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ON A CANONIC ALMOST GEODESIC MAPPINGS OF THE SECOND TYPE OF AFFINE SPACES

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Abstract. In this paper we investigate two kinds of almost geodesic mappings of the second type of affine spaces. Also we find some invariant geometric objects for the canonic almost geodesic mappings.

0. Introduction

Let GA_N be an N-dimensional space with an affine connection L given with the aid of components L^i_{jk} in each local map V on a differentiable manifold. Generally it is $L^i_{jk} \neq L^i_{kj}$.

Generalizing conception of a geodesic mappings for Riemannian and affine spaces Sinyukov introduced [5] following notations:

The curve $l : x^h = x^h(t)$ is called the almost geodesic line if its tangential vector $\lambda^h(t) = dx^h/dt \neq 0$ satisfies the equations

$$\overline{\lambda}^h_{(2)} = \overline{a}(t)\lambda^h + \overline{b}(t)\overline{\lambda}^h_{(1)}, \quad \overline{\lambda}^h_{(1)} = \lambda^h_{||\alpha}\lambda^\alpha, \quad \overline{\lambda}^h_{(2)} = \overline{\lambda}^h_{(1)||\alpha}\lambda^\alpha,$$

where $\overline{a}(t)$ and $\overline{b}(t)$ are functions of a parameter t, and $||$ denotes the covariant derivative with respect to the connection in $\overline{A}_N$.

A mapping f of the affine space A_N onto a space $\overline{A}_N$ is called the almost geodesic mapping if any geodesic line of the space A_N turns into almost geodesic line of the space $\overline{A}_N$.

Sinjukov [5] singled out the three types of the almost geodesic mappings, π_1, π_2, π_3 for spaces without torsion. In the present work we investigate the

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mappings of the type π_2 for spaces with torsion. In a differentiable manifold with nonsymmetric affine connection L^i_{jk}, for a vector exist two kinds of covariant derivative:

$$\lambda^h_{\underset{1}{|}m} = \lambda^h_{,m} + L^h_{pm}\lambda^p, \quad \lambda^h_{\underset{2}{|}m} = \lambda^h_{,m} + L^h_{mp}\lambda^p.$$

Thus, in the case of the space with nonsymmetric affine connection we can define two kinds of almost geodesic lines and two kinds of almost geodesic mappings.

1. Almost geodesic mappings of affine spaces

In an affine spaces GA_N (with nonsymmetric affine connection L) one can define four kinds of a covariant derivative [1,2]. Let us denote by $\underset{\theta}{|}, \underset{\theta}{||}$ a covariant derivative of the kind θ in GA_N and $G\overline{A}_N$ respectively. A curve $l : x^h = x^h(t)$ is called *almost geodesic line of the first kind* if its tangential vector $\lambda^h(t) = dx^h/dt \neq 0$ satisfies the equations

$$(1.1) \qquad \underset{1}{\overline{\lambda}}{}^h_{(2)} = \underset{1}{\overline{a}}(t)\lambda^h + \underset{1}{\overline{b}}(t)\underset{1}{\overline{\lambda}}{}^h_{(1)}, \quad \underset{1}{\overline{\lambda}}{}^h_{(1)} = \lambda^h_{\underset{1}{||}\alpha}\lambda^\alpha, \quad \underset{1}{\overline{\lambda}}{}^h_{(2)} = \underset{1}{\overline{\lambda}}{}^h_{(1)\underset{1}{||}\alpha}\lambda^\alpha,$$

where $\underset{1}{\overline{a}}(t)$ and $\underset{1}{\overline{b}}(t)$ are functions of a parameter t. A curve $l : x^h = x^h(t)$ is called *almost geodesic line of the second kind* if its tangential vector $\lambda^h(t) = dx^h/dt \neq 0$ satisfies the equations

$$(1.2) \qquad \underset{2}{\overline{\lambda}}{}^h_{(2)} = \underset{2}{\overline{a}}(t)\lambda^h + \underset{2}{\overline{b}}(t)\underset{2}{\overline{\lambda}}{}^h_{(1)}, \quad \underset{2}{\overline{\lambda}}{}^h_{(1)} = \lambda^h_{\underset{2}{||}\alpha}\lambda^\alpha, \quad \underset{2}{\overline{\lambda}}{}^h_{(2)} = \underset{2}{\overline{\lambda}}{}^h_{(1)\underset{2}{||}\alpha}\lambda^\alpha,$$

where $\underset{2}{\overline{a}}(t)$ and $\underset{2}{\overline{b}}(t)$ are functions of a parameter t.

A mapping f of the space GA_N onto a space with nonsymmetric affine connection $G\overline{A}_N$ is called *almost geodesic mapping of the first kind* if any geodesic line of the space GA_N turns into almost geodesic line of the first kind of the space $G\overline{A}_N$. A mapping f is called *almost geodesic mapping of the second kind* if any geodesic line of the space GA_N turns into almost geodesic line of the second kind of the space $G\overline{A}_N$.

We can put

$$(1.3) \qquad \overline{L}^h_{ij}(x) = L^h_{ij}(x) + P^h_{ij}(x),$$

where $L^h_{ij}(x)$, $\overline{L}^h_{ij}(x)$ are connection coefficients of the space GA_N and $G\overline{A}_N$, $(N > 2)$, and $P^h_{ij}(x)$ is a deformation tensor. From [7] it follows that the following results hold:

Theorem 1.1. *The mapping f of the space GA_N onto $G\overline{A}_N$ is the almost geodesic mapping of the first kind if and only if the deformation tensor $P^h_{ij}(x)$ satisfies the conditions*

$$(1.4)\qquad (P^h_{\alpha\beta\underset{1}{|}\gamma} + P^h_{\delta\alpha}P^\delta_{\beta\gamma})\lambda^\alpha\lambda^\beta\lambda^\gamma = \underset{1}{b}P^h_{\alpha\beta}\lambda^\alpha\lambda^\beta + \underset{1}{a}\lambda^h$$

identically, where $\underset{1}{a}$ and $\underset{1}{b}$ are invariants.

Theorem 1.2. *The mapping f of the space GA_N onto $G\overline{A}_N$ is the almost geodesic mapping of the second kind if and only if the deformation tensor $P^h_{ij}(x)$ satisfies the conditions*

$$(1.4\,')\qquad (P^h_{\alpha\beta\underset{2}{|}\gamma} + P^h_{\alpha\delta}P^\delta_{\beta\gamma})\lambda^\alpha\lambda^\beta\lambda^\gamma = \underset{2}{b}P^h_{\alpha\beta}\lambda^\alpha\lambda^\beta + \underset{2}{a}\lambda^h$$

identically, where $\underset{1}{a}$ and $\underset{1}{b}$ are invariant.

2. Almost geodesic mappings of the second type

In [5] Sinyukov introduced almost geodesic mapping of the second type π_2 for affine spaces without torsion by condition:

$$b = \frac{b_{\gamma\delta}\lambda^\gamma\lambda^\delta}{\sigma_\alpha\lambda^\alpha}.$$

Analogously, almost geodesic mappings of the first kind of affine space is the second type $\underset{1}{\pi}_2$ if for the function $\underset{1}{b}$ satisfied the following condition:

$$\underset{1}{b} = \frac{\underset{1}{b}_{\gamma\delta}\lambda^\gamma\lambda^\delta}{\sigma_\alpha\lambda^\alpha},$$

where $\sigma_\alpha\lambda^\alpha \neq 0$.

Let there be

$$(2.1)\qquad P^h_{\alpha\beta}\lambda^\alpha\lambda^\beta = 2\sigma_\alpha\lambda^\alpha F^h_\beta\lambda^\beta + 2\psi_\alpha\lambda^\alpha\lambda^h.$$

Then

$$(P^h_{\alpha\beta} - 2\sigma_\alpha F^h_\beta - 2\psi_\alpha\delta^h_\beta)\lambda^\alpha\lambda^\beta \equiv 0,$$

wherefrom

$$(2.2)\qquad P^h_{\underline{ij}} = \psi_i\delta^h_j + \psi_j\delta^h_i + \sigma_i F^h_j + \sigma_j F^h_i.$$

We can put $P^h_{\underset{\vee}{ij}} = \xi^h_{ij}$. In this paper $\underline{ij}$ denotes symmetrization with division and $\underset{\vee}{ij}$ denotes antisymmetrization with division. Then

$$(2.3)\quad P^h_{ij}(x) = \psi_i(x)\,\delta^h_j + \psi_j(x)\,\delta^h_i + \sigma_i(x)F^h_j(x) + \sigma_j(x)F^h_i(x) + \xi^h_{ij}(x)\,.$$

In (2.3) ψ_i, σ_i are vectors, F^h_i is a tensor and ξ^h_{ij} is an antisymmetric tensor.

Substituting (2.3) in (1.4) we obtain:

$$(2.4)\quad \begin{aligned} &F^h_{i\underset{1}{|}j} + F^h_{j\underset{1}{|}i} + F^h_\delta F^\delta_i \sigma_j + F^h_\delta F^\delta_j \sigma_i + \xi^h_{\delta i} F^\delta_j + \xi^h_{\delta j} F^\delta_i \\ &\qquad = \mu_i F^h_j + \mu_j F^h_i + \nu_i \delta^h_j + \nu_j \delta^h_i\,, \end{aligned}$$

where μ_i, ν_i are covariant vectors.

Conditions (2.3) and (2.4) are *the basic equations* of the mapping $\underset{1}{\pi}_2$.

Almost geodesic mapping of the second kind is the second type $\underset{2}{\pi}_2$ if satisfied the following condition for the function $\underset{2}{b}$ in (1.4′):

$$\underset{2}{b} = \frac{\underset{2}{b}_{\gamma\delta}\lambda^\gamma\lambda^\delta}{\sigma_\alpha\lambda^\alpha}$$

where $\sigma_\alpha\lambda^\alpha \neq 0$. Using the methods from the previous case we get

$$(2.4\,')\quad \begin{aligned} &F^h_{i\underset{2}{|}j} + F^h_{j\underset{2}{|}i} + F^h_\delta F^\delta_i \sigma_j + F^h_\delta F^\delta_j \sigma_i + \xi^h_{i\delta} F^\delta_j + \xi^h_{j\delta} F^\delta_i \\ &\qquad = \mu_i F^h_j + \mu_j F^h_i + \nu_i \delta^h_j + \nu_j \delta^h_i\,, \end{aligned}$$

where μ_i, ν_i are covariant vectors.

Conditions (2.3) and (2.4 ′) are *the basic equations* of almost geodesic mappings of $\underset{2}{\pi}_2$.

3. The property of reciprocity of almost geodesic mappings of the second type

The mapping $\underset{1}{\pi}_2$ has *the property of reciprocity*, if its inverse mapping $\underset{1}{\pi}_2^{-1}$ is of $\underset{1}{\pi}_2$ type, and $\underset{1}{\pi}_2^{-1}$ corresponds to the same affinor F^h_i also. Since the inverse mapping $\underset{1}{\pi}_2^{-1} : G\overline{A}_N \to GA_N$ satisfies

$$\overline{P}^h_{ij} = -P^h_{ij},$$

we can put in (2.3) the following:

$$\overline{\psi}_i = -\psi_i, \quad \overline{\sigma}_i = -\sigma_i, \quad \overline{F}_i^h = F_i^h, \quad \overline{\xi}_{ij}^h = -\xi_{ij}^h.$$

The mapping $\underset{1}{\pi}_2$ has the property of reciprocity if and only if the affinor F_i^h of the space $G\overline{A}_N$ satisfies the equation of the form (2.4), i.e.

$$F^h_{i\underset{1}{||}j} - F^h_\alpha F^\alpha_{(i}\sigma_{j)} - \xi^h_{\alpha(i}F^\alpha_{j)} = \overline{\mu}_{(i}F^h_{j)} + \overline{\nu}_{(i}\delta^h_{j)}, \tag{3.1}$$

where (ij) is a symmetrization without division with respect to i and j, and $\underset{1}{||}$ is a covariant derivative of the first kind in $G\overline{A}_N$. Crossing in (3.1) to a covariant derivative of the first kind in GA_N, we get

$$F^h_\alpha F^\alpha_{(i}\sigma_{j)} + \xi^h_{\alpha(i}F^\alpha_{j)} = \overline{\overline{\mu}}_{(i}F^h_{j)} + \overline{\overline{\nu}}_{(i}\delta^h_{j)}, \tag{3.2}$$

where the vectors $\overline{\overline{\mu}}_i$, $\overline{\overline{\nu}}_i$ are expressed by μ_i, ν_i, $\overline{\mu}_i$, $\overline{\nu}_i$, ψ_i, σ_i, F_i^h. Since $\sigma \neq 0$, we get

$$F^h_\alpha F^\alpha_i = p\delta^h_i + qF^h_i, \tag{3.3}$$

where p i q are invariants.

On the base of the facts given above, we have:

Theorem 3.1. *Relation* (3.3) *expressed necessary and sufficient condition that a mapping* $\underset{1}{\pi}_2 : GA_N \to G\overline{A}_N$ *has the property of reciprocity.*

The equations (2.3) and (2.4) are invariant to the mapping of the affinor:

$$\tilde{F}^h_i = rF^h_i + s\delta^h_i \qquad (r \neq 0). \tag{3.4}$$

Then we have

$$\tilde{F}^h_\alpha \tilde{F}^\alpha_i = \tilde{p}\delta^h_i + \tilde{q}\tilde{F}^h_i, \tag{3.5}$$

where are

$$\tilde{p} = r^2p - s^2 - srq, \qquad \tilde{q} = 2s + rq.$$

Here we can select invariants r and s such that

$$\tilde{q} \equiv 0, \qquad \tilde{p} = \tilde{e}\ (= \pm 1, 0).$$

In this case we have

$$\tilde{F}^h_\alpha \tilde{F}^\alpha_i = \tilde{e}\delta^h_i . \tag{3.6}$$

On the base of the facts given above we can put

$$F^h_\alpha F^\alpha_i = e\delta^h_i \qquad (e = \pm 1, 0) \tag{3.7}$$

Using (3.7) in the condition (2.4) we get:

$$F^h_{(i\underset{1}{|}j)} + \xi^h_{\alpha(i} F^\alpha_{j)} = \mu_{(i} F^h_{j)} + \nu_{(i}\delta^h_{j)} \tag{3.8}$$

Hence, the almost geodesic mapping $\underset{1}{\pi}2 : GA_N \to G\overline{A}_N$ which satisfies the property of reciprocity is characterized by equations (2.3), (3.7) and (3.8). This mapping is denoted

$$\underset{1}{\pi}2(e) : GA_N \to G\overline{A}_N .$$

The case of the mappings $\underset{2}{\pi}2$ can be investigated analogously.

4. Canonic almost geodesic mappings

The mapping $\underset{1}{\pi}2(e) : GA_N \to G\tilde{A}_N$ is *canonic* if in by mapping common coordinate system between connection coefficients L^h_{ij} and $\tilde{L}^h_{ij}$ satisfies the relation

$$\tilde{L}^h_{ij} = L^h_{ij} + \sigma_i F^h_j + \sigma_j F^h_i + \xi^h_{ij}, \tag{4.1}$$

where ξ^h_{ij} is an antisymmetric tensor, σ_i a vector, F^h_i is an affinor, and F^h_i satisfies the conditions (3.7) and (3.8).

Theorem 4.1. *Every mapping $\underset{1}{\pi}2$ is canonic or it can be present in form of the product of a geodesic and a canonic mapping.*

Proof. If $\psi_i \equiv 0$ then this mapping is canonic. Let $\psi_i \neq 0$. Suppose that for the mapping $f : GA_N \to G\overline{A}_N$ satisfies

$$\overline{L}^h_{ij} = L^h_{ij} + \sigma_{(i} F^h_{j)} + \overline{\xi}^h_{ij},$$

and for mapping $g : G\overline{A}_N \to G\tilde{A}_N$ satisfies

$$\tilde{L}^h_{ij} = \overline{L}^h_{ij} + \psi_{(i}\delta^h_{j)} + \tilde{\xi}^h_{ij}.$$

Then for the product of the mappings f and g we have

$$\tilde{L}^h_{ij} = L^h_{ij} + \psi_{(i}\delta^h_{j)} + \sigma_{(i}F^h_{j)} + \bar{\xi}^h_{ij} + \tilde{\xi}^h_{ij}.$$

i.e. the mapping $\underset{1}{\pi}2$ is presented as the product of the geodesic f and the canonic mapping g. □

Theorem 4.2. *Geometric objects of the space GA_N*

$$(4.2)\quad \underset{1}{T}^h_{ij} = L^h_{\underline{ij}} + \frac{1}{e - F^2}\left[(FL^\alpha_{\underline{\alpha j}} - F^\alpha_j L^\beta_{\underline{\beta\alpha}})F^h_i + (FL^\alpha_{\underline{\alpha i}} - F^\alpha_i L^\beta_{\underline{\beta\alpha}})F^h_j\right],$$

are invariant of the canonic mapping $\underset{1}{\pi}2(e)$ $(e = \pm 1)$, where F^h_i is an affinor, such that $F = F^\alpha_\alpha$, $e - F^2 \neq 0$ and $\underline{ij}$ is a symmetrization with division.

Proof. Let us conctract in (4.1) with respect to h, i:

$$(4.3)\qquad \tilde{L}^\alpha_{\alpha j} = L^\alpha_{\alpha j} + F\tilde{\sigma}_j + \xi^\alpha_{\alpha j}$$

where

$$(4.4)\qquad \tilde{\sigma}_i = \sigma_\alpha F^\alpha_i, \qquad F = F^\alpha_\alpha.$$

From (3.7) and (4.4) we have

$$(4.5)\qquad F^\beta_j(\tilde{L}^\alpha_{\underline{\alpha\beta}} - L^\alpha_{\underline{\alpha\beta}}) = e\sigma_j + F\tilde{\sigma}_j.$$

Substituting (4.3) in (4.5) we get

$$(4.6)\qquad \sigma_j = \frac{1}{e - F^2}\left[F^\beta_j(\tilde{L}^\alpha_{\underline{\alpha\beta}} - L^\alpha_{\underline{\alpha\beta}}) - F(\tilde{L}^\alpha_{\underline{\alpha j}} - L^\alpha_{\underline{\alpha j}})\right].$$

From (4.6) and (4.1) we have

$$\tilde{L}^h_{\underline{ij}} + \frac{1}{e - F^2}\left[(F\tilde{L}^\alpha_{\underline{\alpha j}} - F^\alpha_j \tilde{L}^\beta_{\underline{\beta\alpha}})F^h_i + (F\tilde{L}^\alpha_{\underline{\alpha i}} - F^\alpha_i \tilde{L}^\beta_{\underline{\beta\alpha}})F^h_j\right]$$
$$= L^h_{\underline{ij}} + \frac{1}{e - F^2}\left[(FL^\alpha_{\underline{\alpha j}} - F^\alpha_j L^\beta_{\underline{\beta\alpha}})F^h_i + (FL^\alpha_{\underline{\alpha i}} - F^\alpha_i L^\beta_{\underline{\beta\alpha}})F^h_j\right],$$

i.e.

$$(4.7)\qquad \underset{1}{\tilde{T}}^h_{ij}(x) = \underset{1}{T}^h_{ij}(x).$$

Theorem is proved. □

From (4.1) and (3.7) we get

$$F^h_{i\underset{1}{\wr}j} = F^h_{i\underset{1}{|}j} + \tilde{\sigma}_i F^h_j - e\sigma_i\delta^h_j + \xi^h_{\alpha j}F^\alpha_i - \xi^\alpha_{ij}F^h_\alpha, \tag{4.8}$$

where $\underset{1}{\wr}$ is a covariant derivative of the first kind in a space $G\tilde{A}_N$. From here we get

$$F^h_\alpha(F^\alpha_{i\underset{1}{\wr}j} - F^\alpha_{i\underset{1}{|}j}) = e(\tilde{\sigma}_i\delta^h_j - \sigma_i F^h_j) + \xi^\alpha_{\beta j}F^\beta_i F^h_\alpha - e\xi^h_{ij}, \tag{4.9}$$

i.e.

$$\sigma_i F^h_j + \sigma_j F^h_i = eF^h_\alpha(F^\alpha_{(i\underset{1}{|}j)} - F^\alpha_{(i\underset{1}{\wr}j)}) + \tilde{\sigma}_{(i}\delta^h_{j)} + e\xi^\alpha_{\beta(j}F^\beta_{i)}F^h_\alpha \tag{4.10}$$

Substituting (4.10) in (4.1) we get

$$\begin{aligned} &\tilde{L}^h_{\underline{ij}} + eF^h_\alpha F^\alpha_{(i\underset{1}{\wr}j)} - e\tilde{L}^\alpha_{\underset{\vee}{\beta(j}}F^\beta_{i)}F^h_\alpha \\ &= L^h_{\underline{ij}} + eF^h_\alpha F^\alpha_{(i\underset{1}{|}j)} - eL^\alpha_{\underset{\vee}{\beta(j}}F^\beta_{i)}F^h_\alpha + \tilde{\sigma}_{(i}\delta^h_{j)}\,. \end{aligned} \tag{4.11}$$

We can put the following relations in (4.11):

$$\begin{aligned} \hat{\tilde{L}}^h_{ij} &= \tilde{L}^h_{\underline{ij}} + eF^h_\alpha F^\alpha_{(i\underset{1}{\wr}j)} - e\tilde{L}^\alpha_{\underset{\vee}{\beta(j}}F^\beta_{i)}F^h_\alpha, \\ \hat{L}^h_{ij} &= L^h_{\underline{ij}} + eF^h_\alpha F^\alpha_{(i\underset{1}{|}j)} - eL^\alpha_{\underset{\vee}{\beta(j}}F^\beta_{i)}F^h_\alpha. \end{aligned} \tag{4.12}$$

Then

$$\hat{\tilde{L}}^h_{ij} = \hat{L}^h_{ij} + \tilde{\sigma}_{(i}\delta^h_{j)}\,. \tag{4.13}$$

We can see that the magnitudes $\hat{\tilde{L}}^h_{ij}$ and $\hat{L}^h_{ij}$ are connection objects of the spaces $\hat{\tilde{A}}_N$ and $\hat{A}_N$ without torsion [5]. From the condition (4.13) we can conclude that the space $\hat{A}_N$ geodesic maps to the space $\hat{\tilde{A}}_N$. Then projective parameters of this spaces are equal,[3] i.e.

$$\underset{2}{\hat{\tilde{T}}}{}^h_{ij}(x) = \underset{2}{\hat{T}}{}^h_{ij}(x). \tag{4.14}$$

They order geometric objects of the spaces GA_N and $G\overline{A}_N$, which are invariant to the canonic almost geodesic mappings. Then we have

$$\begin{aligned}\hat{\underset{2}{T}}{}^h_{ij}(x) &= \hat{L}^h_{\underline{ij}} - \frac{1}{1+N}(\hat{L}^\alpha_{\underline{i\alpha}}\delta^h_j + \hat{L}^\alpha_{\underline{j\alpha}}\delta^h_i) \\ &= L^h_{\underline{ij}} + eF^h_\alpha F^\alpha_{i\underset{1}{|}j} - eL^\alpha_{\beta\underset{\vee}{(j}}F^\beta_{i)}F^h_\alpha \\ &- \frac{1}{1+N}\big[(L^\alpha_{\underline{i\alpha}} + eF^\beta_\alpha F^\alpha_{\beta\underset{1}{|}j} - eL^\alpha_{\beta\underset{\vee}{(\gamma}}F^\beta_{i)}F^\gamma_\alpha)\delta^h_j \\ &+ (L^\alpha_{\underline{j\alpha}} + eF^\beta_\alpha F^\alpha_{\beta\underset{1}{|}i} - eL^\alpha_{\beta\underset{\vee}{(\gamma}}F^\beta_{j)}F^\gamma_\alpha)\delta^h_i\big] \\ &= T^h_{ij} + eF^h_\alpha(F^\alpha_{i\underset{1}{|}j} - L^\alpha_{\beta\underset{\vee}{(j}}F^\beta_{i)} \\ &- \frac{e}{1+N}F^\beta_\alpha\big[F^\alpha_{\beta\underset{1}{|}j} - L^\alpha_{\gamma\underset{\vee}{(\beta}}F^\gamma_{i)})\delta^h_j + (F^\alpha_{\beta\underset{1}{|}i} - L^\alpha_{\gamma\underset{\vee}{(\beta}}F^\gamma_{j)})\delta^h_i\big],\end{aligned}$$

i.e.

$$\begin{aligned}\hat{\underset{2}{T}}{}^h_{ij}(x) &= T^h_{ij} + eF^h_\alpha(F^\alpha_{i\underset{1}{|}j} - L^\alpha_{\beta\underset{\vee}{(j}}F^\beta_{i)}) \\ &- \frac{e}{1+N}F^\beta_\alpha\big[F^\alpha_{\beta\underset{1}{|}j} - L^\alpha_{\gamma\underset{\vee}{(\beta}}F^\gamma_{i)})\delta^h_j + (F^\alpha_{\beta\underset{1}{|}i} - L^\alpha_{\gamma\underset{\vee}{(\beta}}F^\gamma_{j)})\delta^h_i\big],\end{aligned} \tag{4.15}$$

where T^h_{ij} are objects of the projective connection [3]. On the base of the facts given above, we have

Theorem 4.3. *Geometric objects of the space GA_N (4.15) are invariant of the canonic mapping $\underset{1}{\pi}_2(e)$ $(e = \pm 1)$ where F^h_i is an affinor, $F = F^\alpha_\alpha$ and $\underset{\vee}{ij}$ is an antisymmetrization with division.*

Remark. From (4.13) it follows that the space $\hat{A}_N$ geodesic maps to the space $\hat{\tilde{A}}_N$. Also from (4.12) we can see that the spaces $\hat{A}_N$ and $\hat{\tilde{A}}_N$ are with symmetric affine connection.

Now, for example the Weil's tensor [4,6] is an invariant of this mapping, i.e.

$$\hat{\tilde{\underset{2}{W}}}{}^i{}_{jmn}(x) = \hat{\underset{2}{W}}{}^i{}_{jmn}(x) \tag{4.16}$$

where

$$\begin{aligned}\hat{\underset{2}{W}}{}^i{}_{jmn} = \hat{R}^i{}_{jmn} + \frac{1}{1+N}\delta^i_j\,\hat{R}_{[mn]} + \frac{1}{N^2-1}\big[(N\hat{R}_{jn} + \hat{R}_{nj}) \\ -(N\hat{R}_{jm} + \hat{R}_{mj})\big],\end{aligned} \tag{4.17}$$

$\hat{R}^i{}_{jmn}$ is a curvature tensor and $\hat{R}_{jm}$ is a Ricci tensor of the space $\hat{A}_N$.

References

[1] S.M. Minčić, *Ricci identities in the space of non-symmetric affine connexion*, Mat. Vesnik **10**(25)Sv.2 (1973), 161-172.

[2] S.M. Minčić, *New commutation formulas in the non-symetric affine connexion space*, Publ. Inst. Math. (Beograd) (N.S) **22**(36) (1977), 189-199.

[3] S.M. Minčić, M.S. Stanković, *On geodesic mappings of general affine connexion spaces and of generalized Riemannian spaces*, Mat. Vesnik **49** (1997), 27-33.

[4] S.M. Minčić, M.S. Stanković, *Equitorsion geodesic mapings of generalized Riemannian spaces*, Publ. Inst. Math. (Beograd) (N.S) **61 (75)** (1997), 97-104.

[5] Н.С. Синюков, *Геодезические отображеня Римановых пространстe*, "Наука" Москва, 1979.

[6] M.S. Stanković, S.M. Minčić, *New special geodesic mappings of generalized Riemannian space*, Publ. Inst. Math. (Beograd) (to appear).

[7] M.S. Stanković, *Almost geodesic mappings of the first type of affine spaces*, Jour. of Math. (Novi Sad) (to appear).

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