R_{1}$, sending lumped shunt admittance: $C / 2=R_{2}$, the total impedance of the line: $Z=R+x_{l}=R_{3}$, and receiving lumped shunt admittance: $C / 2=R_{2}$, the total impedance of the line: $Z=R+x_{l}=R_{3}$, and receiving lumped shunt admittance: $C / 2=R_{4}$. As shown in Figure (3).


Figure 3: Nominal $\pi$-representation.
In Figure (3) the resistors from $R_{1}$ to $R_{4}$ represent the conditions. Load represents the decision. In the present, case we are trying to study the different effects of the electrical circuit in the case of conduction and non-conductivity if electrical resistors and the impact of the final pregnancy, which is represented here by the decision. Assuming that the status of the resistance is No, suppose the status of the resistance is taken OFF.

Table (1) is considered as decision table which includes four conditional attributes such as $R_{1}, R_{2}, R_{3}$, and $R_{4}$. To simplify operation, we use symbols such as $a, b, c, d$, and $e$ to replace attribute names, use numbers such as $0 ", " 1$ " to substitute for attribute values. They are described as follows: On: " 1 ", and OFF: " 0 ". So the database is converted into a knowledge express system shown in Table (2).

| Attributes |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Decision |  |  |  |  |  |
| $U$ | $R_{1}$ | $R_{2}$ | $R_{3}$ | $R_{4}$ | Load |
| 1 | ON | ON | ON | ON | ON |
| 2 | ON | ON | ON | OFF | ON |
| 3 | ON | ON | OFF | ON | OFF |
| 4 | ON | ON | OFF | OFF | OFF |
| 5 | ON | OFF | ON | ON | ON |
| 6 | ON | OFF | ON | OFF | ON |
| 7 | ON | OFF | OFF | ON | OFF |
| 8 | ON | OFF | OFF | OFF | OFF |
| 9 | OFF | ON | ON | ON | OFF |
| 10 | OFF | ON | ON | OFF | OFF |
| 11 | OFF | ON | OFF | ON | OFF |
| 12 | OFF | ON | OFF | OFF | OFF |
| 13 | OFF | OFF | ON | ON | OFF |
| 14 | OFF | OFF | ON | OFF | OFF |
| 15 | OFF | OFF | OFF | ON | OFF |
| 16 | OFF | OFF | OFF | OFF | OFF |

Table 1

| Attributes |  |  |  |  | Decision |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $U$ | $a$ | $b$ | $c$ | $d$ | Load |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 1 | 0 | 1 |
| 3 | 1 | 1 | 0 | 1 | 0 |
| 4 | 1 | 1 | 0 | 0 | 0 |
| 5 | 1 | 0 | 1 | 1 | 1 |
| 6 | 1 | 0 | 1 | 0 | 1 |
| 7 | 1 | 0 | 0 | 1 | 0 |
| 8 | 1 | 0 | 0 | 0 | 0 |
| 9 | 0 | 1 | 1 | 1 | 0 |
| 10 | 0 | 1 | 1 | 0 | 0 |
| 11 | 0 | 1 | 0 | 1 | 0 |
| 12 | 0 | 1 | 0 | 0 | 0 |
| 13 | 0 | 0 | 1 | 1 | 0 |
| 14 | 0 | 0 | 1 | 0 | 0 |
| 15 | 0 | 0 | 0 | 1 | 0 |
| 16 | 0 | 0 | 0 | 0 | 0 |

Table 2

This table is considered as decision table which includes four conditional attributes such as $R_{1}, R_{2}, R_{3}$,
and $R_{4}$. To simplify operation, we use symbols such as $a, b, c, d$, and $e$ to replace attribute names, use numbers such as 0 ", " 1 " to substitute for attribute values. They are described as follows: On: " 1 ", and OFF: " 0 ". So the database is converted into a knowledge express system shown in Table (2).

## 3. Some Methods of Reduction.

In this section, we aim is to transform any information systems (IS) to different classes of use relations on of its objects, which can form abases of topology to reduce some not necessary knowledge in (IS), this paper focuses on four methods in the following.

### 3.1 Reduction by Using Rough Set Theory.

Rough set theory (RST) is a new mathematical treatment by Z. Pawlak in 1982 [8], that deals with the ambiguity that affects information systems so that each uncertain set can be replaced by a pair of defined sets called lower approximation and upper approximation. Some applications of rough set theory founding the analysis of medical data, pharmaceutical industry, metal engineering, image analysis, and control systems.
Definition 3.1. [8]. Information system (IS) is a pair $(U, A)$ where $U$ is the universe finite set of objects $U=$ $\left\{x_{1}, x_{2}, \ldots, x_{m}\right\}$, and $A$ is the set of attributes (features, variables). Each attribute $a \in A$ defines an information function $f a: U \rightarrow$ Va where Va is the set of values of $a$, called the domain of attribute a. Decision system (DS) is triple information system of the form $(D S)=(U, A, D)$. Where finite $D$ is a non-empty finite set of decision.

Definition 3.2. [8]. Let $U$ be a non-empty finite set of objects called the universe and $R$ be an equivalence relation $U$ named as the indiscernibility relation. The pair $(U, R)$ is called the approximation space.
Let $X$ be a subset of $U$, then:

- The lower approximation of $X$ concerning $R$, is denoted by $R_{*}(X)$,

$$
R_{*}(X)=\cup\{R(X): R(X) \subseteq X\} \text {, where } R(X) \text { an equivalence relation on } U \text {. }
$$

- The upper approximation of $X$ concerning $R$, denoted by $R^{*}(X)$,

$$
R^{*}=\cup\{R(X): R(X) \cap X \neq \phi\}
$$

- The boundary region of $X$ concerning $R$, denoted by $B_{R}(X)$,

$$
B_{R}(X)=R^{*}(X)-R_{*}(X)
$$

The set $X$ is said to be rough concerning $R$ if $R_{*}(X) \neq R^{*}(X)$ that is $B_{R} \neq \phi$.
Let $B \subseteq A, a \in B$, then $a$ is superfluous attributes in $B$ if $U / \operatorname{IND}(B)=U / \operatorname{IND}(B-\{a\})$.
The set $M$ is called a minimal reduction of $B$ if and only if:
(a) $U / \operatorname{IND}(M)=U / \operatorname{IND}(B)$,
(b) $U / \operatorname{IND}(M) \neq U / \operatorname{IND}(M-\{a\}), \forall a \neq M$.

The core of $B$ is the set of all indispensable attribute of $B, \operatorname{Core}(B)=\cap R E D(B)$, where $R E D(B)$ is the set of all reduces of $B$.

Example 3.3. Consider $\pi$-representation which is shown in the above in Table (2).

$$
U=\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16\}
$$

is an universe discourse.

$$
U / a=\{\{1,2,3,4,5,6,7,8\},\{9,10,11,12,13,14,15,16\}\} ;
$$

$U / b=\{\{1,2,3,4,9,10,11,12\},\{5,6,7,8,13,14,15,16\}\} ;$
$U / c=\{\{1,2,5,6,9,10,13,14\},\{3,4,7,8,11,12,15,16\}\} ;$
and
$U / d=\{\{1,3,5,7,9,11,13,15\},\{2,4,6,8,10,12,14,16\}\}$.
Relation $P$ has equivalent classes as follows:

$$
U / I N D(A)=\{\{1\},\{2\},\{3\},\{4\},\{5\},\{6\},\{7\},\{8\},\{9\},\{10\},\{11\},\{12\},\{13\},\{14\},\{15\},\{16\}\},
$$

```
    U/IND (A - {a}) = {{1,9},{2,10},{3,11},{4,12},{5,13},{6,14},{7,15},{8,16}};
    U/IND (A - {b}) = {{1,5},{2,6},{3,7},{4,8},{9,13},{10,14},{11,15},{12,16}};
    U/IND (A - {c}) ={{1,3},{2,4},{5,7},{6,8},{9,11},{10,12},{13,15},{14,16}};
and
    U/IND (A - {d}) ={{1,2},{3,4},{5,6},{7,8},{9,10},{11,12},{13,14},{15,16}}.
And we find that:
    U/IND (A) \not=U/IND (A - {a}),}\quadU/IND (A)\not=U/IND (A - {b})
    U/IND (A)\not=U/IND (A - {c}), and U/IND (A) = U/IND (A - {d}).
```

    Then no attributes are superfluous attributes and we get the reduction of \(A\) is \(\operatorname{RED}(A)=\{a, b, c, d, e\}\), and the
    minimal reduction of $A$ is $M=\{a, b, c, d\}$, and the Core of $(A)$ is $\operatorname{Core}(A)=\{a, b, c, d\}$.

### 3.2 Reduction by Using Nano Topology.

In this section, Thivager, et al. [13], introduced a new topology called nano topology in terms of the lower and upper approximations. The method of reduction of nano topology depends on the division of data through information systems into the collection. In which the categories are elements of the same conditions and the reduction of affects the classification, which makes this method of the best ways to deal with information systems which can be deleted conditions and elements are not influential in the decision.

Remark 3.4. [13]. Let $U$ be the universe of objects and $R$ be an equivalence relation on $U$. For $X \subset U$, we define $\tau_{R}(X)=\left\{U, \phi, R^{*}(X), R_{*}(X), B_{R}(X)\right\}$, where $R^{*}(X), R_{*}(X)$ and $B_{R}(X)$ are, respectively the upper approximation, the lower approximation and the boundary region of $X$ with respect to $R$. We note that $U$ and $\phi \in \tau_{R}(X)$. Since $R_{*}(X) \subseteq$ $\left.R^{*}(X), R_{*}(X)\right) \cup R^{*}(X)=R^{*}(X) \in \tau_{R}(X)$. Also $R_{*}(X) \cup B_{R}(X)=R^{*}(X) \in \tau_{R}(X)$, and $R^{*}(X) \cup B_{R}(X)=R^{*}(X) \in \tau_{R}(X)$. Also, $R_{*}(X) \cap R^{*}(X)=R_{*}(X) \in \tau_{R}(X) ; R^{*}(X) \cap B_{R}(X)=B_{R}(X) \in \tau_{R}(X)$, and $R^{*}(X) \cap B_{R}(X)=\phi \in \tau_{R}(X)$.

Definition 3.5. [13]. Let $U$ be the universe and $R$ be an equivalence relation on $U$ then, $\tau_{R}(X)=\left\{U, \phi, R_{*}(X), R^{*}(X), B_{R}(X)\right\}$,
called Nano topology, where $X \subseteq U$, and $\tau_{R}(X)$ satisfies the following axioms:
(1) $U$ and $\phi \in \tau_{R}(X)$.
(2) The union of the elements of any subcollection $\beta_{R}(X)$ is in $\tau_{R}(X)$.
(3) The intersections of the elements of any subcollection $\beta_{R}(X)$ is in $\tau_{R}(X)$.

Where $R^{*}(X), R_{*}(X)$, and $B_{R}(X)$ are, respectively the upper approximation, the lower approximation, and the boundary region of $X$ concerning $R$.

Proposition 3.6. [13]. If $\tau_{R}(X)$ is a nano topology on $U$ with respect to $X$, then the family $\beta\left(\tau_{R}(X)\right)=$ $\left\{U, R_{*}(X), B_{R}(X)\right\}$ is the basis for $\tau_{R}(X)$.

Definition 3.7. [13]. Let $U$ be the universe and $R$ be an equivalence relation on $U$ and let $\tau_{R}(X)$ be the nano topology on $U$, and $\beta\left(\tau_{R}(X)\right)$ be the basis for $\tau_{R}(X)$. A subset $M$ of $A$ (the set of attributes) is called the Core of $R$ if $\beta_{M} \neq B_{R-(r)}$ for every $r$ in $M$. That is, a Core of $R$ is a subset of attributes, which is such that none of its elements can be removed without affecting the classification power of attributes.

Example 3.8. Consider $\pi$-representation which is shown in the above Table (2)
$U=\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16\}, a, b, c$, and $d$ from condition attributes. Let $X=\{1,2,5,6\}$, be the set of loads (on).

$$
U / I N D(A)=\{\{1\},\{2\},\{3\},\{4\},\{5\},\{6\},\{7\},\{8\},\{9\},\{10\},\{11\},\{12\},\{13\},\{14\},\{15\},\{16\}\} .
$$

The lower and upper approximations of $X$ given by $R_{*}(X)=\{1,2,5,6\}$, and $R^{*}(X)=\{1,2,5,6\}$. Therefore $B_{R}(X)=\phi$. Hence the nano topology on $U$ is $\tau_{R}(X)=\{U, \phi,\{1,2,5,6\}\}$, and its basis,

$$
\beta\left(\tau_{R}(X)\right)=\{U, \phi,\{1,2,5,6\}\} .
$$

If the attribute $a$ is removed from the set of condition attributes, then:
$U /(R-a)=\{\{1,9\},\{2,10\},\{3,11\},\{4,12\},\{5,13\},\{6,14\},\{7,15\},\{8,16\}\}$
$(R-(a))_{*}(X)=\phi ;(R-(a))^{*}(X)=\{1,2,5,6,9,10,13,14\}$,

```
    \(\tau_{R-(a)}(X)=\{U, \phi,\{1,2,5,6,9,10,13,14\}\}\),
and its basis
        \(\left.\beta\left(\tau_{R-(a)}(X)\right)=\{U, 1,2,5,6,9,10,13,14\}\right\} \neq \beta\left(\tau_{R}(X)\right)\).
If the attribute \(b\) is removed, then:
    \(U /(R-(b))=\{\{1,5\},\{2,6\},\{3,7\},\{4,8\},\{9,13\},\{10,14\},\{11,15\},\{12,16\}\}\),
    \((R-(b))_{*}(X)=\{1,2,5,6\} ;(R-(b))^{*}(X)=\{1,2,5,6\}\),
and hence
    \(\tau_{R-(b)}(X)=\{U, \phi,\{1,2,5,6\}\}\),
and its basis
    \(\beta\left(\tau_{R-(b)}(X)\right)=\{U, \phi,\{1,2,5,6\}\}=\beta\left(\tau_{R}(X)\right)\).
If the attribute \(c\) is removed, then:
    \(U /(R-(c))=\{\{1,3\},\{2,4\},\{5,7\},\{6,8\},\{9,11\},\{13,15\},\{14,16\}\}\),
    \((R-(c))_{*}(X)=\phi ;(R-(c))^{*}(X)=\{1,2,3,4,5,6,7,8\}\),
and hence
    \(\tau_{R-(c)}(X)=\{U, \phi,\{1,2,3,4,5,6,7,8\}\}\),
and its basis
    \(\beta\left(\tau_{R-(c)}(X)\right)=\{U, \phi,\{1,2,3,4,5,6,7,8\}\} \neq \beta\left(\tau_{R}(X)\right)\).
```

If the attribute $d$ is removed, then:
$U /(R-(d))=\{\{1,2\},\{3,4\},\{5,6\},\{7,8\},\{9,10\},\{11,12\},\{13,14\},\{15,16\}\}$,
$(R-(d))_{*}(X)=\{1,2,5,6\} ;(R-(d))^{*}(X)=\{1,2,5,6\}$,
and hence
$\tau_{R-(d)}(X)=\{U, \phi,\{1,2,5,6\}\}$,
and its basis
$\beta\left(\tau_{R-(d)}(X)\right)=\{U, \phi,\{1,2,5,6\}\}=\beta\left(\tau_{R}(X)\right)$.
Therefore, $\tau_{R-(d)}(X)=\tau_{R}(X)$, and its basis $\beta\left(\tau_{R(X)-(d)}\right)=\beta\left(\tau_{R(x)}\right)$.


Similarly, if $X$ is taken as the set of (off load), then again $\operatorname{Core}(R)=\{a, c\}$. In the previous example, the conditions that could be eliminated without affecting the total load of the electrical circuit were concluded using nano topology, where resistors $\{b, d\}$ represent conditions that can be deleted.

Remark 3.9. The Core obtained using (RST) method is different from that obtained by Nano topology, using decision elements to set of condition in processing of calculating accuracy in decision making, in the first method we use set of condition only, so the results by use Nano topology better than the results by (RST).

### 3.3 Reduction by Using Discrenibility Matrix and Discernibility Function.

The mathematical method in which the data are represented by a symmetric matrix, elements within the matrix depend on the different objects of the conditions. So, by using Boolean operations, we can reduce any unnecessary conditions in information systems.

Definition 3.10. [9]. A Decision table $S=(U, A, V, f)$ is a knowledge express system, $A=C \cup D, C \cap D=\phi$, and $C$ is a conditional attributes set, $D$ is a decision attribute set. Discernibility matrix of $S$ is a $n \times n$ matrix, each element of discernibility matrix is a $(x, y)=\{a \in C \mid f(x, a) \wedge \omega(x, y)\}$, as to $x, y \in U, \omega(x, y)$ satisfies $x \in \operatorname{pos}_{c}(D)$ or $x \in \operatorname{pos}_{c}(D) \wedge y \in \operatorname{pos}_{c}(D)$ or $x \in \operatorname{pos}_{c}(D) \wedge y \in \operatorname{pos}_{c}(D)$ or $x, y \in \operatorname{pos}_{c}(D) \wedge(x, t) \in \operatorname{Ind}(D)$, where $\operatorname{pos}_{c}(D)$, $\operatorname{pos}_{e}(D)$ are the positive region, and $\operatorname{pos}_{c}(D)=R_{*}(D)$. Discernibility function depends on the Boolean algebra output from discernibility matrix, where it finally shows the core and the elements that can be deleted.

Definition 3.11. [9]. Discermibility function which described as $\Delta^{*}$ is a Boolean function. As to each attribute $a \in A$ in knowledge express system, we assign a Boolean variable " $a$ if $a *(x, y)=\left\{a_{1}, a_{2}, \ldots, a_{n}\right\} \neq \phi$.
We assign a Boolean function $a_{1} \vee a_{2} \vee \cdots \vee a_{k}$ which described as $\sum a *(x, y)=\phi$, define a Boolean discernibility function $\Delta^{*}=\Pi a *(x, y)$, where: $(x, y \in U \times U)$.

The discernibility function $\Delta^{*}$ has the property that all conjunctive normal forms in the minimal disjunctive
normal form of discernibility function $\Delta^{*}$ are all $D$ reducible of $C$. We call algorithm $(A, D)$ is independent where $A$ is a conditional attribute set and $D$ is a decision attribute set. If the algorithm $(R, D)$ is independent and constant, and equivalent class of every attribute for each we call algorithm $(R, D)$ is a reduction of algorithm $(A, D)$ the other is to find out the equivalent class of decision attribute decision rule of the system and find out the minimal attributes set which satisfy relation c. This minimal attribute set is a reduction of the original decision rule.

Example 3.12. Consider $\pi$-representation of an electrical circuit which formed in Table (2). If we study all possible states of the circuit. Suppose that $A=\{a, b, c, d\}$ is a conditional attribute set and $D=\{e\}$ is a decision attribute set, $U=\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16\}$ is a universe of discourse. We construct a discernibility matrix and discernibility function.

| $U \backslash U$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  | $d$ | $c$ | $c d$ | $d$ | $b d$ | $b c$ | $b c d$ | $a$ | $a d$ | $a c$ | $a c d$ | $a b$ | $a b d$ | $a b c$ | $a b c d$ |
| 2 |  |  | $c$ | $c d$ | $b d$ | $d$ | $b c d$ | $b c$ | $a d$ | $a$ | $a c d$ | $a c$ | $a b d$ | $a b$ | $a b c d$ | $a b c$ |
| 3 |  |  |  | $d$ | $b c$ | $b c d$ | $b$ | $b d$ | $a c$ | $a c d$ | $a$ | $a d$ | $a b c$ | $a b c d$ | $a b$ | $a b d$ |
| 4 |  |  |  |  | $b c d$ | $c d$ | $b c$ | $b$ | $a c d$ | $a c$ | $a d$ | $a$ | $a b c d$ | $a b c$ | $a b d$ | $a b$ |
| 5 |  |  |  |  |  | $d$ | $c$ | $c d$ | $a b$ | $a b d$ | $a c d$ | $a b c d$ | $a$ | $a d$ | $a c$ | $a c d$ |
| 6 |  |  |  |  |  |  | $c d$ | $c$ | $a b d$ | $a b$ | $a b c d$ | $a b c$ | $a d$ | $a$ | $a c d$ | $a c$ |
| 7 |  |  |  |  |  |  |  | $d$ | $a b c$ | $a b c d$ | $a b$ | $a b d$ | $a c$ | $a c d$ | $a$ | $a d$ |
| 8 |  |  |  |  |  |  |  |  | $a b c d$ | $a b c$ | $a b d$ | $a b$ | $a c d$ | $a c$ | $a d$ | $a$ |
| 9 |  |  |  |  |  |  |  |  |  | $d$ | $c$ | $c d$ | $b$ | $b d$ | $b c$ | $b c d$ |
| 10 |  |  |  |  |  |  |  |  |  |  | $c d$ | $c$ | $b d$ | $b$ | $b c d$ | $b c$ |
| 11 |  |  |  |  |  |  |  |  |  |  | $d$ | $b c$ | $b c d$ | $b$ | $b d$ |  |
| 12 |  |  |  |  |  |  |  |  |  |  |  | $b c d$ | $b c$ | $b d$ | $d$ |  |
| 13 |  |  |  |  |  |  |  |  |  |  |  |  | $d$ | $c$ | $c d$ |  |
| 14 |  |  |  |  |  |  |  |  |  |  |  |  |  | $c d$ | $c$ |  |
| 15 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $d$ |  |
| 16 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Table 3

Discernibility function corresponding to the discernibility matrix above is as follows:

$$
\begin{aligned}
\Delta^{*}= & (d)(c)(c \vee d)(b)(b \vee d)(b \vee c)(b \vee c \vee d)(a)(a \vee d)(a \vee c)(a \vee c \vee d)(a \vee b)(a \vee b \vee d)(\vee b \vee c)(a \vee b \vee c \vee d) \\
& (c)(c \vee d)(b \vee d)(d)(b \vee c \vee d)(b \vee d)(a \vee d)(a)(a \vee c \vee d)(a \vee c)(a \vee b \vee d)(a \vee d)(a \vee b \vee c \vee d)(a \vee b \vee c) \\
& (d)(b \vee c)(b \vee c \vee d)(b)(b \vee d)(a \vee c)(a \vee c \vee d)(a)(a \vee d)(a \vee b \vee c)(a \vee b \vee c \vee d)(a \vee b)(a \vee b \vee d) \\
& (b \vee c \vee d)(c \vee d)(b \vee d)(a \vee b)(a \vee b \vee d)(a \vee b \vee c)(a \vee b \vee c \vee d)(a)(a \vee d)(a \vee c)(a \vee c \vee d)(c \vee d)(c) \\
& (a \vee b \vee d)(a \vee b)(a \vee b \vee c)(a \vee d)(a)(a \vee c \vee d)(a \vee c)(d)(a \vee b \vee c)(a \vee b \vee c \vee d)(a \vee b)(a \vee b \vee d)(a \vee c) \\
& (a \vee c \vee d)(a)(a \vee d)(a \vee b \vee c \vee d)(a \vee b \vee c)(a \vee b \vee d)(a \vee b)(a \vee c \vee d)(a \vee d)(a)(b)(c)(c \vee d)(b)(b \vee d) \\
& (b \vee c)(b \vee c \vee d)(b)(b \vee d)(b \vee c \vee d)(b \vee c)(b \vee d)(b)(d)(c)(c \vee d)(c)(d)=a b c d .
\end{aligned}
$$

Accordingly, Table (3) has no reduction sets. Because algorithm $(P-\{a\}, A),(P-\{b\}, A),(P-\{c\}, A)$, and $(P-\{d\}, A)$ are constant, we could not leave out $a, b, c$ and $d$. So that $\{a, b, c, d\}$ is Core of $P=\{a, b, c, d\}$. Algorithm $(P, Q)$ has no reductions.

Example 3.13. Consider $\pi$ - representation of an electrical circuit which formed in Table (2), If we use the similar states in the set of objects, we can put the indiscernibility matrix in the following form. Decision table $S=(U, A, V, f)$
is a knowledge express system, $A=C \cup D$ and $C \cap D=\phi, C$ is a conditional attribute set, $D$ is a decision attribute set. An indiscernibility matrix of $S$ is a $n \times n$ matrix, each element of indiscernibility matrix is $a^{*}(x, y)=\{a \in$ $C \mid f(x, a)=f(y, a) \wedge \omega(x, y)\}$, as to $x, y \in U, \omega(x, y)$ satisfie $x \in \operatorname{poc}_{c}(D) \wedge \operatorname{poc}_{c}(D)$ or $x \in \operatorname{poc}_{c}(D) \wedge y \in \operatorname{poc}_{c}(D)$ or $x, y \in \operatorname{poc}_{e}(D) \wedge(x, y) \in \operatorname{Ind}_{c}(D)$ where, $\operatorname{poc}_{c}(D)$ and $\operatorname{poc}_{e}(D)$ are the positive region.

| $U \backslash U$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  | $a b c$ | abd | ad | acd | ac | $b c$ | ad | bcd | $b c$ | $b d$ | $b$ | cd | c | d |  |
| 2 |  |  | $a b d$ | $a b$ | ac | acd | $a$ | ad | $b c$ | bcd | b | $b d$ | c | cd |  | $d$ |
| 3 |  |  |  | $a b c$ | ad | $a$ | acd | $a c$ | $b d$ | $b$ | bcd | $b c$ | $d$ |  | cd | c |
| 4 |  |  |  |  | $a$ | $a b$ | ac | acd | $b$ | $b d$ | $b c$ | $b c d$ |  | $d$ | c | cd |
| 5 |  |  |  |  |  | $a b c$ | $a b d$ | $a b$ | cd | c | $d$ |  | bcd | $b c$ | $b d$ | $b$ |
| 6 |  |  |  |  |  |  | $a b$ | abd | c | cd |  | $d$ | $b c$ | $b c d$ | $d$ | $b d$ |
| 7 |  |  |  |  |  |  |  | $a b c$ | $d$ |  | cd | c | $b d$ | $b$ | $b c d$ | $b c$ |
| 8 |  |  |  |  |  |  |  |  |  | $d$ | c | cd | $b$ | $b d$ | $b c$ | $b c d$ |
| 9 |  |  |  |  |  |  |  |  |  | $a b c$ | $a b d$ | $a b$ | acd | ac | ad | $a$ |
| 10 |  |  |  |  |  |  |  |  |  |  | $a b$ | $a b d$ | ac | acd | $a$ | ad |
| 11 |  |  |  |  |  |  |  |  |  |  |  | $a b c$ | ad | $a$ | acd | ac |
| 12 |  |  |  |  |  |  |  |  |  |  |  |  | $a$ | ad | $a c$ | acd |
| 13 |  |  |  |  |  |  |  |  |  |  |  |  |  | $a b c$ | abd | $a b$ |
| 14 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $a b$ | $a b d$ |
| 15 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $a b c$ |
| 16 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Table 4

Discernibility function corresponding to the indiscernibility matrix above is as follows:

$$
\begin{aligned}
\Delta^{*}= & (a \vee b \vee c)(a \vee b \vee d)(a \vee b)(a \vee c \vee d)(a \vee d)(a)(b \vee c \vee d)(b \vee c)(b \vee d)(b)(c \vee d)(c)(d)(a \vee b \vee d)(a \vee b) \\
& (a \vee c)(\vee c \vee d)(b \vee c \vee d)(b)(b \vee d)(c)(c \vee d)(d)(a \vee b \vee c)(a \vee d)(a)(a \vee c \vee d)(a \vee c)(b \vee d)(b)(b \vee c \vee d) \\
& (b \vee c)(d)(c \vee d)(c)(a)(a \vee b)(a \vee c)(a \vee c \vee d)(b)(b \vee d)(b \vee c)(b \vee c \vee d)(d)(c)(c \vee d)(a \vee c \vee d)(a \vee b \vee d) \\
& (a \vee b)(c \vee d)(c)(d)(b \vee c \vee d)(b \vee c)(b \vee d)(c \vee d)(d)(b \vee c)(b \vee c \vee d 0(b)(b \vee d)(a \vee b \vee c)(a \vee c \vee d)(a \vee c) \\
& (a \vee d)(a)(a \vee c \vee d)(a \vee c)(d)(c \vee d)(c)(b \vee d)(b)(b \vee c \vee d)(b \vee c)(d)(c)(c \vee d)(b)(b \vee d)(b \vee c)(b \vee c \vee d) \\
& (a \vee b)(a)(a \vee d)(a \vee c)(a \vee c \vee d)(a \vee b \vee c)(a \vee b \vee d)(a \vee b)(a \vee b)(a \vee b \vee d)(a \vee b \vee c)=a b c d,
\end{aligned}
$$

Accoringly, Table (4) has no reduction sets. Because of the algorithm $P-\{a\}, A),(P-\{b\}, A) .(P-\{c\}, A)$ and $(P-\{d\}, A)$ are constant, we could not leave out $a, b, c$, and $d$. So that $\{a, b, c, d\}$ is Core of $P=\{a, b, c, d\}$. Algorithm $P, Q$ ) has no reductions.

From the above example, we notice that the indiscernibility matrix may be obtained from the discernibility matrix by using duality properties.

### 3.4 Reduction by Ussing Relations.

We can construct some concepts by using relations to reduce some states of current flow in Nominal- $\pi$ representation of circuit under reduced some not effects resistors in an electrical circuit which in Table(2).

Example 3.14. Consider $\pi$-representation of an electrical circuit which formed in Table (2). Let $U=\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16\}$ be a universe of discourse.
The relation induced from Table (2):

```
R={(1,1),(2,2),(3,3),(4,4),(5,5),(6,6),(7,7),(8, 8),(9,9),(10,10),(11,11),(12,12),(13,13),(14,14),
    (15,15), (16,16)}.
```

The form sets, classification:
$1 R=\{1\}, 2 R=\{2\}, 3 R=\{3\}, 4 R=\{4\}, 5 R=\{5\}, 6 R=\{6\}, 7 R=\{7\}, 8 R=\{8\}, 9 R=\{9\}, 10 R=\{10\}$, $11 R=\{11\}, 12 R=\{12\}, 13 R=\{13\}, 14 R=\{14\}, 15 R=\{15\}, 16 R=\{16\}$.
The collection induced by form set:
$\{\{1\},\{2\},\{3\},\{4\},\{5\},\{6\},\{7\},\{8\},\{9\},\{10\},\{11\},\{12\},\{13\},\{14\},\{15\},\{16\}\}$.
$1^{\text {st }}$ We reduce $A$ :
$R=\{(1,1),(1,9),(2,2),(2,10),(3,3),(3,11),(4,4),(4,12),(9,9),(9,1),(10,10),(10,2),(11,11),(11,3)$, $(12,12),(12,4),(13,13),(13,5),(14,14),(14,6),(15,15),(15,7),(16,16),(16,8)\}$.
The form sets, classification:
$1 R=9 R=\{1,9\}, 2 R=10 R=\{2,10\}, 3 R=11 R=\{3,11\}, 4 R=12 R=\{4,12\}, 5 R=13 R=\{5,13\}$,
$6 R=14 R=\{6,14\}, 7 R=15 R=\{7,15\}, 8 R=16 R=\{8,16\}$.
Then $a$ is not reduced.
$2^{\text {nd }}$ we reduce $b$ :

```
R={(1,1),(1,5),(2, 2),(2,6),(3,3),(3,7),(4,4),(4,8),(5,5),(5,1),(6,6),(6,2),(7,7),(7,3),(8,8),(8,4),
    (9,9), (9,13), (10,10), (10,14), (11,11), (11,15), (12,12), (12,16), (13,13), (13,9), (14,14), (14,10),
    (15,15), (15,11), (16, 16), (16, 12)}.
```

The form sets, classification:
$1 R=5 R=\{1,5\}, 2 R=6 R=\{2,6\}, 3 R=7 R=\{3,7\}, 4 R=8 R=\{4,8\}, 9 R=13 R=\{9,13\}$,
$10 R=14 R=\{10,14\}, 11 R=15 R=\{11,15\}, 12 R=16 R=\{12,16\}$.
Then $b$ is not reduced.
$3^{\text {rd }}$ We reduce c :
$R=\{(1,1),(1,3),(2,2),(2,4),(3,3),(3,1),(4,4),(4,2),(5,5),(5,7),(6,6),(6,8),(7,7),(7,5),(8,8),(8,6)$, $(9,9),(9,11),(10,10),(10,12),(11,11),(11,9),(12,12),(12,10),(13,15),(13,9),(14,14),(14,16)$, $(15,15),(15,13),(16,16),(16,14)\}$.
The form sets, classification:
$1 R=3 R=\{1,3\}, 2 R=4 R=\{2,4\}, 5 R=7 R=\{5,7\}, 6 R=8 R=\{6,8\}, 9 R=11 R=\{9,11\}$,
$10 R=12 R=\{10,12\}, 13 R=15 R=\{13,15\}, 14 R=16 R=\{14,16\}$.
Then $c$ is not reduced.

```
4
    R={(1,1),(1,2),(2,2),(2,1),(3,3),(3,4),(4,4),(4,3),(5,5),(5,6),(6,6),(6,5),(7,7),(7, 8),(8,8),(8,7),
            (9,9), (9,10), (10,10), (10,9), (11,11), (11, 12), (12, 12), (12,11), (13,13), (13,14), (14,14), (14,13),
            (15,15),(15,16), (16, 16), (16, 15)}.
```

The form sets, classification:
$1 R=2 R=\{1,2\}, 3 R=4 R=\{3,4\}, 5 R=6 R=\{5,6\}, 7 R=8 R=\{7,8\}, 9 R=10 R=\{9,10\}$,
$11 R=12 R=\{11,12\}, 13 R=14 R=\{13,14\}, 15 R=16 R=\{15,16\}$.
Then $d$ is not reduced, and from the above, we conclude that $\operatorname{Core}(R)=\{a, b, c, d\}$.

## 4. Data Classification by Reduction Objects.

If all classification in $\pi$-representation by four sets have the same number, then the number of reduces of states equal, number of states divided by the number of elements in each set of each classification. In the following example, we can reduce $R_{1}$ in an electrical circuit, so that the similar repeated objects in the states can be omitted, to given set of objects $\{2,4,5,7,10,12,13,15\}$, and neglected set of objects $\{1,3,6,8,9,11,14,16\}$. For study the second set and neglect the first set.

|  | Attributes |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Decision |  |  |  |  |  |
| $U$ | $R_{1}$ | $R_{2}$ | $R_{3}$ | $R_{4}$ | Load |
| 1 | ON | OFF | ON | ON | ON |
| 2 | ON | ON | OFF | OFF | OFF |
| 3 | ON | ON | ON | OFF | ON |
| 4 | ON | OFF | OFF | ON | OFF |
| 5 | OFF | OFF | ON | ON | ON |
| 6 | OFF | ON | OFF | OFF | OFF |
| 7 | OFF | ON | ON | OFF | OFF |
| 8 | OFF | OFF | OFF | ON | OFF |

Table 5

Table (5) is considered as decision table, which includes four conditional attributes such as $R_{1}, R_{2}, R_{3}$, and $R_{4}$. To simplify operation, we use symbols such as " $a$ ", " $b$ ", " $c$ ", " $d^{\prime \prime}$, and " $e^{\prime \prime}$ to replace attribute names, use numbers such as " 0 ", " 1 ", to substitute for attribute values. They are described as follows: $\mathrm{ON} " 1$ ", and OFF " 0 ". Therefore, the database is converted into a knowledge express system as follows Table (6).

| Attributes |  |  |  |  | Decision |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $U$ | $a$ | $b$ | $c$ | $d$ | $e$ |
| 1 | 1 | 0 | 1 | 1 | 1 |
| 2 | 1 | 1 | 0 | 0 | 0 |
| 3 | 1 | 1 | 1 | 0 | 1 |
| 4 | 1 | 0 | 0 | 1 | 0 |
| 5 | 0 | 0 | 1 | 1 | 1 |
| 6 | 0 | 1 | 0 | 0 | 0 |
| 7 | 0 | 1 | 1 | 0 | 0 |
| 8 | 0 | 0 | 0 | 1 | 0 |

Table 6

Example 4.1. Consider $\pi$-representation of an electrical circuit which formed in Table (6).
Let $U=\{1,2,3,4,5,6,7,8\}$ be a universe of discourse,

$$
\begin{array}{ll}
U / a=\{\{1,2,3,4\},\{5,6,7,8\}\} ; & U / b=\{\{1,4,5,8\},\{2,3,6,7\}\} ; \\
U / c=\{1,3,5,7\},\{2,4,6,8\}\} ; & U / d=\{\{1,4,5,8\},\{2,3,6,7\}\} .
\end{array}
$$

Relation P has equivalent classes as follows:
$U / I N D(A)=\{\{1\},\{2\},\{3\},\{4\},\{5\},\{6\},\{7\},\{8\}\} . \quad U / \operatorname{IND}(A-\{a\})=\{\{1,5\},\{2,6\},\{3,7\},\{4,8\}\} ;$
$U / I N D(A-\{b\})=\{\{1\},\{2\},\{3\},\{4\},\{5\},\{6\},\{7\},\{8\}\} . \quad U / I N D(A-\{c\})=\{\{1,4\},\{2,3\},\{5,8\},\{6,7\}\} ;$
$\operatorname{UIND}(A-\{d\})=\{\{1\},\{2\},\{3\},\{4\},\{5\},\{6\},\{7\},\{8\}\} ;$
and we find that:

$$
\begin{array}{ll}
U / \operatorname{IND}(A) \neq U / \operatorname{IND}(A-\{a\}), & \\
U / \operatorname{IND}(A) \neq U / \operatorname{IND}(A-\{c\}), & \text { and }
\end{array}
$$

Then attributes $b$ and $d$ are superfluous attributes and we get:

$$
U / I N D(A)=U / I N D(A-\{b, d\})
$$

Then the reduction of $A$ is:
$R E D(A)=\{\{a, c, d, e\},\{a, b, c, e\},\{a, c, e\}\}$,
and the minimal reduction of $A$ is: $M=\{a, c, e\}$, and Core of $A$ is: $\operatorname{Core}(A)=\{a, c, e\}$.

Example 4.2. Consider $\pi$-representation of an electrical circuit which formed in Table (6), let $U=\{1,2,3,4,5,6,7,8\}$, and $a, b, c$, and $d$ be the condition attributes, let $X=\{1,3\}$, the set of (on), $U / R=\{\{1\},\{2\},\{3\},\{4\},\{5\},\{6\},\{7\},\{8\}\}$. The lower and upper approximations of $(X)$ with respect to $R$ is given by $R_{*}(X)=\{1,3\}$, and $R^{*}(X)=\{1,3\}$, therefore $B_{R}(X)=\phi$. A nano topology on $U$ is $\tau_{R}=\left\{U, \phi,\{1,3\}\right.$, and its basis $\beta\left(\tau_{R}(X)\right)=\{U, \phi,\{1,3\}\}$.
$1^{\text {st }}$ If the attribute (a) is removed from the set of condition attributes, then:

$$
\begin{array}{lll}
\quad U /(R-\{a\})=\{\{1,5\},\{2,6\},\{3,7\},\{4,8\}\} ; & (R(a))_{*}(X)=\phi ; & (R-(a))^{*}(X)=\{1,3,5,7\}, \text { and hence } \\
& \tau_{R-(a)}(X)=\{U, \phi,\{1,3,5,7\}\}, \text { and its basis } \\
\beta\left(\tau_{R-(a))}(X)=\{U,\{1,3,5,7\}\} \neq \beta\left(\tau_{R}(X)\right) .\right. \\
2^{\text {nd }} \text { If the attribute }(b) \text { removed, then: } & \\
& U /(R(a))=\{\{1\},\{2\},\{3\},\{4\},\{5\},\{6\},\{7\},\{8\}\}, \quad(R-(b))_{*}(X)=\{1,3\} ; \quad(R-(b))^{*}(X)=\{1,3\}, \\
\text { and hence }
\end{array}
$$

$\tau_{R-(b)}(X)=\{U, \phi,\{1,3\}\}$, and its basis
$\beta\left(\tau_{R-(b))}(X)=\{U,\{1,3\}\}=\beta\left(\tau_{R}(X)\right)\right.$.
$3^{\text {rd }}$ If the attribute (c) removed, then:
$U /(R-\{c\})=\{\{1,4\},\{2,3\},\{5,8\},\{6,7\}\} ; \quad(R(c))_{*}(X)=\{1,2,3,4\} ; \quad(R-(a))^{*}(X)=\{1,3,5,7\}$,
and hence
$\tau_{R-(c)}(X)=\{U, \phi,\{1,2,3,4\}\}$, and its basis
$\beta\left(\tau_{R-(c))}(X)=\{U,\{1,2,3,4\}\} \neq \beta\left(\tau_{R}(X)\right)\right.$.
$4^{\text {th }}$ If the attribute (d) removed, then:

$$
U /(R(a))=\{\{1\},\{2\},\{3\},\{4\},\{5\},\{6\},\{7\},\{8\}\}, \quad(R-(d)) *(X)=\{1,3\} ; \quad(R-(d))^{*}(X)=\{1,3\},
$$

and hence
$\tau_{R-(d)}(X)=\{U, \phi,\{1,3\}\}$,
and its basis

$$
\beta\left(\tau_{R-(d))}(X)=\{U, \phi,\{1,3\}\}=\beta\left(\tau_{R}(X)\right) .\right.
$$

Therefore, $\beta\left(\tau_{R(X)-(b)}\right)=\beta\left(\tau_{R}(X)\right)$, and its basis $\beta\left(\tau_{R(X)-(d)}\right)=\beta\left(\tau_{R}(X)\right)$. If $M=\{a, c\}$, we see that $\beta_{M} \neq \beta_{R-(\{a, c\})}$. Therefore, Core $(R)=\{a, c\}$. similarly, if $X$ is taken as the set of (off load), then again Core $(R)=\{a, c\}$.

Example 4.3. Consider $\pi$-representation of an electrical circuit which formed in Table (6), suppose that $A=\{a, b, c, d\}$ is a conditional attribute set and $D=\{e\}$ is a decision attribute set, $U=\{1,2,3,4,5,6,7,8\}$ is a universe of discourse. We construct a discernibility matrix and discernibility function Table (6). $U=\{1,2,3,4,5,6,7,8\}$ is a universe of discourse.

| $U$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  | bcd | b | c |  | abcd | abd | ac |
| 2 |  |  | $c$ | bd | abcd | a | ac | abd |
| 3 |  |  |  | bcd | abd | ac | a | abcd |
| 4 |  |  |  |  | ac | abd | abcd | a |
| 5 |  |  |  |  |  | bcd | bd | $c$ |
| 6 |  |  |  |  |  |  | $c$ | bd |
| 7 |  |  |  |  |  |  |  | bcd |
| 8 |  |  |  |  |  |  |  |  |

Table 7: Discernibility matrix

Discernibility function corresponding to the discernibility matrix above is as follows:

$$
\begin{aligned}
\Delta^{*}= & (b \vee c \vee d)(b \vee d)(c)(a \vee b \vee c \vee d)(a \vee b \vee d)(a \vee c)(c)(b \vee d)(a \vee b \vee c \vee d)(a)(a \vee c)(b \vee c \vee d) \\
& (a \vee b \vee d)(a \vee c)(a)(a \vee b \vee c \vee d)(a \vee c)(a \vee b \vee d)(a \vee b \vee c \vee d)(a)(b \vee c \vee d)(b \vee d)(c)(c)(b \vee d) \\
& (b \vee c \vee d)=a c(b \vee d)=a b c \vee a c d .
\end{aligned}
$$

Accordingly, this Table (5) has two reduction sets such as $\{a, b, c\}$ and $\{a, c, d\}$. Because of algorithm $(P-\{b\}, A)$ and $(P-\{d\}, A)$ are constant, we could leave out b or c (note that they could not leave out at the same time), but we could
not leave out $a$ or $d$. So that $\{a, d\}$ is Core of $P=\{a, b, c, d\}$. Algorithm $(P, Q)$ has two reductions such as $(P-\{b\}, A)$ and $(P-\{c\}, A)$.

Example 4.4. Consider $\pi$-representation of an electrical circuit, which formed in Table (6), if we use the similar states in the set of objects, we can put the indiscernibility matrix in the form.

| $U$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  | a | ac | abd | bcd |  | c | bd |
| 2 |  |  | abc | ac |  | bcd | bd | c |
| 3 |  |  |  | a | c | bd | bcd |  |
| 4 |  |  |  |  | bd | c |  | bcd |
| 5 |  |  |  |  |  | a | ac | abd |
| 6 |  |  |  |  |  |  | abd | ac |
| 7 |  |  |  |  |  |  |  | a |
| 8 |  |  |  |  |  |  |  |  |

Table 8: Discernibility matrix
Discernibility function corresponding to the discernibility matrix above is as follows:
$\Delta^{=(a)(a \vee c)(a \vee b \vee d)(b \vee \vee \vee d)(c)(b \vee d)(a \vee b \vee d)(a \vee c)(b \vee c \vee d)(c)(a)(c)(b \vee d)(b \vee \vee \vee d)(b \vee d)(c)}$
$(b \vee c \vee d)(a)(a \vee c)(a \vee b \vee d)(a \vee b \vee d)(a \vee c)(a 0=a c(b \vee d)=a b c \vee a c d$.
Accordingly, this knowledge express system has two reduction Sets such as $\{a, b, c\}$ and $\{a, c, d\}$. Because algorithm $(P-\{b\}, A)$ and $(P\{d\}, A)$ are constants, we could leave out $b$ or $c$ (note that they could not leave out at the same time), but we could not leave out $a$ or $d$. So that $\{a, d\}$ is Core of $P=\{a, b, c, d\}$. Algorithm $(P, Q)$ has two reductions such as $(P-\{b\}, A)$, and $(P-\{c\}, A)$.

Example 4.5. Consider $\pi$-representation of an electrical circuit which formed in Table (6). We can construct some concepts by use relations to reduce some states of current flow in the circuit under effects resistors in an electrical circuit which in Table (6), $U=\{1,2,3,4,5,6,7,8\}$ is a universe of discourse. The relation induced:

$$
R=\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6),(7,7),(8,8)\}
$$

The form set classification:
$1 R=\{1\}, 2 R=\{2\}, 3 R=\{3\}, 4 R=\{4\}, 5 R=\{5\}, 6 R=\{6\}, 7 R=\{7\}, 8 R=\{8\}$.
The collection induced by form set:
$\{\{1\},\{2\},\{3\},\{4\},\{5\},\{6\},\{7\},\{8\}\}$.
$1^{\text {st }}$ We reduce a:
$R=\{(1,1),(1,5),(2,2),(2,6),(3,3),(3,7),(4,4),(4,8),(5,5),(5,1),(6,6),(6,2),(7,7),(7,3),(8,8),(8,4)\}$.
The form set classification:
$1 R=5 R=\{1,5\}, 2 R=6 R=\{2,6\}, 3 R=7 R=\{3,7\}, 4 R=8 R=\{4,8\}$.
Then (a) is not reduced.
$2^{\text {nd }}$ We reduce b:

$$
R=\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6),(7,7),(8,8)\} .
$$

The form set classification:
$1 R=\{1\}, 2 R=\{2\}, 3 R=\{3\}, 4 R=\{4\}, 5 R=\{5\}, 6 R=\{6\}, 7 R=\{7\}, 8 R=\{8\}$.
Then (b) is reduced.
$3^{\text {rd }}$ We reduce c:
$R=\{(1,1),(1,4),(2,2),(2,3),(3,3),(3,2),(4,4),(4,1),(5,5),(5,8),(6,6),(6,7),(7,7),(7,6),(8,8),(8,5)\}$.
The form set classification:
$1 R=4 R=\{1,4\}, 2 R=3 R=\{2,3\}, 5 R=8 R=\{5,8\}, 6 R=7 R=\{6,7\}$.
Then (c) is not reduced.
$4^{\text {th }}$ We reduce d:

$$
R=\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6),(7,7),(8,8)\}
$$

The form set classification:
$1 R=\{1\}, 2 R=\{2\}, 3 R=\{3\}, 4 R=\{4\}, 5 R=\{5\}, 6 R=\{6\}, 7 R=\{7\}, 8 R=\{8\}$.
Then $(d)$ is reduced. Then from the above, we conclude that $\operatorname{Core}(R)=\{a, c\}$.

## 5. New Method of Reduction.

Let $P \subseteq D$, where $P$ is called a partition and $D$ is the classified sets by decision, and $P=\left\{K_{i}\right\}=1, K_{i}=\left\{\left\{x_{i}\right\},\left\{x_{k}\right\}: D\left(x_{i}\right)=D\left(x_{k}\right), \forall x_{i}, x_{k} \in U\right.$ and $D(x)$ is decision value $\}$.
Let $A$ be called the set of attributes condition, if Core $(A)$ obtained $P / D$, Core $(C)$ is $P / D$, and $C \subseteq A$. Then $C$ is Core of information system.

Example 5.1. Consider $\pi$-representation of an electrical circuit, which formed in Table (9).

$$
U=\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16\}
$$

is a universe of discourse.

|  | $A=$ Attributes |  |  |  | Decision |
| :---: | :---: | :---: | :---: | :---: | :---: |
| U | $a$ | $b$ | c | $d$ | D |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 1 | 0 | 1 |
| 3 | 1 | 1 | 0 | 1 | 0 |
| 4 | 1 | 1 | 0 | 0 | 0 |
| 5 | 1 | 0 | 1 | 1 | 1 |
| 6 | 1 | 0 | 1 | 0 | 1 |
| 7 | 1 | 0 | 0 | 1 | 0 |
| 8 | 1 | 0 | 0 | 0 | 0 |
| 9 | 0 | 1 | 1 | 1 | 0 |
| 10 | 0 | 1 | 1 | 0 | 0 |
| 11 | 0 | 1 | 0 | 1 | 0 |
| 12 | 0 | 1 | 0 | 0 | 0 |
| 13 | 0 | 0 | 1 | 1 | 0 |
| 14 | 0 | 0 | 1 | 0 | 0 |
| 15 | 0 | 0 | 0 | 1 | 0 |
| 16 | 0 | 0 | 0 | 0 | 0 |

Table 9

| $A 1=$ Attributes |  |  |  | Decision |
| :---: | :---: | :---: | :---: | :---: |
| $U$ | $b$ | $c$ | $d$ | $D_{1}$ |
| 1 | 1 | 1 | 1 | 0 |
| 2 | 1 | 1 | 0 | 0 |
| 3 | 1 | 0 | 1 | 0 |
| 4 | 1 | 0 | 0 | 0 |
| 5 | 0 | 1 | 1 | 0 |
| 6 | 0 | 1 | 0 | 0 |
| 7 | 0 | 0 | 1 | 0 |
| 8 | 0 | 0 | 0 | 0 |
| 9 | 1 | 1 | 1 | 0 |
| 10 | 1 | 1 | 0 | 0 |
| 11 | 1 | 0 | 1 | 0 |
| 12 | 1 | 0 | 0 | 0 |
| 13 | 0 | 1 | 1 | 0 |
| 14 | 0 | 1 | 0 | 0 |
| 15 | 0 | 0 | 1 | 0 |
| 16 | 0 | 0 | 0 | 0 |

Table 10: Reduce (a)
$P 1=\{D 1, D 2, D 5, D 6\}$,
where $P 1$ is the set of load on,
$P 2=\{D 3, D 4, D 7, D 8, D 9, D 10, D 11, D 12, D 13, D 14, D 15, D 16\}$,
where $P 2$ is the set of load off, $A=\{a, b, c, d\}$. Let $a, b, c$, and $d$ be all attributes condition, if
$\operatorname{Core}(A)=P 1 / D=4 / 16=1 / 4$.

| $A 2=$ Attributes |  |  |  | Decision |
| :---: | :---: | :---: | :---: | :---: |
| $U$ | $a$ | $c$ | $d$ | $D_{2}$ |
| 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 0 | 1 |
| 3 | 1 | 0 | 1 | 0 |
| 4 | 1 | 0 | 0 | 0 |
| 5 | 1 | 1 | 1 | 1 |
| 6 | 1 | 1 | 0 | 1 |
| 7 | 1 | 0 | 1 | 0 |
| 8 | 1 | 0 | 0 | 0 |
| 9 | 0 | 1 | 1 | 0 |
| 10 | 0 | 1 | 0 | 0 |
| 11 | 0 | 0 | 1 | 0 |
| 12 | 0 | 0 | 0 | 0 |
| 13 | 0 | 1 | 1 | 0 |
| 14 | 0 | 1 | 0 | 0 |
| 15 | 0 | 0 | 1 | 0 |
| 16 | 0 | 0 | 0 | 0 |

Table 11: Reduce (b)

| $A 3=$ Attributes |  |  |  | Decision |
| :---: | :---: | :---: | :---: | :---: |
| $U$ | $a$ | $b$ | $d$ | $D_{3}$ |
| 1 | 1 | 1 | 1 | 0 |
| 2 | 1 | 1 | 0 | 0 |
| 3 | 1 | 1 | 1 | 0 |
| 4 | 1 | 1 | 0 | 0 |
| 5 | 1 | 0 | 1 | 0 |
| 6 | 1 | 0 | 0 | 0 |
| 7 | 1 | 0 | 1 | 0 |
| 8 | 1 | 0 | 0 | 0 |
| 9 | 0 | 1 | 1 | 0 |
| 10 | 0 | 1 | 0 | 0 |
| 11 | 0 | 1 | 1 | 0 |
| 12 | 0 | 1 | 0 | 0 |
| 13 | 0 | 0 | 1 | 0 |
| 14 | 0 | 0 | 0 | 0 |
| 15 | 0 | 0 | 1 | 0 |
| 16 | 0 | 0 | 0 | 0 |

Table 12: Reduce (c)

| $A 4=$ Attributes |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | Decision.

Table 13: Reduce (d)
$1^{\text {st }}$ We reduce (a): the circuit that formed in Table (10): $P 1=\Phi$, where $P 1$ is the set of loads on, $P 2=\{D 1, D 2, D 3, D 4, D 5, D 6, D 7, D 8, D 9, D 10, D 11, D 12, D 13, D 14, D 15, D 16\}$,
where $P 2$ is the set of load off.Then $\operatorname{Core}(A 1)=P 1 / D=\Phi / 16=0 \neq \operatorname{Core}(A)$.Then $a \notin \operatorname{Core}(A)$.
$2^{\text {nd }}$ We reduce $b$ : the circuit that formed in Table (11):
$P 1=\{D 1, D 2, D 5, D 6\}$,
where $P 1$ is the set of loads on,
$P 2=\{D 3, D 4, D 7, D 8, D 9, D 10, D 11, D 12, D 13, D 14, D 15, D 16\}$,
If Core $(A 2)=P 1 / D=4 / 16=1 / 4=\operatorname{Core}(A)$.
Then $b \in \operatorname{Core}(A)$.
$3^{r d}$ We reduce c: the circuit that formed in Table (12):
$P 1=\Phi$,
where $P 1$ is the set of load on,
$P 2=\{D 1, D 2, D 3, D 4, D 5, D 6, D 7, D 8, D 9, D 10, D 11, D 12, D 13, D 14, D 15, D 16\}$,
where $P 2$ is the set of load off.Then Core $(A 3)=P 1 / D=\Phi / 16=0 \neq \operatorname{Core}(A)$. Then $c \notin \operatorname{Core}(A)$.
$4^{\text {th }}$ We reduce $d$ : the circuit that formed in Table (13):
$P 1=\{D 1, D 2, D 5, D 6\}$,
where $P 1$ is the set of loads on,
$P 2=\{D 3, D 4, D 7, D 8, D 9, D 10, D 11, D 12, D 13, D 14, D 15, D 16\}$,
If Core $(A 4)=P 1 / D=4 / 16=1 / 4$ Core $(A)$. Then $d \in \operatorname{Core}(A)$. Then Core $(A 4)=\{a, c\}$.
Similarly, if $P 2$ is taken as the set of (off load), then again Core $(A)=\{a, c\}$.
In the previous example, the conditions that could be eliminated without affecting the total load of the electrical circuit were concluded using New method of reduction, where resistors $b$ and $d$ represent conditions that can be deleted.

## 6. Conclusions and Discussions

Firstly, the laws of the conduction of the resistors have been improved in parallel using rough concepts. It has been shown that the deletion of resistors connected to the parallel does not affect the passage of the current to the load, which were represented by two conditions $b$ and $d$.

Secondly, proving the validity of Nominal- $\pi$ representation method as induction impedance does not affect the passage of the electric current, although it affects the quality of the transmitted power. And also, when all possible cases of an electrical circuit are used, no condition can be detected which can be eliminated by all methods except the method of nano topology. Because it takes into account the decision set. Also, the new method can detect the conditions that can be deleted. In the other case, when using a subset of the general case of Table (2) to Table (6), the conditions that could be deleted from all the mathematical methods used were discovered.

| Reduction methods | Power of set $p(s)$ | Reduction objects |
| :--- | :--- | :--- |
| $[0.5 \mathrm{ex}]$ Rough set | Not reduce | Reduce |
| Nano topology | Reduce | Reduce |
| Discernibility matrix and discernibility function | Not reduce | Reduce |
| Indiscernibility matrix and discernibility function | Not reduce | Reduce |
| Relations | Not reduce | Reduce |
| New method of reduction | Reduce | Reduce |

Table 14: Comparison between different types of reduction

By using the data in the previous Example (5.1), Tables 9, 10, 11, 12, 13, and 14, we can be calculating the Core of electrical circuit.

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| 1 - |  |  | 29- |  |  |
| 2 - | $\mathrm{a}=\left[\begin{array}{llllllllllll}1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\end{array}\right.$ | $000000000] ;$ | $30-$ | alpha1_a=Sko_a/U |  |
| 3 | $\mathrm{b}=\left[\begin{array}{llllllllllll}1 & 1 & 1 & 1 & 0 & 0 & 0 & 0\end{array}\right.$ | $111000000^{\prime} ;$ | $31-$ | alpha 2 _a $=\mathrm{Sk}_{1}$-a/U |  |
| 4 - | $\mathrm{c}=\left[\begin{array}{llllllllll}1 & 1 & 0 & 0 & 1 & 1 & 0 & 0\end{array}\right.$ | $1001100]^{\prime} ;$ | 32 | \%\% - |  |
| 5 | $\mathrm{d}-\left[\begin{array}{lllllllllll}1 & 0 & 1 & 0 & 1 & 0 & 1 & 0\end{array}\right.$ | O101010]'; | 33- | M be[acd] |  |
| 6 | $\mathrm{M}-\left[\mathrm{abcd} \mathrm{c}^{\text {d }}\right.$ |  |  | n_h=[1100110000000000]: |  |
| 7 | $\mathrm{D}=[11001100$ | $00000000]^{\circ}$ | 34- | K0_b=find(D_b=0) |  |
| 8 | \%\% |  | $\begin{aligned} & 35- \\ & 36 \end{aligned}$ | K1_b-find(D_b-1) |  |
| 9 | $\mathrm{KO}=$ find( $\mathrm{D}-0)$ |  | 37- E |  |  |
| 10 | $K 1=$ find ( $\mathrm{D}=1$ ) |  | $38-$ |  |  |
| 11. | $\mathrm{E}=$ zeros( $\mathrm{U}, 1$ ); |  | $39-$ |  |  |
| 12 | $\mathrm{E}(\mathrm{K} 0)-1$; |  | $40-$ | f $\quad \mathrm{b}=$ zeros $(\mathrm{U}, 1)$; |  |
| 13 | Sko $=$ sum(E) |  | 41 - | $\mathrm{f} \mathrm{b}(\mathrm{K} 1 \mathrm{~b})=1$; |  |
| 14 | $\mathrm{f}-\mathrm{zeros}(\mathrm{U}, \mathbf{1})$; |  | 42 - S | Sk1_b-sum(f_b) |  |
| 15 | $\mathrm{f}(\mathrm{K} 1)=1$; |  | 43 - | alpha1_besko_b/U |  |
| 16 | Sk1 $=$ sum $(\mathrm{f})$ |  | 44 - | alpha2_b=Sk1_b/U |  |
| 17 | alphat $=$ Sko/U |  | 45 9 | \%\% - - |  |
| 18 - | alpha2 ${ }_{\text {m }}$ Sk1/U |  | $46-$ | M_c $=\left[\begin{array}{lll}\text { a } & \text { d }\end{array}\right.$ |  |
| 19 | \%\% |  | 47- | D_c $=10000000000000000]$; |  |
| $20-$ |  |  | 48- | K0_c=find( $\mathrm{D}_{2} \mathrm{c}=0$ ) |  |
| 21 | D_a $=10000000$ | $0000000000{ }^{\prime}$ | 49 - | K 1 _c-find( $\left.\mathrm{D}^{-} \mathrm{c}-1\right)$ |  |
| 22 | Ko_a $=$ find( $\mathrm{D}_{\text {- }}$ a $=$ |  | $50-$ | E_c-zeros(U.1): |  |
| 23 | K1 a $=$ find( D a |  |  | E_c(K0_c) $=1$; |  |
| 24 | E_a-zeros(U,1); |  | 51- | Sko_c-sum(E_c) |  |
| 25 | E_a(KO_a)=1; |  | 53 | f_c-zeros(U,1); |  |
| 26 - | Sko_a $\operatorname{sum}^{\text {c }}$ E_a) |  | $54-$ | f_c(K1_c) $=1$; |  |
| $27-$ | f_a=zeros(U,1); |  | $55-$ | Skl_c $=\operatorname{sum}\left(\begin{array}{l}\text { c } \\ \text { c }\end{array}\right.$ |  |
| 28 - | f_a(K1_a)-1; |  | $56-$ | alphal_c-Sko ${ }^{\text {c/ } / \mathrm{U}}$ |  |



Table 15: Calculating the Core by Matlab program.


Table 16: Calculating the Core by Matlab program.

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    Communicated by Biljana Popović
    Email addresses: nasefa50@yahoo.com and arafa.nasef@eng.kfs.edu.eg (A. A. Nasef), mohnayle@yahoo.com (M. Shokry), samirmuk@yahoo.com (S. Mukhtar)

