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Remarks on Fixed Point Theory in Soft Metric Type Spaces

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Abstract. The aim of this paper is to discuss the recent developement regarding fixed point theory in soft metric type spaces such as soft G-metric spaces, soft cone metric spaces, dislocated soft metric spaces and soft b-metric spaces. We show that soft versions of fixed point results proved in such metric type spaces can be directly deduced from the comparable existing results in the literature.

1. Introduction

The role of probability theory, fuzzy set theory, vague sets, rough sets and interval mathematics to deal with uncertainty in data is significant. But there are certain limitations and deficiencies pertaining to the parametrization and useful presentation of data (see [20]). Soft set theory as an extension of fuzzy set theory [34] was initiated by Molodtsov [20] as a mathematical tool to deal with uncertainties associated with real world problems. Soft set theory has been successfully applied in decision making, demand analysis, forecasting, information sciences, mathematics and other disciplines (see for detailed survey [7–9, 12, 13, 18, 24, 25, 35, 36]).

Das and Samanta [5] defined soft real set and soft real number and discussed their properties. Based on these notions, they introduced the concept of a soft metric [4]. Wardowski [32] introduced a notion of soft mapping and obtained its fixed point in the setup of soft topological spaces. Das et al. [6] defined soft linear spaces and soft normed linear spaces. Abbas et al. [1] introduced soft contraction mappings and obtained a soft Banach fixed point theorem in the framework of soft metric spaces. After that, they [2] showed that a soft metric induces a compatible metric and, hence, soft metric extensions of several important fixed point theorems for metric spaces can be directly deduced from comparable existing results.

In the recent years, many researchers proved fixed point results in soft metric type spaces (see, for example, [1–3, 10, 11, 22, 23, 28, 29, 31, 33]).

The purpose of this paper is to reverse the trend of proving fixed point results in soft metric type spaces such as soft G-metric spaces, soft cone metric spaces, dislocated soft metric spaces and soft b-metric spaces. We show that these soft metric type spaces induce compatible metric type spaces and soft versions of fixed point theorems for such metric type spaces can be directly deduced from comparable existing results. In this way, we also provide a correct version of many existing results in the literature having certain flaws.

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2. Preliminaries

Definition 2.1. *[20] Let U be a universe and E a set of parameters. Let $\mathcal{P}(U)$ denote the power set of U and A a nonempty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to \mathcal{P}(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε- approximate elements of the soft set (F, A).*

Definition 2.2. *[19]*

(a) A soft set (F, A) is said to be: (a) an absolute soft set denoted by $\tilde{U}$ if for each $\varepsilon \in A$, $F(\varepsilon) = U$; (b) a null soft set denoted by Φ if for each $\varepsilon \in A$, $F(\varepsilon) = \phi$.

(b) Let (F, A) and (G, B) be soft sets over U. We say that (F, A) is a soft subset of (G, B) or (G, B) is super soft set of (F, A) if $A \subseteq B$ and for all $\varepsilon \in A$, $F(\varepsilon) \subseteq G(\varepsilon)$. We denote it as $(F, A)\tilde{\subset}(G, B)$. (F, A) is said to be soft equal to (G, B), if $(F, A)\tilde{\subset}(G, B)$ and $(G, B)\tilde{\subset}(F, A)$.

(c) Let (F, A) and (G, B) be soft sets over U. Union of (F, A) and (G, B), denoted by $(F, A)\widetilde{\cup}(G, B)$, is a soft set $(H, A \cup B)$ defined as follows:

$$H(\varepsilon) = \begin{cases} F(\varepsilon) & \text{if } \varepsilon \in A - B \\ G(\varepsilon) & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon) & \text{if } \varepsilon \in A \cap B. \end{cases}$$

(d) Intersection of (F, A) and (G, B), denoted by $(F, A)\tilde{\cap}(G, B)$, is the soft set $(H, A \cap B)$ defined as $H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$ for each ε in A.

(e) The complement $(F, A)^c$ of a soft set (F, A), also denoted by (F^c, A), is the multivalued mapping $F^c : A \to P(U)$ defined by $F^c(\varepsilon) = U - F(\varepsilon)$, for each $\varepsilon \in A$.

(f) Difference of (F, A) and (G, B) denoted by $(F, A)\backslash(G, B)$ is the soft set (H, A) defined as $H(\varepsilon) = F(\varepsilon)\backslash G(\varepsilon)$ for each ε in A.

Definition 2.3. *[4] A soft set (P, A) over U is said to be a soft point if there is exactly one $\lambda \in A$, such that $P(\lambda) = \{x\}$ for some $x \in U$ and $P(\mu) = \phi$, $\forall\ \mu \in A\backslash\{\lambda\}$. It will be denoted by x_λ.*

Definition 2.4. *[4] A soft point x_λ is said to belongs to a soft set (F, A) if $\lambda \in A$ and $P(\lambda) = \{x\} \subset F(\lambda)$. It is written as $x_\lambda \tilde{\in} (F, A)$.*

The collection of soft points of a soft set (F, E) will be denoted by $SP(F, E)$.

Definition 2.5. *[4] Two soft points x_λ, y_μ are said to be equal if $\lambda = \mu$ and $P(\lambda) = P(\mu)$ i.e. $x = y$. Thus $x_\lambda \neq y_\mu \Leftrightarrow x \neq y$ or $\lambda \neq \mu$.*

Proposition 2.6. *[4] The union of any collection of soft points can be considered as a soft set and every soft set can be expressed as union of all soft points belonging to it; i.e., $(F, A) = \cup_{x_\lambda \tilde{\in} (F,A)} x_\lambda$.*

Definition 2.7. *[5] If f is a single valued mapping on $A \subset E$ taking values in U, then the pair (f, A), or simply f, is called a soft element of U. A soft element f of U is said to belong to a soft set (F, A), denoted by $f \tilde{\in} (F, A)$, if $f(e) \in F(e)$, for each $e \in A$.*

We will denote $SE(F, E)$ for the collection of soft elements of a soft set (F, E). Also, $\tilde{x}, \tilde{y}, \tilde{z}$ denote soft elements of a soft set.

Proposition 2.8. *[4]*

(a) For any soft sets $(F, A), (G, A)$, we have $(F, A)\tilde{\subseteq}(G, A)$ if and only if every soft element of (F, A) is also a soft element of (G, A).

(b) Any collection of soft elements of a soft set can generate a soft subset of that soft set. The soft set constructed from a collection $\mathcal{B}$ of soft elements is denoted by $SS(\mathcal{B})$.

(c) *For any soft set (F, A), $SS(SE(F, A)) = (F, A)$; whereas for a collection B of soft elements, $SE(SS(\mathcal{B})) \supseteq \mathcal{B}$, but in general, $SE(SS(\mathcal{B})) \neq \mathcal{B}$.*

Definition 2.9. *[5] Let A be a nonempty subset of E. A soft real set is a mapping $F : A \to B(\mathbb{R})$. It is denoted by (F, A). If in particular (F, A) is a singleton soft set, then identifying (F, A) with the corresponding soft element, it will be called a soft real number. If F is a single valued mapping on $A \subset E$ taking values in the set $\mathbb{R}^+$ of non negative real numbers, then a pair (F, A), or simply F, is called a non negative soft real number. We shall denote the set of non negative soft real numbers by $\mathbb{R}(A)^*$. A null soft number $\overline{0}$ is a soft real number defined by $\overline{0}(e) = 0$ for all $e \in A$. A unit soft number $\overline{1}$ is a soft real number defined by $\overline{1}(e) = 1$ for all $e \in A$. A constant soft real number $\overline{c}$ is a soft real number such that for each $e \in A$, we have $\overline{c}(e) = c$, where c is some real number*

Definition 2.10. *[4] For two soft real numbers $\tilde{r}, \tilde{s}$, we say that*

(i) $\tilde{r} \tilde{\leq} \tilde{s}$ *if* $\tilde{r}(e) \leq \tilde{s}(e)$, *for all* $e \in A$,
(ii) $\tilde{r} \tilde{\geq} \tilde{s}$ *if* $\tilde{r}(e) \geq \tilde{s}(e)$, *for all* $e \in A$,
(iii) $\tilde{r} \tilde{<} \tilde{s}$ *if* $\tilde{r}(e) < \tilde{s}(e)$, *for all* $e \in A$, *and*
(iv) $\tilde{r} \tilde{>} \tilde{s}$ *if* $\tilde{r}(e) > \tilde{s}(e)$, *for all* $e \in A$.

We will use the same operations (stated above) for soft elements.

Initially Das and Samanta [4] defined soft metric as a function on the set of soft points but later on they used soft metric as a function on the set of soft elements (see [6]) in the following manner:

Definition 2.11. *A mapping $d : SE(\tilde{X}) \times SE(\tilde{X}) \to \mathbb{R}(A)^*$ is said to be a soft metric on $\tilde{X}$ if for any $\tilde{x}, \tilde{y}, \tilde{z} \tilde{\in} \tilde{X}$, the following hold*

M1. $d(\tilde{x}, \tilde{y}) \tilde{\geq} \bar{0}$,
M2. $d(\tilde{x}, \tilde{y}) = \bar{0}$ *if and only if* $\tilde{x} = \tilde{y}$,
M3. $d(\tilde{y}, \tilde{x}) = d(\tilde{x}, \tilde{y})$,
M4. $d(\tilde{x}, \tilde{z}) \tilde{\leq} d(\tilde{x}, \tilde{y}) + d(\tilde{y}, \tilde{z})$.

A soft metric space is a pair $(\tilde{X}, d)$ such that $\tilde{X}$ is a soft set and d is a soft metric on $\tilde{X}$.

3. Soft G-metric space and fixed point result

We begin this section by recalling some definitions and fixed point results in soft G-metric spaces in [10]. Throughout this section, we will use E as a set of parameters. The concept of soft G-metric space is defined as follows:

Definition 3.1. *[10] A mapping $\tilde{G} : SE(\tilde{X}) \times SE(\tilde{X}) \times SE(\tilde{X}) \to \mathbb{R}(A)^*$ is said to be a soft G-metric on $\tilde{X}$ if for any soft elements $\tilde{x}, \tilde{y}, \tilde{z}$, the following hold*

1. $\tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}) = \bar{0}$ *if* $\tilde{x} = \tilde{y} = \tilde{z}$;
2. $\bar{0} \tilde{<} \tilde{G}(\tilde{x}, \tilde{x}, \tilde{y})$ *for all* $\tilde{x}, \tilde{y} \in SE(\tilde{X})$ *with* $\tilde{x} \neq \tilde{y}$;
3. $\tilde{G}(\tilde{x}, \tilde{x}, \tilde{y}) \tilde{\leq} \tilde{G}(\tilde{x}, \tilde{y}, \tilde{z})$ *for all* $\tilde{x}, \tilde{y}, \tilde{z} \in SE(\tilde{X})$ *with* $\tilde{y} \neq \tilde{z}$;
4. $\tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}) = \tilde{G}(\tilde{x}, \tilde{z}, \tilde{y}) = \tilde{G}(\tilde{y}, \tilde{z}, \tilde{x}) = \cdots$;
5. $\tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}) \tilde{\leq} \tilde{G}(\tilde{x}, \tilde{a}, \tilde{a}) + \tilde{G}(\tilde{a}, \tilde{y}, \tilde{z})$ *for all* $\tilde{x}, \tilde{y}, \tilde{z}, \tilde{a} \in SE(\tilde{X})$.

The soft set $\tilde{X}$ with a soft G-metric $\tilde{G}$ on $\tilde{X}$ is said to be a soft G-metric space and is denoted by $(\tilde{X}, \tilde{G}, E)$.

Definition 3.2. *[10] A soft G-metric space $(\tilde{X}, \tilde{G}, E)$ is symmetric if $\tilde{G}(\tilde{x}, \tilde{y}, \tilde{y}) = \tilde{G}(\tilde{x}, \tilde{x}, \tilde{y})$ holds for all $\tilde{x}, \tilde{y} \in SE(\tilde{X})$.*

Definition 3.3. *[10] Let $(\tilde{X}, \tilde{G}, E)$ be a soft G-metric space. A sequence $\{\tilde{x}_n\}_n$ of soft elements in $\tilde{X}$ is said to be soft G-convergent to $\tilde{x}$ in $\tilde{X}$ if for every $\tilde{\varepsilon} \tilde{>} \bar{0}$, chosen arbitrary, there exists a natural number $N = N(\tilde{\varepsilon})$ such that $\bar{0} \tilde{\leq} \tilde{G}(\tilde{x}_n, \tilde{x}_n, \tilde{x}) \tilde{<} \tilde{\varepsilon}$, whenever $n > N$. We denote this by $\tilde{x}_n \to \tilde{x}$ as $n \to \infty$ or by $\lim_{n\to\infty} \tilde{x}_n = \tilde{x}$.*

Definition 3.4. *[11] Let $(\tilde{X}, \tilde{G}, E)$ be a soft G-metric space. A sequence $\{\tilde{x}_n\}_n$ of soft elements in $\tilde{X}$ is said to be soft G-Cauchy if for every $\tilde{\varepsilon}\tilde{>}\bar{0}$, chosen arbitrary, there exists a natural number k such that $\tilde{G}(\tilde{x}_n, \tilde{x}_m, \tilde{x}_l)\tilde{<}\tilde{\varepsilon}$, whenever $n, m, l \geq k$. A soft G-metric space $(\tilde{X}, \tilde{G}, E)$ is said to be soft G-complete if every soft G-Cauchy sequence in $(\tilde{X}, \tilde{G}, E)$ is soft G-convergent in $(\tilde{X}, \tilde{G}, E)$.*

Definition 3.5. *[10] Let $(\tilde{X}, \tilde{G}, E)$ be a soft G-metric space. Let $T : (\tilde{X}, \tilde{G}, E) \to (\tilde{X}, \tilde{G}, E)$ be a mapping. If there exists a soft element $\tilde{x}_0$ in $\tilde{X}$ such that $T(\tilde{x}_0) = \tilde{x}_0$, then $\tilde{x}_0$ is called a fixed point of T.*

Theorem 3.6. *[11] Let $(\tilde{X}, \tilde{G}, E)$ be a soft G-complete space and $T : (\tilde{X}, \tilde{G}, E) \to (\tilde{X}, \tilde{G}, E)$ a mapping that satisfies the following condition*

$$\tilde{G}(T\tilde{x}, T\tilde{y}, T\tilde{z})\tilde{\leq}\bar{a}\tilde{G}(\tilde{x}, T\tilde{x}, T\tilde{x}) + \bar{b}\tilde{G}(\tilde{y}, T\tilde{y}, T\tilde{y}) + \bar{c}\tilde{G}(\tilde{z}, T\tilde{z}, T\tilde{z}) + \bar{d}\tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}) \tag{1}$$

for any $\tilde{x}, \tilde{y}, \tilde{z} \in SE(\tilde{X})$, where $\bar{0}\tilde{\leq}\bar{a}+\bar{b}+\bar{c}+\bar{d}\tilde{<}\bar{1}$. Then T has a unique fixed point, say $\tilde{u}$, and T is soft G-continuous at $\tilde{u}$.

We now show that the Theorem 3.6 fails to hold if the set of parameter is not finite. For this we will provide two examples: one for the symmetric soft G-metric space and other for non symmetric soft G-metric space. At the end of this section, we see that if the set of parameter is finite then we may obtain this fixed point result (when the set of parameter is finite) withe help of a soft metric.

Example 3.7. *Let $X = E = \{1/n : n \in \mathbb{N}\}$. Consider the soft G-metric space $(\tilde{X}, \tilde{G}, E)$, where*

$$\tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}) = \overline{\frac{1}{3}}\{|\bar{x} - \bar{y}| + |\bar{y} - \bar{z}| + |\bar{x} - \bar{z}|\}$$

for $\tilde{x}, \tilde{y}, \tilde{z} \in SE(\tilde{X})$. Note that, it is a symmetric soft G-metric ([10], Proposition 2.11). Now we show that $(\tilde{X}, \tilde{G}, E)$ is soft G-complete. For this suppose that $\{\tilde{x}_n\}_n$ is a soft G-Cauchy sequence of soft elements in $(\tilde{X}, \tilde{G}, E)$. Take the soft real number $\widetilde{\varepsilon}$ such that $\widetilde{\varepsilon}(\lambda) = \lambda$ for all $\lambda \in E$, i.e., $\widetilde{\varepsilon}(1/h) = 1/h$ for all $h \in \mathbb{N}$. Then, there is $k \in \mathbb{N}$ such that

$$\tilde{G}(\tilde{x}_n, \tilde{x}_m, \tilde{x}_l)\tilde{<}\widetilde{\varepsilon},$$

for all $n, m, l \geq k$. This implies that $\tilde{G}(\tilde{x}_n, \tilde{x}_m, \tilde{x}_l)(1/h) < \widetilde{\varepsilon}(1/h)$ for all $h \in \mathbb{N}$. Hence

$$\overline{\frac{1}{3}}(|\bar{x}_n - \bar{x}_m| + |\bar{x}_m - \bar{x}_l| + |\bar{x}_n - \bar{x}_l|)(\frac{1}{h}) < \frac{1}{h},$$

for all $n, m, l \geq k$ and for all $h \in \mathbb{N}$. Consequently

$$\frac{1}{3}(|x_n - x_m| + |x_m - x_l| + |x_n - x_l|) < \frac{1}{h},$$

for all $n, m, l \geq k$ and for all $h \in \mathbb{N}$. This implies that the sequence $\{\tilde{x}_n\}_n$ is eventually constant, and hence soft G-convergent. Hence $(\tilde{X}, \tilde{G}, E)$ is soft G-complete.

Now let $T : (\tilde{X}, \tilde{G}, E) \to (\tilde{X}, \tilde{G}, E)$ be defined as $T(\tilde{x}) = \tilde{x}/8$ for all $\tilde{x} \in SE(\tilde{X})$. Note that T satisfies (1). Indeed, for any $\tilde{x}, \tilde{y}, \tilde{z} \in SE(\tilde{X})$ and for $\eta \in E$, we have

$$\begin{aligned}
\tilde{G}(T\tilde{x}, T\tilde{y}, T\tilde{z})(\eta) &= (\frac{\bar{1}}{3}\{\left|\frac{\bar{x}}{8} - \frac{\bar{y}}{8}\right| + \left|\frac{\bar{y}}{8} - \frac{\bar{z}}{8}\right| + \left|\frac{\bar{x}}{8} - \frac{\bar{z}}{8}\right|\})(\eta) \\
&= \frac{1}{24}\{\left|x - y\right| + \left|y - z\right| + |x - z|\} \\
&= (\frac{\bar{1}}{24}(d(\tilde{x}, \tilde{y}) + d(\tilde{y}, \tilde{z}) + d(\tilde{x}, \tilde{z})))(\eta) = \frac{\bar{1}}{8}\tilde{G}(\tilde{x}, \tilde{y}, \tilde{z})(\eta) \\
&\leq \frac{\bar{1}}{8}\{\tilde{G}(\tilde{x}, T\tilde{x}, T\tilde{x}) + \tilde{G}(\tilde{y}, T\tilde{y}, T\tilde{y}) + \tilde{G}(\tilde{z}, T\tilde{z}, T\tilde{z}) + \tilde{G}(\tilde{x}, \tilde{y}, \tilde{z})\}(\eta).
\end{aligned}$$

Hence,

$$\tilde{G}(T\tilde{x}, T\tilde{y}, T\tilde{z})\tilde{\leq}\frac{\bar{1}}{8}\tilde{G}(\tilde{x}, T\tilde{x}, T\tilde{x}) + \frac{\bar{1}}{8}\tilde{G}(\tilde{y}, T\tilde{y}, T\tilde{y}) + \frac{\bar{1}}{8}\tilde{G}(\tilde{z}, T\tilde{z}, T\tilde{z}) + \frac{\bar{1}}{8}\tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}).$$

All conditions of the Theorem 3.6 are satisfied but T is a fixed point free map.

Example 3.8. *Let $X = E = \{1/n : n \in \mathbb{N}\}$. Define the soft G-metric by*

$$\tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}) = \begin{cases} \tilde{x} + \tilde{y} + \tilde{z} + \bar{1}, & \text{when } \tilde{x} \neq \tilde{y} \neq \tilde{z} \text{ and all are different from } \bar{0}, \\ \tilde{x} + \tilde{z} + \bar{1}, & \text{when } \tilde{x} = \tilde{y} \neq \tilde{z} \text{ and all are different from } \bar{0}, \\ \tilde{y} + \tilde{z} + \bar{2}, & \text{when } \tilde{x} = \bar{0},\ \tilde{y} \neq \tilde{z} \text{ and } \tilde{y}, \tilde{z} \text{ are different from } \bar{0}, \\ \tilde{y} + \bar{3}, & \text{when } \tilde{x} = \bar{0},\ \tilde{y} = \tilde{z} \text{ and } \tilde{y} \text{ is different from } \bar{0}, \\ \tilde{z} + \bar{2}, & \text{when } \tilde{x} = \tilde{y} = \bar{0} \text{ and } \tilde{z} \text{ is different from } \bar{0}, \\ \bar{0}, & \text{if } \tilde{x} = \tilde{y} = \tilde{z}, \end{cases}$$

and extend the definition by symmetry in its arguments. It is easy to show that $(\tilde{X}, \tilde{G}, E)$ is a soft G-metric which is not symmetric. Now we show that $(\tilde{X}, \tilde{G}, E)$ is soft G-complete. Suppose that $\{\tilde{x}_n\}_n$ is a soft G-Cauchy sequence of soft elements in $(\tilde{X}, \tilde{G}, E)$. Take the soft real number $\widetilde{\varepsilon}$ such that $\widetilde{\varepsilon}(\lambda) = \lambda$ for all $\lambda \in E$, i.e., $\widetilde{\varepsilon}(1/h) = 1/h$ for all $h \in \mathbb{N}$. Then, there is $k \in \mathbb{N}$ such that

$$\tilde{G}(\tilde{x}_n, \tilde{x}_m, \tilde{x}_l)\tilde{<}\widetilde{\varepsilon},$$

for all $m, n, l \geq k$. This implies that $\tilde{G}(\tilde{x}_n, \tilde{x}_m, \tilde{x}_l)(1/h) < \widetilde{\varepsilon}(1/h)$ for all $h \in \mathbb{N}$. Hence

$$\tilde{G}(\tilde{x}_n, \tilde{x}_m, \tilde{x}_l)(1/h) < \frac{1}{h},$$

for all $m, n, l \geq k$ and for all $h \in \mathbb{N}$. And this is possible only if the sequence $\{\tilde{x}_n\}_n$ is constant, and therefore soft G-convergent. We conclude that $(\tilde{X}, \tilde{G}, E)$ is soft G-complete.

Now let $T : (\tilde{X}, \tilde{G}, E) \to (\tilde{X}, \tilde{G}, E)$ be defined as $T(\tilde{x}) = \tilde{x}/8$ for all $\tilde{x} \in SE(\tilde{X})$. Note that, it satisfies (1). Indeed, for any $\tilde{x}, \tilde{y}, \tilde{z} \in SE(\tilde{X})$ and for each $\eta \in E$ we have

$$\begin{aligned} \tilde{G}(T\tilde{x}, T\tilde{y}, T\tilde{z})(\eta) &= \frac{\bar{1}}{8}\tilde{G}(\tilde{x}, \tilde{y}, \tilde{z})(\eta) \\ &\leq \frac{\bar{1}}{8}\{\tilde{G}(\tilde{x}, T\tilde{x}, T\tilde{x}) + \tilde{G}(\tilde{y}, T\tilde{y}, T\tilde{y}) \\ &\quad +\tilde{G}(\tilde{z}, T\tilde{z}, T\tilde{z}) + \tilde{G}(\tilde{x}, \tilde{y}, \tilde{z})\}(\eta). \end{aligned}$$

Hence we have

$$\tilde{G}(T\tilde{x}, T\tilde{y}, T\tilde{z})\tilde{\leq}\frac{\bar{1}}{8}\tilde{G}(\tilde{x}, T\tilde{x}, T\tilde{x}) + \frac{\bar{1}}{8}\tilde{G}(\tilde{y}, T\tilde{y}, T\tilde{y}) + \frac{\bar{1}}{8}\tilde{G}(\tilde{z}, T\tilde{z}, T\tilde{z}) + \frac{\bar{1}}{8}\tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}).$$

All conditions of the Theorem 3.6 are satisfied but we can see that T has no fixed point.

Güler and Yildirim [11] established fixed point result for complete soft G-metric spaces but they did not consider the finiteness of set of parameters. The cardinality of set of parameter plays a significant role in this regard as evident from the counter examples given above.

Next, we see that keeping the set of parameters finite, one can obtain the same result by soft metric if we define

$$d(\tilde{x}, \tilde{y}) = \max\{\tilde{G}(\tilde{x}, \tilde{y}, \tilde{y})\tilde{G}(\tilde{y}, \tilde{x}, \tilde{x})\}.$$

Proof and details are similar to the results given in [15].

4. Soft cone metric space and fixed point results

Altıntaş and Taşköprü [3] defined the soft cone metric space and obtained soft versions of some fixed point results. But keeping in view the scalarization method defined in the paper [30], all results in soft cone metric space can be obtained by an ordinary soft metric.

5. Dislocated Soft metric space and fixed point result

We start this section by recalling a definition of a dislocated soft metric space and some fixed point results in such spaces.

Definition 5.1. *[28] A mapping $\tilde{\rho} : SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}(E)^*$ is said to be a dislocated soft metric on $\tilde{X}$ if for any soft points x_λ, y_μ, z_η, the following hold*

1. $\tilde{\rho}(x_\lambda, y_\mu) = \bar{0}$ *then* $x_\lambda = y_\mu$;
2. $\tilde{\rho}(x_\lambda, y_\mu) = \tilde{\rho}(y_\mu, x_\lambda)$;
3. $\tilde{\rho}(x_\lambda, y_\mu) \tilde{\leq} \tilde{\rho}(x_\lambda, z_\eta) + \tilde{\rho}(z_\eta, y_\mu)$.

The soft set $\tilde{X}$ with a dislocated soft metric $\tilde{\rho}$ on $\tilde{X}$ is said to be a dislocated soft metric space and is denoted by $(\tilde{X}, \tilde{\rho}, E)$.

For sake of completeness, we state the following result from [16].

Theorem 5.2. *[16] Let A and B be two nonempty closed subsets of a complete dislocated metric space (X, d). Suppose that $T : A \cup B \to A \cup B$ satisfies the following condition*

$$d(Tx, Ty) \leq kd(x, y),$$

for $x \in A$, and $y \in B$, where $0 < k < 1$. Then T has a unique fixed point that belongs to $A \cap B$.

Wadkar et al. [28] gave the following fixed point result in the context of a dislocated soft metric space.

Theorem 5.3. *[28] Let A and B be two non-empty closed subsets of a complete dislocated soft metric space $(\tilde{X}, \tilde{\rho}, E)$. Suppose that $(f, \varphi) : A \cup B \to A \cup B$ satisfies the following condition*

$$\tilde{\rho}((f, \varphi)x_\lambda, (f, \varphi)y_\mu) \leq k\tilde{\rho}(x_\lambda, y_\mu)$$

for all $x_\lambda \in A, y_\mu \in B$, where $0 < k < 1$. Then (f, φ) has a unique fixed point that belongs to $A \cap B$.

The above theorem is not well presented in the context of a soft set theory.
First, we restate the above theorem and then discuss its limitations with the help of an example.

Theorem 5.4. *Let F and G be two non-null soft closed subsets of a complete dislocated soft metric space $(\tilde{X}, \tilde{\rho}, E)$. Suppose that $T : F\tilde{\cup}G \to F\tilde{\cup}G$ satisfies the following condition*

$$\tilde{\rho}(Tx_\lambda, Ty_\mu) \tilde{\leq} \bar{k}\tilde{\rho}(x_\lambda, y_\mu), \tag{2}$$

for any $x_\lambda \tilde{\in} F$, and $y_\mu \tilde{\in} G$, where $\bar{0} \tilde{<} \bar{k} \tilde{<} \bar{1}$. Then T has a unique fixed point that belongs to $F\tilde{\cap}G$.

In the following example we see that the revised version also has some limitations. Actually the above theorem is inconsistent with arbitrary set of parameters.

Example 5.5. *Let $X = E = \{1/n : n \in \mathbb{N}\}$. Let us define $\tilde{\rho}(x_\lambda, y_\mu) = \max\{|\bar{x}|, |\bar{y}|\} + \max\{|\bar{\lambda}|, |\bar{\mu}|\}$ for $x_\lambda, y_\mu \in SP(\tilde{X})$. It is easy to check that $(\tilde{X}, \tilde{\rho}, E)$ is a dislocated soft metric space. We show that $(\tilde{X}, \tilde{\rho}, E)$ complete. Suppose that $\{x_{\lambda,n}\}_n$ is a Cauchy sequence of soft points in $(\tilde{X}, \tilde{\rho}, E)$. Take the soft real number $\widetilde{\varepsilon}$ such that $\widetilde{\varepsilon}(\lambda) = \lambda$ for all $\lambda \in E$, i.e. $\widetilde{\varepsilon}(1/k) = 1/k$ for all $k \in \mathbb{N}$. Then, there is $m \in \mathbb{N}$ such that*

$$\tilde{\rho}(x_{\lambda,i}, x_{\lambda,j}) \tilde{<} \widetilde{\varepsilon},$$

for all $i, j \geq m$. We have $\tilde{\rho}(x_\lambda^{(i)}, x_\lambda^{(j)})(1/k) < \widetilde{\varepsilon}(1/k)$ for all $k \in \mathbb{N}$. Hence

$$(\max\{|\bar{x}_i|, |\bar{x}_j|\} + \max\{|\bar{\lambda}_i|, |\bar{\lambda}_j|\})(\frac{1}{k}) < \frac{1}{k},$$

for all $i, j \geq m$ and for all $k \in \mathbb{N}$. Consequently

$$\max\{|x_i|, |x_j|\} + \max\{|\lambda_i|, |\lambda_j|\} < \frac{1}{k},$$

for all $i, j \geq m$ and for all $k \in \mathbb{N}$. In particular, for any $j \geq m$,

$$\max\{|x_j|, |x_{j+1}|\} + \max\{|\lambda_j|, |\lambda_{j+1}|\} < \frac{1}{k},$$

for all $k \in \mathbb{N}$.

Therefore $x_j = x_{j+1}$ and $\lambda_j = \lambda_{j+1}$ for all $j \geq m$. We deduce that $x_j = x_m$ for all $j \geq m$. Thus the sequence $\{x_{\lambda,n}\}_n$ is eventually constant, and hence convergent. We conclude that $(\tilde{X}, \tilde{\rho}, E)$ is complete.

Now let $T : (\tilde{X}, \tilde{\rho}, E) \to (\tilde{X}, \tilde{\rho}, E)$ be defined as $T(x_\lambda) = (x/2)_{\lambda/2}$ for all $x_\lambda \in SP(\tilde{X})$. It satisfies equation (2). Indeed, fix $x_\lambda, y_\mu \in SP(\tilde{X})$, then for each $\eta \in E$ we have

$$\begin{aligned}
\tilde{\rho}(Tx_\lambda, Ty_\mu)(\eta) &= (\max\{|\overline{x/2}|, |\overline{y/2}|\} + \max\{|\overline{\lambda/2}|, |\overline{\mu/2}|\})(\eta) \\
&= \frac{1}{2}(\max\{|x|, |y|\} + \max\{|\lambda|, |\mu|\}) \\
&= \frac{\bar{1}}{2}\{\tilde{\rho}(x_\lambda, y_\mu)\}(\eta).
\end{aligned}$$

Hence we have

$$\tilde{\rho}(Tx_\lambda, Ty_\mu) = \frac{\bar{1}}{2}\tilde{\rho}(x_\lambda, y_\mu).$$

By taking $F = G = \tilde{X}$, all the conditions of the Theorem 5.4 are satisfied but T has no fixed point.

We now present the following theorem to show the role of cardinality of a set of parameters in case of fixed point results in soft dislocated soft metric spaces.

Theorem 5.6. *Let $(\tilde{X}, \tilde{\rho}, E)$ be a dislocated soft metric space with E a finite set. Define a function $m_d : SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}^+$ as*

$$m_d(x_\lambda, y_\mu) = \max_{\eta \in E} \tilde{\rho}(x_\lambda, y_\mu)(\eta),$$

for all $x_\lambda, y_\mu \in SP(\tilde{X})$. Then the following hold:

(1) m_d is a dislocated metric on $SP(\tilde{X})$.

(2) For any sequence $\{x_{\lambda,n}\}_n$ of soft points and a soft points x_μ, we have

(2a) $\{x_{\lambda,n}\}_n$ is a Cauchy sequence in $(\tilde{X}, \tilde{\rho}, E)$ if and only if it is a Cauchy sequence in $(SP(\tilde{X}), m_d)$.

(2b) $\lim_{n\to\infty} x_{\lambda,n} = x_\mu$ *in dislocated soft metric space* $(\tilde{X}, \tilde{\rho}, E)$ *if and only if* $\lim_{n\to\infty} x_{\lambda,n} = x_\mu$ *in dislocated metric space* $(SP(\tilde{X}), m_d)$.

(3) $(\tilde{X}, \tilde{\rho}, E)$ *is complete if and only if* $(SP(\tilde{X}), m_d)$ *is complete.*

Proof. (1) Let $x_\lambda, y_\mu \in SP(\tilde{X})$. Then we have:

(i) $m_d(x_\lambda, y_\mu) = \bar{0}$ then $x_\lambda = y_\mu$ holds by the condition (1) of Definition 5.1.

(ii)$m_d(x_\lambda, y_\mu) = m_d(y_\mu, x_\lambda)$ holds by the condition (2) of Definition 5.1.

(iii) $m_d(x_\lambda, y_\mu) \leq m_d(x_\lambda, z_v) + m_d(z_v, y_\mu)$ for all $x_\lambda, y_\mu, z_v \in SP(\tilde{X})$ holds by the condition (3) of Definition 5.1.

(2a) Suppose that $\{x_{\lambda,n}\}_n$ is a Cauchy sequence in $(\tilde{X}, \tilde{\rho}, E)$. Given $\varepsilon > 0$, take the constant soft real number $\bar{\varepsilon}\tilde{>}\bar{0}$. Then there is a $m \in \mathbb{N}$ such that $\tilde{\rho}(x_{\lambda,i}, x_{\lambda,j})\tilde{<}\bar{\varepsilon}$ for all $i, j \geq m$. Hence $\tilde{\rho}(x_{\lambda,i}, x_{\lambda,j})(\eta) < \varepsilon$ for all $i, j \geq m$. Thus we have $m_d(x_{\lambda,i}, x_{\lambda,j}) < \varepsilon$ for all $i, j \geq m$. This implies that $\{x_{\lambda,n}\}_n$ is a Cauchy sequence in $(SP(\tilde{X}), m_d)$.

Conversely, let $\{x_{\lambda,n}\}_n$ be a Cauchy sequence in $(SP(\tilde{X}), m_d)$. Given $\tilde{\varepsilon}\tilde{>}\bar{0}$, there exists $\varepsilon = \min_{\eta\in E} \tilde{\varepsilon}(\eta) > 0$, because E is finite. Then there is a $m \in \mathbb{N}$ such that $m_d(x_{\lambda,i}, x_{\lambda,j}) < \varepsilon$ for all $i, j \geq m$. Hence $\tilde{\rho}(x_{\lambda,i}, x_{\lambda,j})(\eta) < \varepsilon \leq \tilde{\varepsilon}(\eta)$ for all $\eta \in E$ and $i, j \geq m$. We deduced that $\{x_{\lambda,n}\}_n$ is a Cauchy sequence in $(\tilde{X}, \tilde{\rho}, E)$.

(2b) follows from (2a) and (3) is a consequence of (2a) and (2b). □

As every soft set can be viewed as a union of soft points, soft closedness is equivalent to the closedness in compatible metric keeping in view the previous theorem.

In the next theorem we will prove that having finite set of parameters, we can obtain the same result by compatible dislocated metric on the set of soft points.

Theorem 5.7. *Let F and G be two non-null soft closed subsets of a complete dislocated soft metric space* $(\tilde{X}, \tilde{\rho}, E)$ *with E a finite set of parameters. Suppose that* $T : F\tilde{\cup}G \to F\tilde{\cup}G$ *satisfies the following condition*

$$\tilde{\rho}(Tx_\lambda, Ty_\mu)\tilde{\leq}\bar{k}\tilde{\rho}(x_\lambda, y_\mu),$$

for all $x_\lambda\tilde{\in}F, y_\mu\tilde{\in}G$, *where* $\bar{0}\tilde{\leq}\bar{k}\tilde{<}\bar{1}$. *Then T has a unique fixed point which belongs to* $F\tilde{\cap}G$.

Proof. Let us consider the dislocated metric m_d on $SP(\tilde{X})$ as constructed in the previous theorem. Since $(\tilde{X}, \tilde{\rho}, E)$ is a complete dislocated soft metric space, it follows from the Theorem 5.6(3) that $(SP(\tilde{X}), m_d)$ is a complete dislocated metric space. We can view T as a self mapping on $SP(\tilde{X})$. Also the real number k generating the constant soft real number $\bar{k}$ satisfies $0 \leq k < 1$. Note that for each $x_\lambda, y_\mu \in SP(\tilde{X})$, we have

$$\begin{aligned} m_d(Tx_\lambda, Ty_\mu) &= \max_{\eta\in E} \tilde{\rho}(Tx_\lambda, Ty_\mu)(\eta) \\ &\leq \max_{\eta\in E} \bar{k}\tilde{\rho}(x_\lambda, y_\mu)(\eta) \\ &= k \max_{\eta\in E} \tilde{\rho}(x_\lambda, y_\mu)(\eta) = k m_d(x_\lambda, y_\mu). \end{aligned}$$

Hence T has a unique fixed point by the Theorem 5.2, and it concludes the proof of the theorem. □

6. Soft b-metric spaces and fixed point result

Finally we discuss soft set theory in soft b-metric spaces. The definition of soft b-metric was given by Wadkar et al. in [29] and [31]. They also established some fixed point results in the framework of soft b-metric spaces.

First we recollect the Banach contraction theorem in b-metric spaces

Theorem 6.1. *[17] Let* (X, d) *a complete b-metric space with constant* $s \geq 1$ *such that b-metric is a continuous functional. If* $T : X \to X$ *is a contraction mapping with contraction constant* $k \in [0, 1)$ *satisfying the condition* $ks < 1$, *then T has a unique fixed point.*

We first rephrase the definition of a soft b-metric given in [29] and [31] as follows:

Definition 6.2. *Let $\bar{s}\tilde{\geq}\bar{1}$ be a given constant soft real number. A mapping $\tilde{\rho} : SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}(E)^*$ is said to be a soft b-metric on $\tilde{X}$ if for any soft points x_λ, y_μ, z_η, the following hold*

(1) $\tilde{\rho}(x_\lambda, y_\mu) = \bar{0}$ *then* $x_\lambda = y_\mu$;
(2) $\tilde{\rho}(x_\lambda, y_\mu) = \tilde{\rho}(y_\mu, x_\lambda)$;
(3) $\tilde{\rho}(x_\lambda, y_\mu)\tilde{\leq}\bar{s}\{\tilde{\rho}(x_\lambda, z_\eta) + \tilde{\rho}(z_\eta, y_\mu)\}$.

The soft set $\tilde{X}$ with a soft b-metric $\tilde{\rho}$ on $\tilde{X}$ is said to be a soft b-metric space and is denoted by $(\tilde{X}, \tilde{\rho}, E)$.

The following theorem is a soft version of Banach theorem in soft b-metric space.

Theorem 6.3. *Let $(\tilde{X}, \tilde{\rho}, E)$ a complete soft b-metric space with soft constant $\bar{s}\tilde{\geq}\bar{1}$ and E a finite set of parameters, and $T : (\tilde{X}, \tilde{\rho}, E) \to (\tilde{X}, \tilde{\rho}, E)$ satisfies the following contraction condition*

$$\tilde{\rho}(Tx_\lambda, Ty_\mu) \leq \bar{k}\tilde{\rho}(x_\lambda, y_\mu) \tag{3}$$

where $\bar{0}\tilde{\leq}\bar{k}\tilde{<}\bar{1}$ such that $\bar{k}\bar{s}\tilde{<}\bar{1}$. Then T has a unique fixed point.

In the following example we will see that the condition "E is finite" cannot be omitted in the preceding theorem.

Example 6.4. *Let $X = E = \{1/n : n \in \mathbb{N}\}$. Lets us define $\tilde{\rho}(x_\lambda, y_\mu) = |\bar{x} - \bar{y}|^2 + |\bar{\lambda} - \bar{\mu}|^2$ for $x_\lambda, y_\mu \in SP(\tilde{X})$. It is easy to verify that $(\tilde{X}, \tilde{\rho}, E)$ is a soft b-metric space with soft constant $\bar{s} = \bar{2}$. First we show that $(\tilde{X}, \tilde{\rho}, E)$ complete. Suppose that $\{x_{\lambda,n}\}_n$ is a Cauchy sequence of soft points in $(\tilde{X}, \tilde{\rho}, E)$. Take the soft real number $\widetilde{\varepsilon}$ such that $\widetilde{\varepsilon}(\lambda) = \lambda$ for all $\lambda \in E$, i.e. $\widetilde{\varepsilon}(1/k) = 1/k$ for all $k \in \mathbb{N}$. Then, there is $m \in \mathbb{N}$ such that*

$$\tilde{\rho}(x_{\lambda,i}, x_{\lambda,j})\tilde{<}\widetilde{\varepsilon},$$

for all $i, j \geq m$. We have $\tilde{\rho}(x^{(i)}_\lambda, x^{(j)}_\lambda)(1/k) < \widetilde{\varepsilon}(1/k)$ for all $k \in \mathbb{N}$. Hence

$$(|\bar{x}_i - \bar{x}_j|^2 + |\bar{\lambda}_i - \bar{\lambda}_j|^2)(\frac{1}{k}) < \frac{1}{k},$$

for all $i, j \geq m$ and for all $k \in \mathbb{N}$. Consequently

$$|x_i - x_j|^2 + |\lambda_i - \lambda_j|^2 < \frac{1}{k},$$

for all $i, j \geq m$ and for all $k \in \mathbb{N}$. In particular, for any $j \geq m$,

$$|x_j - x_{j+1}|^2 + |\lambda_j - \lambda_{j+1}|^2 < \frac{1}{k},$$

for all $k \in \mathbb{N}$. Therefore $x_j = x_{j+1}$ and $\lambda_j = \lambda_{j+1}$ for all $j \geq m$. We deduce that $x_j = x_m$ for all $j \geq m$. Thus the sequence $\{x_{\lambda,n}\}_n$ is eventually constant, and hence convergent. We conclude that $(\tilde{X}, \tilde{\rho}, E)$ is complete.

Let $T : (\tilde{X}, \tilde{\rho}, E) \to (\tilde{X}, \tilde{\rho}, E)$ be defined as $T(x_\lambda) = (x/4)_1$ for all $x_\lambda \in SP(\tilde{X})$. Note that for $x_\lambda, y_\mu \in SP(\tilde{X})$ and $\eta \in E$, we have

$$\begin{aligned}
\tilde{\rho}(Tx_\lambda, Ty_\mu)(\eta) &= (|\overline{x/4} - \overline{y/4}|^2)(\eta) \\
&= \frac{1}{4}|x - y|^2 \\
&\leq \frac{1}{4}(|x - y|^2 + |\lambda - \mu|^2) \\
&\leq \frac{\bar{1}}{4}\{\tilde{\rho}(x_\lambda, y_\mu)\}(\eta).
\end{aligned}$$

Hence we have

$$\tilde{\rho}(Tx_\lambda, Ty_\mu)\tilde{\leq}\frac{\bar{1}}{4}\tilde{\rho}(x_\lambda, y_\mu).$$

Also $\bar{s}.\overline{(1/4)}\tilde{<}\bar{1}$. Thus all conditions of the Theorem 6.3 are satisfied but T has no fixed point.

Theorem 6.5. *Let $(\tilde{X}, \tilde{\rho}, E)$ be a soft b-metric space with soft constant $\bar{s}\tilde{\geq}\bar{1}$ and finite set of parameters E. Define a function $m_b : SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}^+$ as*

$$m_b(x_\lambda, y_\mu) = \max_{\eta \in E} \tilde{\rho}(x_\lambda, y_\mu)(\eta),$$

for all $x_\lambda, y_\mu \in SP(\tilde{X})$. Then the following hold:

(1) *m_b is a b-metric on $SP(\tilde{X})$.*

(2) *For any sequence $\{x_{\lambda,n}\}_n$ of soft points and a soft point x_μ, we have*

(2a) *$\{x_{\lambda,n}\}_n$ is a Cauchy sequence in $(\tilde{X}, \tilde{\rho}, E)$ if and only if it is a Cauchy sequence in $(SP(\tilde{X}), m_b)$.*

(2b) *$\lim_{n\to\infty} x_{\lambda,n} = x_\mu$ in soft b-metric space $(\tilde{X}, \tilde{\rho}, E)$ if and only if $\lim_{n\to\infty} x_{\lambda,n} = x_\mu$ in b-metric space $(SP(\tilde{X}), m_b)$.*

(3) *$(\tilde{X}, \tilde{\rho}, E)$ is complete if and only if $(SP(\tilde{X}), m_b)$ is complete.*

Proof. (1) It is easy to see that $(SP(\tilde{X}), m_b)$ a b-metric space with constant $s \geq 1$.

(2a) Suppose that $\{x_{\lambda,n}\}_n$ is a Cauchy sequence in $(\tilde{X}, \tilde{\rho}, E)$. Given $\varepsilon > 0$, take the constant soft real number $\bar{\varepsilon}\tilde{>}\bar{0}$. Then there is a $m \in \mathbb{N}$ such that $\tilde{\rho}(x_{\lambda,i}, x_{\lambda,j})\tilde{<}\bar{\varepsilon}$ for all $i, j \geq m$. Hence $\tilde{\rho}(x_{\lambda,i}, x_{\lambda,j})(\eta) < \varepsilon$, whenever $i, j \geq m$. Thus we have $m_b(x_{\lambda,i}, x_{\lambda,j}) < \varepsilon$ for all $i, j \geq m$. This implies that $\{x_{\lambda,n}\}_n$ is a Cauchy sequence in $(SP(\tilde{X}), m_b)$.

Conversely, suppose that $\{x_{\lambda,n}\}_n$ is a Cauchy sequence in $(SP(\tilde{X}), m_b)$. Given $\tilde{\varepsilon}\tilde{>}\bar{0}$, there exists $\varepsilon = \min_{\eta\in E} \tilde{\varepsilon}(\eta) > 0$, because E is finite. Then there exists a $m \in \mathbb{N}$ such that $m_b(x_{\lambda,i}, x_{\lambda,j}) < \varepsilon$ for all $i, j \geq m$. Hence $\tilde{\rho}(x_{\lambda,i}, x_{\lambda,j})(\eta) < \varepsilon \leq \tilde{\varepsilon}(\eta)$ for all $\eta \in E$ and $i, j \geq m$. We deduced that $\{x_{\lambda,n}\}_n$ is a Cauchy sequence in $(\tilde{X}, \tilde{\rho}, E)$.

(2b) follows from (2a) and (3) is a consequence of (2a) and (2b). □

In the following theorem we show that having finite set of parameters, we can obtain the same result by compatible b-metric on the set of soft points.

Theorem 6.6. *Let $(\tilde{X}, \tilde{\rho}, E)$ a complete soft b-metric space with soft constant $\bar{s}\tilde{\geq}\bar{1}$ and E a finite set of parameters, and $T : (\tilde{X}, \tilde{\rho}, E) \to (\tilde{X}, \tilde{\rho}, E)$ satisfies the following contraction condition*

$$\tilde{\rho}(Tx_\lambda, Ty_\mu) \leq \bar{k}\tilde{\rho}(x_\lambda, y_\mu),$$

where $\bar{0}\tilde{\leq}\bar{k}\tilde{<}\bar{1}$ such that $\bar{k}\bar{s}\tilde{<}\bar{1}$. Then T has a unique fixed point.

Proof. Let us consider the b-metric m_b on $SP(\tilde{X})$ as constructed in the previous theorem. Since $(\tilde{X}, \tilde{\rho}, E)$ is a soft b-complete space, it follows from the Theorem 6.5(3) that $(SP(\tilde{X}), m_b)$ is a complete b-metric space with constant s. We can view T as a self mapping on $SP(\tilde{X})$. Also the real number k generating the constant soft real number $\bar{k}$ satisfies $0 \leq k < 1$ and $ks < 1$. Note that, for each $x_\lambda, y_\mu \in SP(\tilde{X})$ we have

$$\begin{aligned} m_b(Tx_\lambda, Ty_\mu) &= \max_{\eta\in E} \tilde{\rho}(Tx_\lambda, Ty_\mu)(\eta) \\ &\leq \max_{\eta\in E} \bar{k}\tilde{\rho}(x_\lambda, y_\mu)(\eta) \\ &= k \max_{\eta\in E} \tilde{\rho}(x_\lambda, y_\mu)(\eta) = k m_b(x_\lambda, y_\mu). \end{aligned}$$

Hence T has a unique fixed point by the Theorem 6.1. □

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