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From Graph Theory to Nano Topology

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Abstract. The basic objective of this paper is to study and investigate some properties of nano-topological space induced by a graph theory .We discuss a new measure generated by nano topology. In addition, we apply the connections between a digraph theory and a nano topology in the urinary system as one medical application.

1. Introduction

Pawalak [14] introduced mathematical rough set theory in the early 1980's. The theory was based on the discernibility of objects. Rough set theory provides systems designers with the ability to handle uncertainty. If a concept is 'not definable' in a given knowledge base, rough sets can 'approximate' with respect to that knowledge. From a medical point of view, the attribute-value boundaries are usually vague.

Graph theory can describe a lot of cases such that network, electrical circuits and information systems as vertices and edges which representing the nature of the trend to be studied. One of the most important issues in the process of blood flow and dependence of the diseases of some body organs to others.

The theory of nano topology introduced by Lellis Thivagar and Richard [9] which was defined in terms of approximations and boundary region of a subset of a universe using an equivalence relation on it and also defined nano closed sets, nano interior and nano closure operators. The elements of a nano topological space are called nano open sets. It originates from the Greek word"Nanos" which means "Dwarf" in its modern scientific sense, an order to magnitude-one billionth. The topology is named as nano topology so because of its size since it has at most five elements.

In 2016, Lellis Thivagar et al. [7] defined the concept of nano topological space via graphs. In this paper, the main concept based upon converting a map to graph, it must take different colors to illustrate regions and countries for the vertices [5].So, the new methods of choosing the vertices to form nano topology may be used to getting properties on colors of some maps. In [11] Generating and discussions topological graph by three methods. On the other hand, it also performed comparisons and introduced independence and dependence of two graphs through a new scale. In [16] Applied both graph and topology on some of medical application such as the blood circulation in the human heart.

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The main contribution of the present work is that we provide a new method form to generate nano topological induced by the vertices of the graph and study some properties on it. Throughout the paper, we put forward the dependence scale by three comparisons. With the medical application the urinary system in the human body, the inequality between the medical view and the results of the topological structure on graphs investigated. In this paper, we introduce some topological concepts by using graph theory as follows: In Section 2, we present a new method to construct nano topology induced by the graph and improve the boundary region. In Section 3, we can put forward the new concept of independence of two graphs by three methods. Finally, we offer practical application applied to previous studies and we recognize its effectiveness on the essence of the healthy body (the urinary system).

2. Preliminaries

In this section, we link the applied application between rough set that convert information system to graph theory. Some important properties studied in information system redact non important elements from a set of object or condition, also obtained the core of set of condition. Any information system can formed to graph, so deletion of edges and vertices in the graph corresponding in reduction in information systems .Dependence of part of the graph corresponding to some condition on the other in the information system [5, 7, 8, 14].

2.1. Pawlak's Rough Sets and Nano Topology

Definition 2.1. [14] *Let U be a nonempty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes .Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair* (U, R) *is said to* be *the approximation space. Let* $\mathrm{X} \subseteq \mathrm{U}$

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certainly classified as X with respect to R and it is denoted by $L_R(X)$, that is $L_R(X) = \cup_{x \in U}\{R(x) : R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by $x \in X$.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $H_R(X)$, that is $H_R(X) = \cup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}$

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not- X with respect to R and it is denoted by $B_R(X)$, $B_R(X) = H_R(X) \backslash L_R(X)$

Proposition 2.2. *[9]If* (U, R) *is an approximation space and* $\mathrm{X}, \mathrm{Y} \subseteq \mathrm{U}$*, then*

(i) $L_R(X) \subseteq \mathrm{X} \subseteq H_R(X)$

(ii) $L_R(\phi) = H_R(\phi) = \phi$ *and* $L_R(U) = H_R(U) = U$

(iii) $H_R(X \cup Y) = H_R(X) \cup H_R(Y)$

(iv) $H_R(X \cap Y) = H_R(X) \cap H_R(Y)$

(v) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$

(vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$

(vii) $L_R(X) \subseteq L_R(Y)$ *and* $H_R(X) \subseteq H_R(Y)$ *whenever* $X \subseteq Y$

(viii) $H_R\,(X^c) = [L_R(\mathrm{X})]^c$ *and* $L_R\,(X^c) = [H_R(\mathrm{X})]^c$

(ix) $H_R\,(H_R(X)) = L_R\,(H_R(X)) = H_R(X)$

(x) $L_R\,(L_R(X)) = H_R\,(L_R(X)) = L_R(X)$

Study of intelligent systems characterized by in sufficient and incomplete information. Also, we construct a topological structure based information system, called nano topology.

Definition 2.3. [8] *Let U be nonempty finite universe of objects and R be an equivalence relation on U. Let* $X \subseteq U, \tau_R(X) = \left\{U, \phi, L_R(X), H_R(X), B_R(X)\right\}$. *Then* $\tau_R(X)$ *is topology on* U, *called as the nano topology with respect to X Elements of the nano topology are known as the nano- open sets in U and the ordered pair* $(U, \tau_R(X))$ *is called the nano topological space. But* $[\tau_R(X)]^c$ *is called as the dual nano topology of* $\tau_R(X)$ *and the elements of* $[\tau_R(X)]^c$ *are called as nano closed sets.*

Definition 2.4. [9] *If* $\tau_R(X)$ *is the nano topology on U with respect to* X, *then the set* $\beta = \{U, L_R(X), B_R(X)\}$ *is the basis for* $\tau_R(X)$.

Definition 2.5. [9] *If* $(U, \tau_R(X))$ *is a nano topological space with respect to X where* $X \subseteq U$ *and if* $A \subseteq U$, *then the nano interior of* A *is defined as the union of all nano- open subsets of* A *and it is denoted by* nInt(A). *That is,* nInt(A) *is the largest nano open subset of A .Also, The nano closure of A is defined as the intersection of all nano- closed sets containing A and is denoted by* nCl(A). *That is,* nCl(A) *is the smallest nano closed set containing A.*

Example 2.6. *Let* $U = \{a, b, c, d\}$ *with* $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ *and let* $X = \{a, c\} \subseteq U$. *Then one can deduce that,* $L_R(X) = \{a\}, H_R(X) = \{a, c, d\}, B_R(X) = \{c, d\}$ *Then the nano topology is defined as* $\tau_R(X) = \{U, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$

2.2. Bipartite Graphs

A graph is a pair $G = (V, E)$ consisting of a set V of vertices and a set E of edges such that $E \subseteq V \times V$ Two vertices are adjacent if there is an edge that has them as ends. An isolated vertex is a vertex that is not an end of any edge. An independent set is a set of pairwise nonadjacent vertices. A graph G is simple if every edge links a unique pair of distinct vertices. Much of graph theory is concerned with the study of simple graph [2].

In graph theory, there are two primary ways to represent a graph: adjacency matrix and adjacency list [1]. The adjacency matrix of G is the $|V| \times |V|$ matrix $M = (a_{uv})$ where $a_{uv} = 1$ if $(u, v) \in E$ and $a_{uv} = 0$ otherwise. The adjacency list is a linked list to vertices that are adjacent to it. The choice between the two representations will be due to the information of graphs that will be input to the algorithm. For example, if a graph has many vertices, an adjacency list would be best since it uses less spaces .While an adjacency matrix would be best when a graph has many edges since there would be few empty entries.

A graph $G = (V, E)$ is called bipartite if the vertex set V can be partitioned into two sets, X and Y, such that each edge has one end in X and one end in Y . In other words, a graph G is bipartite if and only if the vertex set V of G can be partitioned into two independent sets.

Bipartite graph play prominent roles in applications of graph theory [4,15] .For example, bipartite graphs are useful for modeling matching problems, such as job matching problem. Bipartite graph also play important roles in theoretical considerations. For instance, multigraphs can be described by bipartite graphs. Therefore, it is important to characterize bipartite graphs and test bipartiteness in graph theory.

Lemma 2.7. [15] *A graph is bipartite if and only if it contains no odd length cycle.*

Theoretically, lemma2.7 provides a simple tool to check if a graph is bipartite. The above method may need to find all the cycles of a graph. However, it is not an easy task [12, 18]. Traditionally, the Breadth First Search algorithm [12] is used to check whether or not a graph is bipartite [1].

A directed graph is defined as an ordered triple $G = (V, E, f)$, where f is a function that maps each element in E to an ordered pair of vertices in V. The ordered pairs of vertices are called directed edges, arcs or arrows. An edge $E = (i, j)$ is considered to have direction from i to j. Directed graphs are mostly suitable for the representation of schemas describing biological pathways or procedures which show the sequential interaction of elements at one or multiple time points and the flow of information throughout the network. These are mainly metabolic, signal transduction or regulatory networks [6].

3. Nano Topology on Graph Vertices and Scales

3.1. New View of Lower and Upper Approximation of Subgraphs

Improving information derived from uncertainty information, it is desirable for increasing the region confirmed. Through this part, we provide a new method to construct a nano topological structure based on the concept of view interior, closure and boundary of the graph.

Definition 3.1. [7] *Let $G = (V, E)$ be a graph and $v \in V(G)$, then the neighbourhood of v defined as follows:* $N(v) = \{v\} \cup \{u \in V(G) : \overrightarrow{uv} \in E(G)\}$

Definition 3.2. *Let $G = (V, E)$ be a graph ,S be a subgraph of G and $N(V(G))$ be a neighbourhood of v in V. Then the lower approximation operation defined as follows:*

$$L_N(V(S)) = \left\{\{v_i\} \cup \left\{v_j\right\} : e_{ij} \in E(S); v_i, v_j \in E(S)\right\}$$

.The upper approximation operation defined as follows

$$H_N(V(S)) = \left\{\left\{v_i, v_j\right\} : e_{ij} \in E(S), v_i, v_j \in E(S)\right\} \cup \{v_k : v_k \in v(G - S) \text{ and } e_{ik} \in E(G)\}.$$

The boundary region defined as follows: $B_N(V(S)) = H_N(V(S)) \backslash L_N(V(S))$

Remark 3.3. *Let $G = (V, E)$ be a graph, H is a subgraph of G. Then:*

(i) $L_N(V(H)) \subseteq V(H) \subseteq U_N(V(H))$

(ii) $L_N(V(G)) = V(G) = U_N(V(G)$

(iii) *If* K *is the empty graph, graph with no vertices, then* $L_N(K) = \phi = U_N(K)$.

Proposition 3.4. *Let $G = (V, E)$ be a graph, H and K are two subgraphs of a graph G. Then:*

(i) *If* $V(H) \subseteq V(K)$ *then ,* $L_N(V(H)) \subseteq L_N(V(K))$ *and* $U_N(V(H)) \subseteq U_N(V(K))$

(ii) $L_N(V(H)) \cup L_N(V(K)) \subseteq L_N(V(H) \cup V(K))$

(iii) $L_N(V(H) \cap V(K)) = L_N(V(H)) \cap L_N(V(K))$

(iv) $U_N(V(H) \cup V(K)) = U_N(V(H)) \cup U_N(V(K))$

(v) $U_N(V(H) \cap V(K)) \subseteq U_N(V(H)) \cap U_N(V(K))$

Proof. From Proposition 2.2 that is obvious. □

Proposition 3.5. *Let $G = (V, E)$ be a graph, H a subgraph of G. Then:*

$$L_N(L_N(V(H))) \subseteq L_N(V(H)) \subseteq U_N(L_N(V(H))) \subseteq V(H) \subseteq L_N(U_N(V(H))) \subseteq U_N(V(H)) \subseteq U_N(U_N(V(H))).$$

Proof. We will prove that $U_N(L_N(V(H))) \subseteq V(H) \subseteq L_N(U_N(V(H)))$. Let $v_1 \in U_N(L_N(V(H)))$. Implies, there exist $v_2 \in L_N(V(H))$ such that $v_1v_2 \in E(G)$ Therefore, $v_1 \in N(v) \subseteq V(H)$. Now, since$v_1 \in V(H)$ then $N(v_1) \subseteq U_N(V(H))$. Implies, $v_1 \in L_N(U_N(V(H)))$ □

Proposition 3.6. *Let $G = (V, E)$ be a graph, H a subgraph of G. Then*

(i) $L_N(U_N(L_N(V(H)))) = L_N(V(H))$

(ii) $U_N(L_N(U_N(V(H)))) = U_N(V(H))$

Proof. **Necessity**, let $v_1 \in L_N(\mathrm{V(H)})$, implies $\underline{\mathrm{N}}(v_1) \subseteq \mathrm{U}_N(L_N(\mathrm{V(H)}))$. So $v_1 \in L_N(\mathrm{U}_N(L_N(\mathrm{V(H)})))$.
Sufficiency, from Proposition 3.6 we get $\mathrm{U}_N(L_N(\mathrm{V(H)})) \subseteq \mathrm{V(H)}$. Implies, $L_N(\mathrm{U}_N(L_N(\mathrm{V(H)}))) \subseteq \mathrm{U}_N(\mathrm{V(H)})$
Therefore, $L_N(\mathrm{U}_N(L_N(\mathrm{V(H)}))) = L_N(\mathrm{V(H)})$
Necessity, form Proposition 3.6 we get $\mathrm{U}_N(L_N(\mathrm{V(H)})) \subseteq \mathrm{V(H)}$. Implies, $\mathrm{U}_N(L_N(\mathrm{U}_N(\mathrm{V}))) \subseteq \mathrm{U}_N(\mathrm{V(H)})$
Sufficiency, form Proposition 3.6 we get $\mathrm{V(H)} \subseteq L_N(\mathrm{U}_N(\mathrm{V(H)}))$. Implies, $\mathrm{U}_N(\mathrm{V(H)}) \subseteq \mathrm{U}_N(L_N(\mathrm{U}_N(\mathrm{V(H)})))$
Therefore, $\mathrm{U}_N(L_N(\mathrm{U}_N(\mathrm{V(H)}))) = \mathrm{U}_N(\mathrm{V(H)})$ □

Definition 3.7. *Let G be a graph, N(v) be a neighborhood of v in V and S be a subgraph of G, then*

$$\tau_N(V(S)) = \left\{V(G), \phi, L_N(V(S)), H_N(V(S)), B_N(V(S))\right\}$$

forms a topology called the nano topology on $V(G)$ with respect to $V(S)$ .We call $(\mathrm{V(G)}, \tau_N(V(S)))$ *as the nano topological space induced by a graph.*

Proposition 3.8. *Let $G = (V, E)$ be a graph and H and K are two subgraphs of a graph G, then the following properties holds:*

(i) $\mathrm{Int}_N(\mathrm{G}) \subseteq \mathrm{Cl_N(G)}$

(ii) *If* $\mathrm{H} \subseteq \mathrm{G}$ *then* $\ln \mathrm{t}_N(\mathrm{H}) \subseteq \mathrm{In\, t}_N(\mathrm{G}) \subseteq \mathrm{V(G)}$

(iii) $\mathrm{Int}_N(\mathrm{H} \cap \mathrm{K}) = \mathrm{Int}_N(\mathrm{H}) \cap \mathrm{In\, t}_N(\mathrm{K})$

(iv) $\mathrm{Int}_N(\mathrm{H} \cup \mathrm{K}) \supset \mathrm{Int}_N(\mathrm{H}) \cup \mathrm{In\, t}_N(\mathrm{K})$

(v) *If* $\mathrm{H} \subseteq G$ *then* $\mathrm{Cl_N(H)} \subset \mathrm{V(G)} \subset \mathrm{Cl_N(G)}$

(vi) $\mathrm{Cl_N(H \cap K)} \subset \mathrm{Cl_N(H)} \cap \mathrm{Cl_N(K)}$

(vii) $\mathrm{Bd_N(H)} \supseteq \mathrm{Bd_N(G)}$

Proof. Obviously. □

As an illustration, consider the following example.

Example 3.9. *Consider the graph $G = (V, E)$ as shown in Figure1. where $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$*

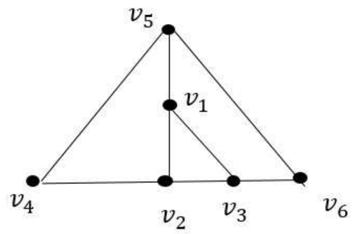

Figure 1:

(1) Let S be a subgraph of G such that $V(S) = \{v_1, v_2, v_3, v_4\}$ then one can deduce that

$L_N(V(S)) = \{v_1, v_2, v_3, v_4\}$,
$U_N(V(S)) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$
$B_N(V(S)) = \{v_5, v_6\}$,
$\tau_N(V(S)) = \left\{V(G), \phi, \{v_1, v_2, v_3, v_4\}, \{v_5, v_6\}\right\}$

(2) Let K *be a subgraph of G such that* $V(K) = \{v_1, v_2, v_3, v_6\}$ *then we can check that*

$$L_N(V(K)) = \{v_1, v_2, v_3, v_6\},$$
$$U_N(V(K)) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$$
$$B_N(V(K)) = \{v_4, v_5\},$$
$$\tau_N(V(K)) = \Big\{V(G), \phi, \{v_1, v_2, v_3, v_6\}, \{v_4, v_5\}\Big\}$$

3.2. Dependence scales of Subgraphs

The usefulness of any part of system realize how indispensable in it. We can identify this from scale called dependence coefficient, comparisons between two maps or two electrical systems or the way to connect water networks with each other.

Definition 3.10. *If* $G_1 = (V_1, E_1), G_2 = (V_2, E_2)$ *), are simple graphs, then dependence scales of nano topology induced by a graph, are denoted by* $\boldsymbol{D_{\tau_N}}$*, is defined as:*

$$D^{(1)}\tau_N(G_1, G_2) = \begin{cases} \frac{|\mathrm{Int}_{\tau_N}(G_1)\cap \mathrm{In}\, t_{\tau_N}(G_2)|}{|\mathrm{Cl}_{\tau_N}(G_1)\cap \mathrm{Cl}_{\tau_N}(G_2)|}, & \textit{if}\ \mathrm{Cl}_{\tau_N}(G_1)\cap \mathrm{Cl}_{\tau_N}(G_2) \neq 0 \\ 0, & \textit{otherwise.} \end{cases}$$

$$D^{(2)}_{\tau_N}(G_1, G_2) = \begin{cases} \frac{|\mathrm{In}\ \mathrm{t}_{\tau_N}(G_1\cap G_2)|}{|\mathrm{Cl}_{\tau_N}(G_1\cap G_2)|}, & \textit{if}\ \mathrm{Cl}_{\tau_N}(G_1\cap G_2) \neq 0 \\ 0, & \textit{otherwise.} \end{cases}$$

$$D^{(3)}\tau_N(G_1, G_2) = \begin{cases} \frac{|Cl_{\tau_N}\ \mathrm{Int}_{\tau_N}(G_1)\cap Cl_{\tau_N}\ \mathrm{In}\ \mathrm{t}_N(G_2)|}{|\mathrm{ln}\ \mathrm{t}_N \mathrm{Cl}_N(G_1)\cap \mathrm{Int}_{\tau_N}\ \mathrm{Cl}_N(G_2)|}, & \textit{if}\ \mathrm{Int}_{\tau_N}\ \mathrm{Cl}_N(G_1)\cap \mathrm{Int}_{\tau_N}\ \mathrm{Cl}_{\tau_N}(G_2) \neq 0 \\ 0, & \textit{otherwise.} \end{cases}$$

Example 3.11. *Consider the graph* $G = (V, E)$ *as shown in Figure 1. Let* $\mathrm{G} = \{v_1, v_2, v_3, v_4, v_5, v_6\}$, $\mathrm{ln}\ \mathrm{t_N(H)} = \{\mathrm{v_1, v_2, v_3, v_4}\}$, $Cl_{\tau_N}(H) = \{v_1, v_2, v_3, v_4\}$ $\mathrm{Int}_{\mathcal{T}_N}(K) = \{v_1, v_2, v_3, v_6\}$, $Cl_{\tau_N}(K) = \{v_1, v_2, v_3, v_6\}$*, then one can deduce that* $D^{(1)}\tau_N(\mathrm{H, K}) = \frac{3}{6} = 0.5, D^{(2)}\tau_N(\mathrm{H, K}) = \frac{3}{6} = 0.5, D^{(3)}\tau_\mathrm{N}(\mathrm{H, K}) = \frac{6}{6} = 1$

From the previous example, we measure dependence scales by three ways.
Studying this scale on parts of networks, circuits, management system by two different topological structures (nano topology, relative topology). Relative topologies, we suppose throughout this subsection that (X, τ) is a topological space and $\mathrm{A} \subset \mathrm{X}$. We let $\tau_A(\tau) = \{\mathrm{A} \cap \mathrm{U} : \mathrm{U} \in \tau\}$ One easily verifies that τ_A is a topology for A which we call the relative topology for A. We say a subset β of A is open relative to A if $\beta \in \tau_A$. We say a subset β of A is closed relative to A if $\mathrm{A} \sim \mathrm{B}$ is relatively open [7]. A connected graph that not contains any cycle called tree graph [9].

To present an important of dependence scales, we study and explain it on some types of graph as follows:

(i) Dependence scales of topological structure of tree.

(ii) Dependence scales of topological structure on path.

(iii) Dependence scales of topological structure on bipartite graph.

Case I: Dependence scales of topological structure of tree.
Trees belong to the simplest class of graphs. Despite their simplicity, they have a rich structure.It provide a range of useful applications as simple as a family tree to as complex as trees in data structures of computer science.

Proposition 3.12. *Let G be tree,* T_n, T_m *are subtree of tree, then the following properties hold:*

(i) $\boldsymbol{D}^{(1)}\tau_A(T_n, T_m) \leq \boldsymbol{D}^{(1)}\boldsymbol{\tau}_N(T_n, T_m)$

(ii) $\boldsymbol{D}^{(2)}\tau_A(T_n, T_m) \geq \boldsymbol{D}^{(2)}\tau_N(T_n, T_m)$

(iii) $\boldsymbol{D}^{(3)}_{\tau_A}(T_n, T_m) \leq \boldsymbol{D}^{(3)}_{\tau_N}(T_n, T_m)$

Proof. From Definition 3.10 □

Proposition 3.13. *Let G be tree, T_n, T_m are subtree of tree then the following properties hold:*

(i) $D^{(1)}_{\tau_A}(T_n, T_m) \leq D^{(2)}\tau_A(T_n, T_m) \leq D^{(3)}_{\tau_A}(T_n, T_m)$

(ii) $D^{(1)}\tau_N(T_n, T_m) \leq D^{(2)}_{\tau_N}(T_n, T_m) \leq D^{(3)}_{\tau_N}(T_n, T_m)$

Proof. Obviously. □

Example 3.14. *Consider the graph* T(V.E) *as shown in Figure* 2. *Let* $V(T) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$

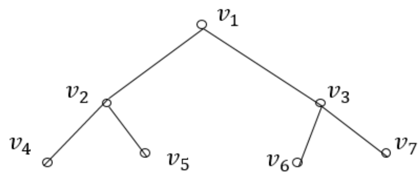

Figure 2:

$N(v_1) = \{v_1, v_2, v_3\}, N(v_2) = \{v_1, v_2, v_4, v_5\}, N(v_3) = \{v_1, v_2, v_6, v_7\}, N(v_4) = \{v_2, v_4\}, N(v_5) = \{v_2, v_5\}, N(v_6) = \{v_3, v_6\}, N(v_7) = \{v_3, v_7\}$.
The induced topology by $N(T)$ is

$$S\beta_N(T) = \{\{v_1, v_2, v_3\}, \{v_1, v_2, v_4, v_5\}, \{v_1, v_2, v_6, v_7\}, \{v_2, v_4\}, \{v_2, v_5\}, \{v_3, v_6\}, \{v_3, v_7\}\}$$

$$\beta_N(G) = \Big\{\varphi, \{v_2\}, \{v_3\}, \{v_1, v_3\}, \{v_1, v_2, v_3\}, \{v_1, v_2, v_5, v_5\}, \{v_6, v_7\}, \{v_2, v_4\}, \{v_2, v_5\}, \{v_3, v_6\}, \{v_3, v_7\}\Big\}$$

$$\tau_N(T) = \Big\{V(T), \phi, \{v_2\}, \{v_3\}, \{v_1, v_3\}, \{v_2, v_4\}, \{v_3, v_5\}, \{v_3, v_6\}, \{v_3, v_7\}, \{v_1, v_2, v_3\}, \{v_1, v_2, v_4, v_5\}, \{v_1, v_2, v_6, v_7\}, \{v_2, v_3\}, \{v_2, v_3, v_6\}, \{v_2, v_3, v_7\}, \{v_2, v_3, v_4\}, \{v_2, v_3, v_5\}, \{v_1, v_2, v_3, v_4, v_5\}, \{v_1, v_2, v_3, v_6, v_7\}, \{v_1, v_2, v_3, v_4\}, \{v_1, v_2, v_3, v_5\}, \{v_1, v_3, v_6\}, \{v_1, v_3, v_7\}, \{v_1, v_2, v_3, v_6, v_7\}, \{v_1, v_2, v_3, v_6\}, \{v_1, v_2, v_3, v_7\}, \{v_2, v_4, v_5\}, \{v_2, v_3, v_4, v_6\}, , \{v_2, v_3, v_4, v_7\}, \{v_1, v_2, v_4, v_6, v_7\}, \{v_2, v_3, v_4, v_5\}, \{v_1, v_2, v_3, v_4, v_6, v_7\}, \{v_1, v_2, v_3, v_4, v_6\}, \{v_1, v_2, v_3, v_4, v_7\}, \{v_2, v_3, v_5, v_6\}, \{v_2, v_3, v_5, v_7\}, \{v_1, v_2, v_5, v_6, v_7\}, \{v_2, v_3, v_5, v_6\}, \{v_1, v_2, v_3, v_5, v_6, v_7\}, \{v_1, v_2, v_3, v_5, v_6\}, \{v_1, v_2, v_3, v_5, v_7\}, \{v_3, v_6, v_7\}, \{v_2, v_3, v_6, v_7\}, \{v_1, v_2, v_3, v_4, v_5, v_6\}, \{v_1, v_3, v_6, v_7\}, \{v_2, v_3, v_4, v_5, v_6\}, \{v_2, v_3, v_4, v_6, v_7\}, \{v_1, v_2, v_3, v_4, v_6, v_7\}, \{v_2, v_3, v_5, v_6, v_7\}, \{v_1, v_2, v_3, v_4, v_5, v_7\}, \{v_2, v_3, v_4, v_5, v_7\}\Big\}$$

Let $V(T_1) = A = \{v_2, v_4, v_5\}$

$$\tau_A(T_1) = \Big\{V(T), \phi, \{v_2\}, \{v_2, v_4\}, \{v_2, v_4, v_5\}\Big\}$$

$$\tau^c_A(T_1) = \Big\{V(T), \phi, \{v_1, v_3, v_6, v_7\}, \{v_1, v_3, v_5, v_6, v_7\}, \{v_1, v_3, v_4, v_5, v_6, v_7\}\Big\}$$

Let $V(T_2) = B = \{v_2, v_4\}$

$$\tau_A(T_2) = \Big\{V(G), \phi, \{v_2\}, \{v_2, v_4\}\Big\}$$

$$\tau^c_A(T_2) = \Big\{V(G), \phi, \{v_1, v_3, v_5, v_6, v_7\}, \{v_1, v_3, v_4, v_5, v_6, v_7\}\Big\}$$

$$\mathrm{Int}_{\tau_A}(T_1) = \{v_2, v_4, v_5\}, Cl_{\tau_A}(T_1) = \mathrm{V(T)}, Cl_{\tau_A}\mathrm{Int}_{\tau_A}(T_1) = V(T), \mathrm{In}\,t_{\tau_A}Cl_{\tau_A}(T_1) = \mathrm{V(T)}$$

$$\mathrm{Int}_{\tau_A}(T_2) = \mathrm{V(G)}, Cl_{\tau_A}(T_2) = \mathrm{V(T)}, Cl_{\tau_A}\mathrm{Int}_{\tau_A}(T_2) = \mathrm{V(T)}, \mathrm{In}\,t_{\mathrm{t}_A}Cl_{\tau_A}(T_2) = \mathrm{V(T)}$$

Then we can deduce that dependence scales relative topological tree

$$D^{(1)}\tau_A(T_1, T_2) = \frac{2}{7}, D^{(2)}\tau_A(T_1, T_2) = \frac{2}{7}, D^{(3)}\tau_A(T_1, T_2) = \frac{7}{7} = 1$$

Determined dependance scales using nano topological graph
Let T_1 be a subgraph of G such that $V(T_1) = \{v_2, v_4, v_5\}$ then

$$\begin{aligned}
&L_N(V(T_1)) = \{v_2, v_4, v_5\},\\
&U_N(V(T_1)) = \{v_1, v_2, v_3, v_4, v_5\},\\
&B_N(V(T_1)) = \{v_1, v_3\}\\
&\tau_N(V(T_1)) = \left\{V(T), \phi, \{v_2, v_4, v_5\}, \{v_1, v_3\}, \{v_1, v_2, v_3, v_4, v_5\}\right\}\\
&\tau_N^c(V(T_1)) = \left\{V(T), \phi, \{v_2, v_4, v_5, v_6, v_7\}, \{v_1, v_3, v_6, v_7\}, \{v_6, v_7\}\right\}
\end{aligned}$$

Let G_2 be a subgraph of G *such that $V(T_2) = \{v_2, v_4\}$ then*

$$\begin{aligned}
&L_N(V(T_2)) = \{v_2, v_4\},\\
&U_N(V(T_2)) = \{v_1, v_2, v_3, v_4, v_5\},\\
&B_N(V(T_2)) = \{v_1, v_3, v_5\}\\
&\tau_N(V(T_2)) = \left\{V(T), \phi, \{v_1, v_3, v_5\}, \{v_2, v_4\}, \{v_1, v_2, v_3, v_4, v_5\}\right\}\\
&\tau_N^c(V(T_2)) = \left\{V(T), \phi, \{v_6, v_7\}, \{v_2, v_4, v_6, v_7\}, \{v_1, v_3, v_5, v_6, v_7\}\right\}
\end{aligned}$$

$\mathrm{Int}_{\tau_N}(T_1) = \{v_2, v_4, v_5\}, Cl_{\tau_N}(T_1) = \{v_2, v_4, v_5, v_6, v_7\}, Cl_{\tau_N}\mathrm{Int}_{\tau_N}(T_1) = \{v_2, v_4, v_5, v_6, v_7\},$
$\mathrm{In}\,t_{\tau_N}Cl_{\tau_N}(T_1) = \mathrm{V(T)}$
$\mathrm{In}\,t_{\tau_N}(T_2) = \{v_2, v_4\}, \quad, Cl_{\tau_N}(T_2) = \{v_2, v_4, v_6, v_7\}, Cl_{\tau_N}\mathrm{In}\,t_{\tau_N}(T_2) = \{v_2, v_4, v_6, v_7\}, \mathrm{In}\,t_{_N}Cl_{\tau_N}(T_2) = \mathrm{V(T)}.$
Then we can deduce that

$$D^{(1)}\tau_N(T_1, T_2) = \frac{2}{4}, D^{(2)}\tau_N(T_1, T_2) = \frac{2}{4}, D^{(3)}_{\tau_N}(T_1, T_2) = \frac{4}{7}$$

Case II: Dependence scales of topological structure on path graph.
A path graph or linear graph is a graph whose vertices can be listed in the order $v_1, v_2, \ldots, v_n$ such that the edges are $\{v_i, v_{i+1}\}$ where $i = 1, 2, \ldots, n-1$. Paths are often important in their role as subgraphs of other graphs, in which case they are called paths in that graph. A path is a particularly simple example of a tree, and in fact the paths are exactly the trees in which no vertex has degree 3 or more. A disjoint union of paths is called a linear forest.

Proposition 3.15. *Let* $\mathrm{P} = (\mathrm{V}, \mathrm{E})$ *be path graph with* $\mathrm{n} \geq 1$ *vertices and P_n, P_m subgraph of* P*, then the following properties hold:*

(i) $\boldsymbol{D}^{(1)}_{\tau_A}(P_n, P_m) \leq \boldsymbol{D}^{(1)}_{\tau_N}(P_n, P_m)$

(ii) $\boldsymbol{D}^{(2)}_{\tau_A}(P_n, P_m) \geq \boldsymbol{D}^{(2)}_{\tau_N}(P_n, P_m)$

(iii) $\boldsymbol{D}^{(3)}_{\tau_A}(P_n, P_m) \leq \boldsymbol{D}^{(3)}_{\tau_N}(P_n, P_m)$

Proof. Follows from the fact that $\mathrm{In}\,t_{\tau_A} \geq \mathrm{Int}_{\tau_N}$ □

Proposition 3.16. *Let* $\mathrm{P} = (\mathrm{V}, \mathrm{E})$ *be path graph with* $\mathrm{n} \geq 1$ *vertices and* P_n, P_m *subgraph of* P*, then the following properties hold:*

(i) $D^{(1)}_{\tau_A}(P_n, P_m) \leq D^{(2)}_{\tau_A}(P_n, P_m) \leq D^{(3)}_{\tau_A}(P_n, P_m)$

(ii) $D^{(1)}\tau_N(P_n, P_m) \leq D^{(2)}_{\tau_N}(P_n, P_m) \leq D^{(3)}\tau_N(P_n, P_m)$

Proof. Follows from Definition 3.10 □

Proposition 3.17. *Let* $P = (V, E)$ *be a path graph with length=* n *, then the following properties hold:*

(i) $|Cl_{\tau_N}(\textit{boundary of vertices})| \leq \textit{length}(\mathrm{P})$

(ii) If $\textit{length}(P) = 4, |Cl_{\tau_N}(\textit{boundary of vertices})| = \textit{length}(\mathrm{P})$

Proof. Obviously. □

Case III: Dependence scale of topological structure on bipartite graph.

Proposition 3.18. *Let* $BP = (V, E)$ *be bipartite graph with* $V = (A, B)$ *and* H *subgraph of* BP*, then the following properties hold:*

(i) If $\mathrm{H} = \{v_1\} \subseteq A$ *then* $L_N(H) = \phi$ *and conversly then* $D^{(1,2,3)}\tau_N(H, BP) = \mathbf{0}$

(ii) The elements of closure of $\{v_1\} \subseteq A$ *are belong to set* B

Proof. Obviously. □

Proposition 3.19. *Let* $BP = (V, E)$ *be bipartite graph with* $V = (A, B)$ *and* H, K *subgraphs of* BP*,* $|\,|$ *is cardinality measure. Then the following properties hold:*

(i) If $\mathrm{H} = \{v_1\} \subseteq A$ *then* $L_N(H) \leq |Cl_N(H)| \leq |B|$ *and vice versa*

(ii) If $\mathrm{H} = \{v_1\}, K = \{v_2\} \subseteq A$ *then* $|Cl_N(H) \cup Cl_N(K)| \leq |B|$ *and vice versa.*

(iii) From $|E| \leq C_2^{|V|}$*. If* $\mathrm{H} = \{v_1\} \subseteq A$ *then* $|E(Cl_N(v_1))| \leq C_2^{|B|}$ *and vice versa.*

Proof. Directly from properties of bipartite graph. □

4. Medical Application via Graph Theory

Understanding complex systems often require a bottom-up analysis towards a systems biology approach. The need to investigate a system, not only as individual components but also as a whole, emerges. This can be done by examining the elementary constituents individually and then how these are connected. The myriad components of a system and their interactions are best characterized as networks and they are mainly represented as graphs where thousands of nodes are connected with thousands of vertices. In this section, we demonstrate approaches, models and methods from the graph theory universe and we discuss ways in which they can be used to reveal hidden properties and features of a network. This network profiling combined with knowledge extraction will help us to better understand the biological significance of the system.

4.1. The Urinary System in Human Body[20]

The urinary system is a group of organs in the body concerned with filtering out excess fluid and other substances from the bloodstream. The substances are filtered out from the body in the form of urine.

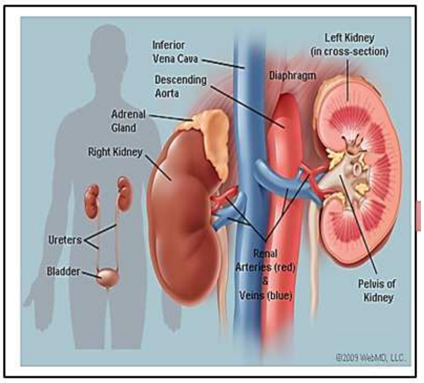

Figure 3: Urinary System in Human Body

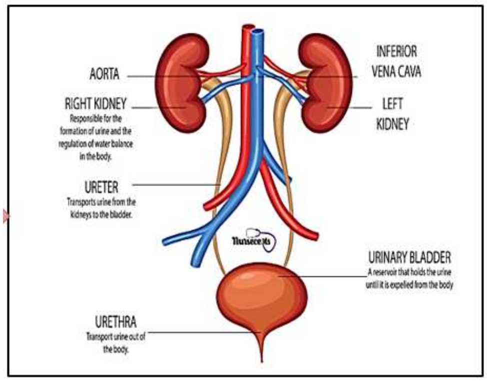

Figure 4: Structure of Urinary System

The major structures of the urinary system and functions are:

- Two kidneys extract wastes from the blood, balance body fluids, and form urine.
- Two ureters conduct urine from the kidneys to the urinary bladder.
- The urinary bladder serves as a reservoir for urine.
- The urethra conducts urine from the bladder to the outside of the body for elimination.

Medical tests play a prominent role in the life of rights to make sure that the retreat of diseases, perhaps the most prominent of those analyzes macroeconomic analysis functions. It plays a key role in the evaluation of kidney functions and identify the diseases that cause an imbalance in the performance of those functions or to predict the emergence of a satisfactory situation .

Creatinine is the producer of the waste are produced continuously during the collapse of normal muscle. The total liquidation of creatinine from the blood in the urine, and does not absorb any part thereof. Checking the proportion of creatinine in the blood, it is usually the total liquidation of creatinine from the blood. If the high level of creatinine in the blood, it indicates a problem in the function of kidneys. Is the measurement of creatinine is optimal to test kidney functions.
• Creatinine and Urea Blood levels of creatinine and urea reflects the function of the kidneys. Creatinine and urea are two by- products that are normally removed from the blood by the kidney. When the kidney function slows down, the blood levels of creatinine and urea increase. Normal value of serum creatinine is 0.9 to 1.4 mg/dl and normal value of blood urea nitrogen (BUN) is 20 to 40 mg/dl. Higher values suggest damage to the kidneys. Creatinine level is a more reliable guide of kidney function as compared to BUN.
• ABG- Arterial Blood Gases Test (High ph in the blood). Any degree between 7.35-7.4. This condition is safety region.
• Glumerular filtration rate GFR Test is an indication of the kidney's condition and its role in renal physiology.
(GFR) describes the flow rate of filtered fluid through the kidney 90-120mg/m.

4.2. Representing a Graph on the Urinary System in Human Body

Through the medical application, we can a new clarifying graph model. Generating two topological graph structure on the application with discussing a new scale explaining the importance of each organ in

the system.
Considering every organ as vertices (Renal Artery as v_1, Renal Vein as v_2,Right Kidney as v_3, Left Kidney as v_4, Bladder as v_5). Considering the channels of liquid between the organs as edges.

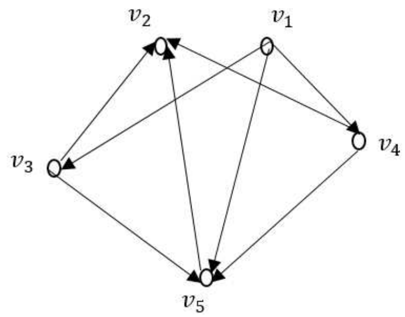

Figure 5: Conversion of the urinary system to mathematical model

The vast majority of algorithms of interest operate on data. Therefore, there are particular ways of organizing data that play a critical role in the design and analysis of algorithms. From that, we can say that data structures are simply ways of organizing data[18]. Based on the mathematical model of urinary system and the corresponding nano topological graph, we will detect and predict the diseases of the urinary by the new algorithm. Similarity, detect the diseases using new algorithm induced by the relative topological graph.

Algorithm of nano topology to detect diseases of the urinary system:
Step 1: By taking Definition 3.7 generates nano topological graph.
Step 2: From results of tests measuring serum, construct subgraphs from graph model.
Step 3: To follow diseases of the urinary system, we divide the urinary system to subsets and study the corresponding relative topology.
Step 5: Check dependence scales by definition 3.10 is called safety numbers.
Step 6: From result, we can conclude verifies the medical results.

Initiate of the implementation of the statements
Step 1: $N(v_1) = \{v_1, v_3, v_4, v_5\}, N(v_2) = \{v_2\}, N(v_3) = \{v_2, v_3, v_5\}, N(v_4) = \{v_2, v_4, v_5\}\, N(v_5) = \{v_2, v_5\}$. The induced topology by N(G)

$$S\beta_N(G) = \{\{v_2\}, \{v_2, v_5\}, \{v_2, v_3, v_5\}, \{v_2, v_4, v_5\}, \{v_1, v_3, v_4, v_5\}\}$$
$$\beta_N(G) = \left\{\phi, \{v_2\}, \{v_5\}, \{v_2, v_5\}, \{v_2, v_4, v_5\}, \{v_2, v_3, v_5\}, \{v_1, v_3, v_4, v_5\}, \{v_4, v_5\}, \{v_3, v_5\}\right\}$$
$$\tau_N(G) = \Big\{V(G), \phi, \{v_5\}, \{v_5\}, \{v_2, v_5\}, \{v_4, v_5\}, \{v_2, v_5\}, \{v_2, v_4, v_5\}, \{v_3, v_4, v_5\}, \{v_2, v_3, v_5\}, \{v_1, v_3, v_4, v_5\}, \{v_2, v_3, v_4, v_5\}\Big\}$$

Step 2: Study some tests on the urinary system, abnormal rate of the blood levels of creatinine and urea.
Step 3: Because of observed only with marked damage to functioning nephrons of right or left kidney. It is a gift from God to us that we can live with only one kidney .

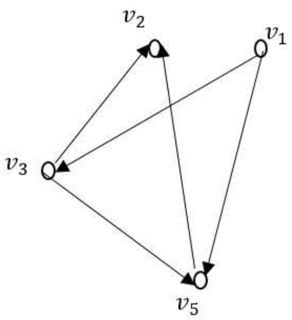

Figure 6: The left kidney failure

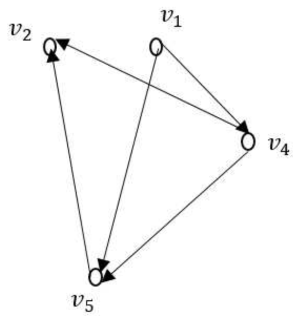

Figure 7: The right kidney failure

Case I) If the right kidney failure from Figure 6.Then

$$V(G_1) = A = \{v_1, v_2, v_3, v_5\}$$
$$\tau_A(G_1) = \left\{V(G), \phi, \{v_2\}, \{v_5\}, \{v_2, v_5\}, \{v_3, v_5\}, \{v_2, v_3, v_5\}, \{v_1, v_3, v_5\}\right\}$$

Case II) If the left kidney failure from Figure 7. Then

$$V(G_2) = B = \{v_1, v_2, v_4, v_5\}$$
$$\tau_A(G_2) = \left\{V(G), \phi, \{v_2\}, \{v_5\}, \{v_2, v_5\}, \{v_2, v_5, v_5\}, \{v_1, v_4, v_5\}\right\}$$

$$\mathrm{Int}_{\tau_N}(G_1) = \{v_1, v_2, v_3, v_5\}, Cl_{\tau_N}(G_1) = V(G), \mathrm{Cl}_{\mathrm{rN}}\,\mathrm{Int}_{\tau_N}(G_1) = \mathrm{V(G)}, \mathrm{Int}_{\mathrm{TN}}\,\mathrm{Cl}_{\mathrm{N}}(G_1) = \mathrm{V(G)}$$

$$\ln \mathrm{t}_{\tau_N}(\mathrm{G}_2) = \{\mathrm{v}_1, \mathrm{v}_2, \mathrm{v}_4, \mathrm{v}_5\}, \mathrm{Cl}_{\mathrm{T_N}}(\mathrm{G}_2) = \mathrm{V(G)}, \mathrm{Cl}_{\mathrm{T_N}}\,\mathrm{lnt}_{\mathrm{T_N}}(\mathrm{G}_2) = \mathrm{V(G)}, \ln \mathrm{t}_{\tau_N}\mathrm{Cl}_{\mathrm{T_N}}(\mathrm{G}_2) = \mathrm{V(G)}$$

Case III) Abnormal of ABG- Arterial Blood Gases Test

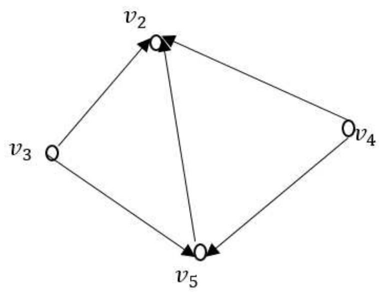

Figure 8: Increasing the toxicity in Renal Artery

The following tables observe all potential diseases of the urinary system from the point of the nano topological graph. Also, it presents dependence (safety) scales by three methods.

Table1: Nano Topological Graph				
$\boldsymbol{V(G)}$	$L_N(\boldsymbol{V}(G))$	$H_N(V(G))$	$B_N(V(G))$	$\tau_N(V(G))$*Nano* topologies
$\{v_1\}$	ϕ	$V(G)$	$V(G)$	$\{V(G), \phi\}$
$\{v_2\}$	(v_2)	$V(G)$	$\{v_1, v_3, v_4, v_5\}$	$\left\{V(G), \phi, \{v_2\}, \{v_1, v_3, v_4, v_5\}\right\}$
$\{v_3\}$	ϕ	$V(G)$	$V(G)$	$\{V(G), \phi\}$
$\{v_4\}$	ϕ	$V(G)$	$V(G)$	$\{V(G), \phi\}$
$\{v_5\}$	ϕ	$V(G)$	$V(G)$	$\{V(G), \phi\}$
$\{v_1, v_2\}$	(v_2)	$V(G)$	$\{v_1, v_3, v_4, v_5\}$	$\left\{V(G), \phi, \{v_2\}, \{v_1, v_3, v_4, v_5\}\right\}$
$\{v_1, v_3\}$	ϕ	$V(G)$	$V(G)$	$\{V(G), \phi\}$
$\{v_1, v_4\}$	ϕ	$V(G)$	$V(G)$	$\{V(G), \phi\}$
$\{v_1, v_5\}$	ϕ	$V(G)$	$V(G)$	$\{V(G), \phi\}$
$\{v_2, v_3\}$	(v_2)	$V(G)$	$\{v_1, v_3, v_4, v_5\}$	$\left\{V(G), \phi, \{v_2\}, \{v_1, v_3, v_4, v_5\}\right\}$
$\{v_2, v_4\}$	(v_2)	$V(G)$	$\{v_1, v_3, v_4, v_5\}$	$\left\{V(G), \phi, \{v_2\}, \{v_1, v_3, v_4, v_5\}\right\}$
$\{v_2, v_5\}$	$\{v_2, v_5\}$	$V(G)$	$\{v_1, v_3, v_4\}$	$\left\{V(G), \phi, \{v_2, v_5\} \{v_1, v_3, v_4\}\right\}$
$\{v_3, v_4\}$	ϕ	$V(G)$	$V(G)$	$\{V(G), \phi\}$
$\{v_3, v_5\}$	ϕ	$V(G)$	$V(G)$	$\{V(G), \phi\}$
$\{v_4, v_5\}$	ϕ	$V(G)$	$V(G)$	$\{V(G), \phi\}$
$[v_1, v_2, v_3\}$	(v_2)	$V(G)$	$\{v_1, v_3, v_4, v_5\}$	$\left\{V(G), \phi, \{v_2\}, \{v_1, v_3, v_4, v_5\}\right\}$
$[v_1, v_2, v_4\}$	(v_2)	$V(G)$	$\{v_1, v_3, v_4, v_5\}$	$\left\{V(G), \phi, \{v_2\}, \{v_1, v_3, v_4, v_5\}\right\}$
$[v_1, v_2, v_5\}$	$\{v_2, v_5\}$	$V(G)$	$\{v_1, v_3, v_4\}$	$\left\{V(G), \phi, \{v_2, v_5\} \{v_1, v_3, v_4\}\right\}$
$[v_1, v_3, v_4\}$	ϕ	$V(G)$	$V(G)$	$\{V(G), \phi\}$
$[v_1, v_3, v_5\}$	ϕ	$V(G)$	$V(G)$	$\{V(G), \phi\}$
$[v_1, v_4, v_5\}$	ϕ	$V(G)$	$V(G)$	$\{V(G), \phi\}$
$[v_2, v_3, v_4\}$	(v_2)	$V(G)$	$\{v_1, v_3, v_4, v_5\}$	$\left\{V(G), \phi, \{v_2\}, \{v_1, v_3, v_4, v_5\}\right\}$
$[v_2, v_3, v_5\}$	$\{v_2, v_3, v_5\}$	$V(G)$	$\{v_1, v_4\}$	$\left\{V(G), \phi, \{v_1, v_4\}, \{v_2, v_3, v_5\}\right\}$
$[v_2, v_4, v_5\}$	$[v_2, v_4, v_5\}$	$V(G)$	$\{v_1, v_3\}$	$\left\{V(G), \phi, \{v_1, v_3\}, \{v_2, v_4, v_5\}\right\}$
$[v_3, v_4, v_5\}$	ϕ	$V(G)$	$V(G)$	$\{V(G), \phi\}$
$\{v_1, v_2, v_3, v_4\}$	(v_2)	$V(G)$	$\{v_1, v_3, v_4, v_5\}$	$\left\{V(G), \phi, \{v_2\}, \{v_1, v_3, v_4, v_5\}\right\}$
$\{v_1, v_2, v_3, v_5\}$	$\{v_2, v_3, v_5\}$	$V(G)$	$\{v_1, v_4\}$	$\left\{V(G), \phi, \{v_1, v_4\}, \{v_2, v_3, v_5\}\right\}$
$\{v_1, v_2, v_4, v_5\}$	$\{v_2, v_4, v_5\}$	$V(G)$	$\{v_1, v_3\}$	$\left\{V(G), \phi, \{v_1, v_3\}, \{v_2, v_4, v_5\}\right\}$
$\{v_1, v_3, v_4, v_5\}$	$\{v_1, v_3, v_4, v_5\}$	$V(G)$	$\{v_2\}$	$\{V(G), \varphi, \{v_2\} \{v_1, v_3, v_4, v_5\}\}$
$\{v_2, v_3, v_4, v_5\}$	$\{v_2, v_3, v_4, v_5\}$	$V(G)$	$\{v_1\}$	$[V(G), \varphi, \{v_1\}, \{v_2, v_3, v_4, v_5\}\}$
$V(G)$	$V(G)$	$V(G)$	ϕ	$\{V(G), \phi\}$
ϕ	ϕ	ϕ	ϕ	$\{V(G), \phi\}$

Table 2: Dependence (safety) Scales of Nano Topological Graph.

$\boldsymbol{V(G)}$	$D^{(1)}\tau_N\,(G_1, G)$	$D^{(2)}\tau_N\,(G_1, G)$	$D^{(3)}_{\tau_N}\,(G_1, G)$
$\{v_1\}$	0	0	0
$\{v_2\}$	1/2	2/5	1/2
$\{v_3\}$	0	0	0
$\{v_4\}$	0	0	0
$\{v_5\}$	0	0	0
$\{v_1, v_2\}$	1/5	1/5	1/5
$\{v_1, v_3\}$	0	0	0
$\{v_1, v_4\}$	0	0	0
$\{v_1, v_5\}$	0	0	0
$\{v_2, v_3\}$	1/5	1/5	1/5
$\{v_2, v_4\}$	1/5	1/5	1/5
$\{v_2, v_5\}$	2/5	2/5	2/5
$\{v_3, v_4\}$	0	0	0
$\{v_3, v_5\}$	0	0	0
$\{v_4, v_5\}$	0	0	0
$[v_1, v_2, v_3\}$	1/5	1/5	1/5
$[v_1, v_2, v_4\}$	1/5	1/5	1/5
$[v_1, v_2, v_5\}$	2/5	2/5	2/5
$[v_1, v_3, v_4\}$	0	0	0
$[v_1, v_3, v_5\}$	0	0	0
$[v_1, v_4, v_5\}$	0	0	0
$[v_2, v_3, v_4\}$	1/5	1/5	1/5
$[v_2, v_3, v_5\}$	3/5	3/5	3/5
$[v_2, v_4, v_5\}$	3/5	3/5	3/5
$[v_3, v_4, v_5\}$	0	0	0
$\{v_1, v_2, v_3, v_4\}$	1/5	1/5	1/5
$\{v_1, v_2, v_3, v_5\}$	3/5	3/5	3/5
$\{v_1, v_2, v_4, v_5\}$	3/5	3/5	3/5
$\{v_1, v_3, v_4, v_5\}$	4/5	4/5	4/5
$\{v_2, v_3, v_4, v_5\}$	4/5	4/5	4/5
$V(G)$	1	1	1
ϕ	0	0	0

The graph medical results from nano topological graph

(i) There symmetric between safety coefficient by the nano topological graph on urinary system based on right kidney and left kidney .

$$D^{(1)}\tau_N \geq 0.5, \quad D^{(1)}\tau_N\,(G_1, G) = 0.6, D^{(1)}\tau_N\,(G_2, G) = 0.6$$

(ii) There is no effect between the failure of left or right kidney about another.

$$\mathrm{D}_{\tau_N} \leq 0.5$$

(iii) The urinary system cannot operate by singleton organ.

$$\mathrm{D}_{\tau_N} = 0$$

(iv) There is not overlap between the renal Artery and renal Vein

$$\mathrm{D}_{\tau_N} = 0$$

(v) Tumor eradication or malformation defect operation of the urinary system

$$D_{\tau_N} \leq 0.5$$

.

Determine safety coefficient $\mu_{\tau N}(G_1, G)$ using a relative topological graph

Algorithm of relative topology to detect diseases of the urinary system:
Step 1: By taking Definition 3.7 of the neighbourhood of vertices, generate general topological graphs.
Step 2: From results of tests measuring serum, construct subgraphs from graph model.
Step 3: To follow diseases of the urinary system, we divide the urinary system to subsets and study the corresponding relative topology.
Step 4: Check dependence scales by Definition 3.10 is called safety numbers.
Step 5: From result, we can conclude verifies the medical results.

Apply the five steps of the algorithm on the urinary system. We obtain measurements of the safety scales in the following table.

Table 3: Safety Scales of Relative Topological Graph.

$\boldsymbol{V(G)}$	$D^{(1)}\tau_N(G_1, G)$	$D^{(2)}\tau_N(G_1, G)$	$D^{(3)}_{\tau_N}(G_1, G)$
$\{v_1\}$	1/5	1	1/5
$\{v_2\}$	1/5	1	1/5
$\{v_3\}$	1/5	1	1/5
$\{v_4\}$	1/5	1	1/5
$\{v_5\}$	1/5	1	1/5
$\{v_1, v_2\}$	2/5	1	2/5
$\{v_1, v_3\}$	2/5	1	2/5
$\{v_1, v_4\}$	2/5	1	2/5
$\{v_1, v_5\}$	2/5	1	2/5
$\{v_2, v_3\}$	2/5	1	2/5
$\{v_2, v_4\}$	2/5	1	2/5
$\{v_2, v_5\}$	2/5	1	2/5
$\{v_3, v_4\}$	2/5	1	2/5
$\{v_3, v_5\}$	2/5	1	2/5
$\{v_4, v_5\}$	2/5	1	2/5
$[v_1, v_2, v_3\}$	3/5	1	3/5
$[v_1, v_2, v_4\}$	3/5	1	3/5
$[v_1, v_2, v_5\}$	3/5	1	3/5
$[v_1, v_3, v_4\}$	3/5	1	3/5
$[v_1, v_3, v_5\}$	3/5	1	3/5
$[v_1, v_4, v_5\}$	3/5	1	3/5
$[v_2, v_3, v_4\}$	3/5	1	3/5
$[v_2, v_3, v_5\}$	3/5	1	3/5
$[v_2, v_4, v_5\}$	3/5	1	3/5
$[v_3, v_4, v_5\}$	3/5	1	3/5
$\{v_1, v_2, v_3, v_4\}$	4/5	1	4/5
$\{v_1, v_2, v_3, v_5\}$	4/5	1	4/5
$\{v_1, v_2, v_4, v_5\}$	4/5	1	4/5
$\{v_1, v_3, v_4, v_5\}$	4/5	1	4/5
$\{v_2, v_3, v_4, v_5\}$	4/5	1	4/5
$V(G)$	1	1	1
ϕ	0	0	0

The graph medical results from relative topology induced by graph

(i) There symmetric between safety coefficient by the nano topological graph on urinary system based on right kidney and left kidney . $D^{(1)}\tau_A \geq 0.5, D^{(1)}_{\tau_A}(G_1, G) = 0.8, D^{(1)}_{\tau_A}(G_2, G) = 0.8$

(ii) There is no effect between the failure of left or right kidney about another.

$D^{(1)}\tau_A \leq 0.5, D^{(1)}\tau_A\,(G_1, G_2) = 0.4$

(iii) The urinary system cannot operate by singleton organ. $\mathrm{D}_{\tau_N} \leq 0.2$

(iv) There is not overlap between the renal Artery and renal Vein $\mathrm{D}_{\tau_N} \leq 0.2$

(v) Tumor eradication or malformation defect operation of the urinary system $D_{\tau_N} \leq 0.5$.

Reduce cost, time and effort from the most important goals, from the previous tables' results; we can obtain the core of important of medical tests expressive of the case of urinary system in Table 4. The new scale is called (PRED) that indicates necessary medical tests to organs of the urinary system as a percentage value.

Table 4: Medical tests corresponding analysis safety scales.

$V(G)$	**PRED%**
$\{v_1\}$	47%
$\{v_2\}$	47%
$\{v_3\}$	47%
$\{v_4\}$	47%
$\{v_5\}$	47%
$\{v_1, v_2\}$	60%
$\{v_1, v_3\}$	60%
$\{v_1, v_4\}$	60%
$\{v_1, v_5\}$	60%
$\{v_2, v_3\}$	60%
$\{v_2, v_4\}$	60%
$\{v_2, v_5\}$	60%
$\{v_3, v_4\}$	60%
$\{v_3, v_5\}$	60%
$\{v_4, v_5\}$	60%
$[v_1, v_2, v_3\}$	73%
$[v_1, v_2, v_4\}$	73%
$[v_1, v_2, v_5\}$	73%
$[v_1, v_3, v_4\}$	73%
$[v_1, v_3, v_5\}$	73%
$[v_1, v_4, v_5\}$	73%
$[v_2, v_3, v_4\}$	73%
$[v_2, v_3, v_5\}$	73%
$[v_2, v_4, v_5\}$	73%
$[v_3, v_4, v_5\}$	73%
$\{v_1, v_2, v_3, v_4\}$	87%
$\{v_1, v_2, v_3, v_5\}$	87%
$\{v_1, v_2, v_4, v_5\}$	87%
$\{v_1, v_3, v_4, v_5\}$	87%
$\{v_2, v_3, v_4, v_5\}$	87%
$V(G)$	100%
ϕ	0%

Conclusions

In the present application, we give a digraph of the urinary system .Results of application techniques of the digraph, which we obtained are most useful in solving the liquid flow system in the urinary system. We investigate nano topological properties in that we can deduce some diseases that affect the urinary system in the human body.

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