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Comments on "New Hybrid Conjugate Gradient Method as a Convex Combination of FR and PRP Methods"

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Abstract. In this note, we present a new theory as a modification and an alternative to S.Djordjević's Theorem (2.2), Here we rephrase the text of theory (2.2) by deleting condition (2.16), Notations and equation numbers as in S.Djordjević.

Theorem 2.2 (S.Djordjević's [5]). *Assume that (2.12) and (2.13) hold and let strong Wolfe conditions (1.4)-(1.6) hold with* $\sigma < \frac{1}{2}$. *Also, let* $\{||s_k||\}$ *tend to zero, and let there exist some nonnegative constants* η_1, η_2 *such that*

$$||g_k||^2 \geq \eta_1 ||s_k||^2, \tag{2.15}$$

$$||g_{k+1}||^2 \leq \eta_2 ||s_k||. \tag{2.16}$$

Then d_k^{hyb} *satisfies the sufficient descent condition for all* k.

We suggest a new formula of S.Djordjević's Theorem (2.2).

Theorem 2.2*. *Assume that Assumption (2.12) and (2.13) hold, let strong Wolfe conditions (1.4)-(1.6) held with* $\sigma < \frac{1}{2}$, *and there exists* $\eta_1 > 0$ *such that*

$$||g_k||^2 \geq \eta_1 ||s_k||^2, \; L \leq \eta_1.$$

Then d_k^{hyb} *satisfies the sufficient descent condition for all* k.

Proof. We have $d_0 = -g_0$. So, for $k = 0$, it holds $g_0^T d_0 = -||g_0||^2$.
If $\theta_k = 0$ then

$$d_{k+1}^{hyb} = -g_{k+1} + \beta_k^{PRP} s_k. \tag{0.1}$$

Multiplying (0.1) by g_{k+1}^T, we get

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$$\begin{aligned} g_{k+1}^T d_{k+1}^{hyb} &= -\|g_{k+1}\|^2 + \beta_k^{PRP} g_{k+1}^T s_k \\ &= -\|g_{k+1}\|^2 + \frac{(g_{k+1}^T y_k)(g_{k+1}^T s_k)}{\|g_k\|^2} \\ &\leq -\|g_{k+1}\|^2 + \frac{\|g_{k+1}\|^2\|y_k\|\|s_k\|}{\|g_k\|^2} \\ &\leq -\|g_{k+1}\|^2 + \frac{\|g_{k+1}\|^2 L\|s_k\|^2}{\|g_k\|^2} \\ &\leq -\|g_{k+1}\|^2 + \frac{L}{\eta_1}\|g_{k+1}\|^2. \end{aligned}$$

Hence

$$g_{k+1}^T d_{k+1}^{hyb} \leq -(1 - \frac{L}{\eta_1})\|g_{k+1}\|^2. \tag{0.2}$$

If $\theta_k = 1$ then

$$d_{k+1}^{hyb} = -g_{k+1} + \beta_k^{FR} s_k. \tag{0.3}$$

However, according to the strong Wolfe line search, FR method satisfies the sufficient descent condition [1].
Now, let $0 < \theta_k < 1$
There exist two real numbers μ_1, μ_2 such that $0 < \mu_1 \leq \theta_k \leq \mu_2 < 1$. Then

$$\begin{aligned} g_{k+1}^T d_{k+1}^{hyb} &= \theta_k g_{k+1}^T d_{k+1}^{FR} + (1 - \theta_k) g_{k+1}^T d_{k+1}^{PRP} \\ &\leq \mu_1 g_{k+1}^T d_{k+1}^{FR} + (1 - \mu_2) g_{k+1}^T d_{k+1}^{PRP}. \end{aligned}$$

Hence

$$g_{k+1}^T d_{k+1}^{hyb} \leq -K\|g_{k+1}\|^2. \tag{0.4}$$

Where $K = \mu_1(\frac{1-2\sigma}{1-\sigma}) + (1 - \mu_2)(1 - \frac{L}{\eta_1})$. □

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