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Bounds of Coefficients for Classes of Analytic Functions Related to Hypergeometric Functions

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Abstract. The main purpose of the present paper is to give some sharp coefficients bounds for a certain class of univalent analytic functions in unit open disk, which was defined by using principle of differential subordination and generalized hypergeometric function. As applications, we investigate the almost starlike-type functions, parabolic starlike-type functions and uniformly convex-type functions with conic domain. Our results extend some earlier works related to Ma-Minda starlike and convex functions.

1. Introduction

Let $\mathcal{A}$ be the family of all analytic functions from $\mathbb{U} := \{z \in \mathbb{C} : |z| < 1\}$ into $\mathbb{C}$ and $\mathcal{S}$ be the subclass of $\mathcal{A}$ consisting of functions which are univalent in $\mathbb{U}$. If f and g are analytic in $\mathbb{U}$, we say that f is subordinate to g, written $f(z) \prec g(z)$, provided there exists a analytic function $\omega(z)$ defined on $\mathbb{U}$ with $\omega(0) = 0$ and $|\omega(z)| < 1$ satisfying $f(z) = g(\omega(z))$. Let h be an analytic univalent function with positive real part and normalized by $h(0) = 1, h'(0) > 0$ and h maps $\mathbb{U}$ on to domains symmetric with respect to the real axis and starlike with respect to 1. With this h, Ma-Minda [16] introduced the following Ma-Minda starlike and convex classes:

$$f \in \mathcal{S}^*(h) \Longleftrightarrow f \in \mathcal{A},\ \frac{zf'(z)}{f(z)} \prec h(z),\ z \in \mathbb{U} \tag{1}$$

$$f \in \mathcal{K}(h) \Longleftrightarrow f \in \mathcal{A},\ 1 + \frac{zf''(z)}{f'(z)} \prec h(z),\ z \in \mathbb{U}, \tag{2}$$

which envelope kinds of subclasses as special cases (see, e.g. [5, 6, 8, 9, 21, 22, 24, 25, 27]). If $f_i = z + \sum_{n=2}^{\infty} a_{n,i} z^n \in \mathcal{A}$ $(i = 1, 2)$, then the Hadamard product (or convolution) of f_1 and f_2 is defined by

$$(f_1 * f_2)(z) = z + \sum_{n=2}^{\infty} a_{n,1} a_{n,2} z^n,\ z \in \mathbb{U}.$$

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Let $(a)_k$ be the Pochhammer symbol given as

$$(a)_k = \frac{\Gamma(a+k)}{\Gamma(k)} = \begin{cases} 1, & k = 0, \\ a(a+1)...(a+k-1), & k \in \{1, 2, ...\}. \end{cases}$$

For $\{\alpha_i\}_{i=1}^{i=q}, \{\beta_j\}_{j=1}^{j=s} \in \mathbb{C} - \{0, -1, -2, -3, ...\}$, define the generalized hypergeometric function ${}_qF_s(z)$ by

$${}_qF_s(z) =_q F_s(\alpha_1, \alpha_2, ..., \alpha_q; \beta_1, \beta_2, ..., \beta_s; z) = \sum_{n=0}^{\infty} \frac{(\alpha_1)_n...(\alpha_q)_n}{(\beta_1)_n...(\beta_s)_n} \frac{z^n}{n!},$$

where $q \leqslant s+1$, $q, s \in \{0, 1, 2, ...\}, z \in \mathbb{U}$ (see, e.g. [14, 15, 18, 26]).
Assume that $f(z) = z + \sum_{n=2}^{\infty} a_n z^n \in \mathcal{A}$. Dziok-Srivastava [3] introduced the following linear operator $\mathcal{H}(\alpha_1, ...\alpha_q; \beta_1, ..., \beta_s) : \mathcal{A} \to \mathcal{A}$ by

$$\mathcal{H}(\alpha_1, ...\alpha_q; \beta_1, ..., \beta_s) f(z) = z_q F_s(\alpha_1, ..., \alpha_q; \beta_1, ..., \beta_s; z) * f(z) = z + \sum_{n=2}^{\infty} \frac{(\alpha_1)_{n-1}...(\alpha_q)_{n-1}}{(\beta_1)_{n-1}...(\beta_q)_{n-1}} \frac{a_n z^n}{(n-1)!}.$$

In literature, the above operator is called Dziok-Srivastava operator.

The following definition associated with the class $\mathcal{P}_{\alpha_1,...\alpha_q;\beta_1,...,\beta_s}(h, \lambda)$ of analytic functions was given by Xu-Xiao-Srivastava [28].

Definition 1.1 (Xu-Xiao-Srivastava, [28]). *Let h satisfies the conditions as in* (1)-(2) *and* $0 \leqslant \lambda \leqslant 1$. *Then*

$$f \in \mathcal{P}_{\alpha_1,...\alpha_q;\beta_1,...,\beta_s}(h, \lambda) \Longleftrightarrow \frac{z(\mathbb{H}f)'(z) + \lambda z^2 (\mathbb{H}f)''(z)}{(1-\lambda)(\mathbb{H}f)(z) + \lambda z (\mathbb{H}f)'(z)} \prec h(z), \tag{3}$$

where $f \in \mathcal{A}$, $\mathbb{H} = \mathcal{H}(\alpha_1, ...\alpha_q; \beta_1, ..., \beta_s)$ and $z \in \mathbb{U}$.

In [28], Xu-Xiao-Srivastava obtained some inclusion relationships and convolution results on $\mathcal{P}_{\alpha_1,...\alpha_q;\beta_1,...,\beta_s}(h, \lambda)$. In particular, we can note that some examples with Definition 1.1.

Example 1.2. *If $\lambda = 0$, $h(z) = \left(\frac{1+z}{1-z}\right)^{\beta}$ in Definition 1.1, where $0 < \beta \leqslant 1$, then*

$$\mathcal{SP}(\alpha_1, ..., \alpha_q; \beta_1, ..., \beta_s; \beta) = \left\{ f \in \mathcal{A} : \left| \arg \frac{z(\mathbb{H}f)'(z)}{(\mathbb{H}f)(z)} \right| < \frac{\beta}{2}\pi \right\}$$

is class of strongly starlike-type functions of order β (see, Orhan-Răducanu [18]).

Example 1.3. *If $\lambda = 0$, $h(z) = 1 + \frac{2}{\pi^2}\left(\log \frac{1+\sqrt{z}}{1-\sqrt{z}}\right)^2$ in Definition 1.1, then*

$$\mathcal{P}_p(\alpha_1, ..., \alpha_q; \beta_1, ..., \beta_s) = \left\{ f \in \mathcal{A} : \Re\left(\frac{z(\mathbb{H}f)'(z)}{(\mathbb{H}f)(z)}\right) > \left| \frac{z(\mathbb{H}f)'(z)}{(\mathbb{H}f)(z)} - 1 \right| \right\}$$

is class of parabolic starlike-type functions (see, Deniz-Budak [6]).

Example 1.4. *If $\lambda = 1$, $h(z) = 1 + \frac{2}{\pi^2}\left(\log \frac{1+\sqrt{z}}{1-\sqrt{z}}\right)^2$ in Definition 1.1, then*

$$\mathcal{CP}(\alpha_1, ..., \alpha_q; \beta_1, ..., \beta_s) = \left\{ f \in \mathcal{A} : \Re\left(1 + \frac{z(\mathbb{H}f)''(z)}{(\mathbb{H}f)'(z)}\right) > \left| \frac{z(\mathbb{H}f)''(z)}{(\mathbb{H}f)'(z)} \right| \right\}$$

is class of uniformly convex-type functions (see, Kanas-Wiśniowska [11]).

Example 1.5. *If $\lambda = 0$, $h(z) = \frac{1+z}{1+(2\gamma-1)z}$ in Definition 1.1, where $\gamma \in [0,1)$, then*

$$\mathcal{AP}_\gamma(\alpha_1, ..., \alpha_q; \beta_1, ..., \beta_s) = \left\{ f \in \mathcal{A} : \Re\Big(\frac{(\mathbb{H}f)(z)}{z(\mathbb{H}f)'(z)}\Big) > \gamma \right\}$$

is class of almost starlike-type functions of order γ.

Let $f(z) = z + \sum_{n=1}^{\infty} a_n z^n \in \mathcal{A}$, $z \in \mathbb{U}$. Estimating for the upper bound of $|a_3 - \mu a_2^2|$ is known as the Fekete-Szegö problem. Until now, there are several results related to this problem (see, e.g. [1, 2, 4, 7, 10, 12, 13, 17, 20, 23]).

In this paper, we give the sharp bounds for the first two coefficients of the class $\mathcal{P}_{\alpha_1,...\alpha_q;\beta_1,...,\beta_s}(h, \lambda)$, and the Fekete-Szegö problem is solved when $\mu \in \mathbb{R}$. Some applications associated with the classes $\mathcal{SP}(\alpha_1, ..., \alpha_q; \beta_1, ..., \beta_s; \beta), \mathcal{P}_p(\alpha_1, ..., \alpha_q; \beta_1, ..., \beta_s), \mathcal{CP}(\alpha_1, ..., \alpha_q; \beta_1, ..., \beta_s)$ and $\mathcal{AP}_\gamma(\alpha_1, ..., \alpha_q; \beta_1, ..., \beta_s)$ were showed.

2. Preliminaries

The class $\mathcal{P}$ of functions consists of all analytic functions p with $\Re(p) > 0$ for $z \in \mathbb{U}$. For this class, we need the following lemma.

Lemma 2.1 (Pommerenke, [19]). *Suppose that $p(z) = 1 + c_1 z + c_2 z^2 + \cdots \in \mathcal{P}$, then $|c_n| \leqslant 2$ for $n \geqslant 1$. If $|c_1| = 2$ then $p(z) \equiv p_1(z) = \frac{1+\gamma_1 z}{1-\gamma_1 z}$ with $\gamma_1 = c_1/2$. Conversely, if $p(z) \equiv p_1(z)$ for some $|\gamma_1| = 1$, then $c_1 = 2\gamma_1$ and $|c_1| = 2$. Furthermore we have*

$$|c_2 - \frac{c_1^2}{2}| \leqslant 2 - \frac{|c_1|^2}{2}.$$

If $|c_1| < 2$ and $|c_2 - \frac{c_1^2}{2}| = 2 - \frac{|c_1|^2}{2}$, then $p(z) \equiv p_2(z)$, where $p_2(z) = \frac{1+z\frac{\gamma_2 z+\gamma_1}{1+\overline{\gamma_1}\gamma_2 z}}{1-z\frac{\gamma_2 z+\gamma_1}{1+\overline{\gamma_1}\gamma_2 z}}$ and $\gamma_1 = \frac{c_1}{2}, \gamma_2 = \frac{2c_2-c_1^2}{4-|c_1|^2}$. Conversely if $p(z) = p_2(z)$ for some $|\gamma_1| < 1$ and $|\gamma_2| = 1$, then $\gamma_1 = \frac{c_1}{2}, \gamma_2 = \frac{2c_2-c_1^2}{4-|c_1|^2}$ and $|c_2 - \frac{c_1^2}{2}| = 2 - \frac{|c_1|^2}{2}$.

3. Main Theorems

Theorem 3.1. *Let $\{\alpha_i\}_{i=1}^{i=q}, \{\beta_j\}_{j=1}^{j=s} \in \mathbb{C} - \{0, -1, -2, ...\}$ and h satisfies the condition as in (1)-(2). If $f = z + \sum_{n=2}^{\infty} a_n z^n \in \mathcal{P}_{\alpha_1,...\alpha_q;\beta_1,...,\beta_s}(h, \lambda)$, then*

$$|a_n| \leqslant \begin{cases} \dfrac{\prod_{j=1}^{s}|\beta_j|}{(1+\lambda)\prod_{i=1}^{q}|\alpha_i|}|h'(0)|, & n = 2, \\ \dfrac{\left|(h'(0))^2 + \frac{1}{2}h''(0)\right| \prod_{j=1}^{s}|\beta_j||\beta_j + 1|}{(1 + 2\lambda)\prod_{i=1}^{q}|\alpha_i||\alpha_i + 1|}, & n = 3, |h'(0)| \leqslant \left|(h'(0))^2 + \frac{1}{2}h''(0)\right|, \\ \dfrac{|h'(0)| \prod_{j=1}^{s}|\beta_j||\beta_j + 1|}{(1 + 2\lambda)\prod_{i=1}^{q}|\alpha_i||\alpha_i + 1|}, & n = 3, |h'(0)| \geqslant \left|(h'(0))^2 + \frac{1}{2}h''(0)\right|. \end{cases}$$

The results are sharp.

Proof. Let $f \in \mathcal{P}_{\alpha_1,...\alpha_q;\beta_1,...,\beta_s}(h, \lambda)$. By (3), then there is a function $\mathrm{w}(z)$, such that

$$\mathfrak{M} = \frac{z(\mathbb{H}f)'(z) + \lambda z^2(\mathbb{H}f)''(z)}{(1-\lambda)(\mathbb{H}f)(z) + \lambda z(\mathbb{H}f)'(z)} = h(\mathrm{w}(z)),\ z \in \mathbb{U}. \tag{4}$$

where $\mathbb{H} = \mathcal{H}(\alpha_1, ...\alpha_q; \beta_1, ..., \beta_s)$. Take the function $\mathbb{p}(z)$ by

$$\mathbb{p}(z) = \frac{1 + \mathbb{w}(z)}{1 - \mathbb{w}(z)} = 1 + c_1 z + c_2 z^2 + \cdots \prec \frac{1+z}{1-z}, \; z \in \mathbb{U}, \tag{5}$$

thus, $\mathbb{p}(0) = 1$ and $\mathbb{p} \in \mathcal{P}$. In fact, using the (5), it is easy to know that

$$\mathbb{w}(z) = \frac{\mathbb{p}(z) - 1}{\mathbb{p}(z) + 1} = \frac{1}{2}\Big(c_1 z + \Big(c_2 - \frac{c_1^2}{2}\Big)z^2 + ...\Big). \tag{6}$$

Following (4) and (6), then it gives that

$$\mathfrak{M} = 1 + \frac{1}{2}h'(0)c_1 z + \Big(\frac{1}{2}h'(0)\Big(c_2 - \frac{c_1^2}{2}\Big) + \frac{1}{8}h''(0)c_1^2\Big)z^2 + \cdots \tag{7}$$

Suppose that

$$\mathcal{H}(\alpha_1, ...\alpha_q; \beta_1, ..., \beta_s) f(z) = z + F_2 z^2 + \frac{1}{2!}F_3 z^3 + ..., \; z \in \mathbb{U}, \tag{8}$$

where

$$F_2 = \frac{(\alpha_1)_1 ... (\alpha_q)_1}{(\beta_1)_1(\beta_2)_1 ... (\beta_s)_1} a_2 = \frac{\prod_{i=1}^{q} \alpha_i}{\prod_{j=1}^{s} \beta_j} a_2, \; F_3 = \frac{(\alpha_1)_2 ... (\alpha_q)_2}{(\beta_1)_2(\beta_2)_2 ... (\beta_s)_2} a_3 = \frac{\prod_{i=1}^{q} \alpha_i(\alpha_i + 1)}{\prod_{j=1}^{s} \beta_j(\beta_j + 1)} a_3.$$

By (4) and (8), a computation shows that

$$\mathfrak{M} = 1 + (1+\lambda)F_2 z + \Big[(1+2\lambda)F_3 - (1+\lambda)^2 F_2^2\Big]z^2 + \cdots \tag{9}$$

The equations (7) and (9) yield

$$(1+\lambda)F_2 = \frac{1}{2}h'(0)c_1, \tag{10}$$

$$(1+2\lambda)F_3 - (1+\lambda)^2 F_2^2 = \frac{1}{2}h'(0)\Big(c_2 - \frac{c_1^2}{2}\Big) + \frac{1}{8}h''(0)c_1^2. \tag{11}$$

Let $\mathfrak{P}_{\alpha_i}^{\beta_j}(\lambda) = \dfrac{\prod_{j=1}^{s} \beta_j(\beta_j + 1)}{(1+2\lambda)\prod_{i=1}^{q} \alpha_i(\alpha_i + 1)}$. Then (10) and (11) imply that

$$a_2 = \frac{\prod_{j=1}^{s} \beta_j}{2(1+\lambda)\prod_{i=1}^{q} \alpha_i} h'(0)c_1, \tag{12}$$

$$a_3 = \mathfrak{P}_{\alpha_i}^{\beta_j}(\lambda)\Big[\frac{1}{2}h'(0)\Big(c_2 - \frac{1}{2}c_1^2\Big) + \Big(\frac{1}{4}(h'(0))^2 + \frac{1}{8}h''(0)\Big)c_1^2\Big] \tag{13}$$

It follows (12) and Lemma 2.1 that

$$|a_2| \leqslant \frac{\prod_{j=1}^{s} |\beta_j|}{(1+\lambda)\prod_{i=1}^{q} |\alpha_i|} |h'(0)|. \tag{14}$$

Furthermore, making use of (13) with Lemma 2.1, then we obtain

$$\begin{aligned}|a_3| &\leqslant |\mathfrak{P}_{\alpha_i}^{\beta_j}(\lambda)|\left[\frac{1}{2}|h'(0)|\left|c_2-\frac{1}{2}c_1^2\right|+\left|\frac{1}{4}(h'(0))^2+\frac{1}{8}h''(0)\right||c_1|^2\right]\\ &\leqslant |\mathfrak{P}_{\alpha_i}^{\beta_j}(\lambda)|\left[\frac{1}{2}|h'(0)|\left(2-\frac{1}{2}|c_1|^2\right)+\left|\frac{1}{4}(h'(0))^2+\frac{1}{8}h''(0)\right||c_1|^2\right]\\ &\leqslant |\mathfrak{P}_{\alpha_i}^{\beta_j}(\lambda)|\left[|h'(0)|+\left(\left|\frac{1}{4}(h'(0))^2+\frac{1}{8}h''(0)\right|-\frac{1}{4}|h'(0)|\right)|c_1|^2\right] \qquad (15)\end{aligned}$$

It is easy to observe that the (15) reduces to the bounds of $|a_3|$.

An examination of proof shows the first and second equalities hold if $c_1 = 2$, thus, we have $\mathbb{p}_1(z) = \frac{1+z}{1-z}$ by Lemma 2.1. The extremal function in $\mathcal{P}_{\alpha_1,\dots\alpha_q;\beta_1,\dots,\beta_s}(h,\lambda)$ is given by

$$\frac{z(\mathbb{H}f)'(z)+\lambda z^2(\mathbb{H}f)''(z)}{(1-\lambda)(\mathbb{H}f)(z)+\lambda z(\mathbb{H}f)'(z)} = h\left(\frac{\mathbb{p}_1(z)-1}{\mathbb{p}_1(z)+1}\right) = h(z),\ z\in\mathbb{U}.$$

The third equality holds if $c_1 = 0$ and $c_2 = 2$, thus, we have $\mathbb{p}_2(z) = \frac{1+z^2}{1-z^2}$ by Lemma 2.1. The extremal function in $\mathcal{P}_{\alpha_1,\dots\alpha_q;\beta_1,\dots,\beta_s}(h,\lambda)$ is given by

$$\frac{z(\mathbb{H}f)'(z)+\lambda z^2(\mathbb{H}f)''(z)}{(1-\lambda)(\mathbb{H}f)(z)+\lambda z(\mathbb{H}f)'(z)} = h\left(\frac{\mathbb{p}_2(z)-1}{\mathbb{p}_2(z)+1}\right) = h(z^2),\ z\in\mathbb{U}.$$

This completes the proof of Theorem 3.1.

□

Theorem 3.2. *Let* $\{\alpha_i\}_{i=1}^{i=q}, \{\beta_j\}_{j=1}^{j=s} \in \mathbb{R}^+ - \{0\}$. *If h satisfies the condition as in* (1)-(2) *and* $f = z+\sum_{n=2}^{\infty}a_nz^n \in \mathcal{P}_{\alpha_1,\dots\alpha_q;\beta_1,\dots,\beta_s}(h,\lambda)$, *then for any* $\mu\in\mathbb{R}$, *we have*

$$|a_3-\mu a_2^2| \leqslant \begin{cases}\dfrac{(1+\lambda)^2|h'(0)|\prod\limits_{i=1}^{q}\prod\limits_{j=1}^{s}\alpha_i\beta_j(\beta_j+1)+\mathcal{J}_1}{(1+2\lambda)(1+\lambda)^2\prod\limits_{i=1}^{q}\alpha_i^2(\alpha_i+1)}, & \mu\leqslant M_1\mathcal{N},\\[2ex] \dfrac{|h'(0)|\prod\limits_{j=1}^{s}\beta_j(\beta_j+1)}{(1+2\lambda)\prod\limits_{i=1}^{q}\alpha_i(\alpha_i+1)}, & M_1\mathcal{N}\leqslant\mu\leqslant M_2\mathcal{N},\\[2ex] \dfrac{(1+\lambda)^2|h'(0)|\prod\limits_{i=1}^{q}\prod\limits_{j=1}^{s}\alpha_i\beta_j(\beta_j+1)+\mathcal{J}_2}{(1+2\lambda)(1+\lambda)^2\prod\limits_{i=1}^{q}\alpha_i^2(\alpha_i+1)}, & \mu\geqslant M_2\mathcal{N},\end{cases}$$

where $M_1, M_2, \mathcal{J}_1, \mathcal{J}_2$ *and* $\mathcal{N}$ *are defined as the following* (19). *The above estimate are sharp for each* μ.

Proof. Let

$$\mathfrak{U}_{\alpha_i}^{\beta_j}(h,\lambda) = \frac{h'(0)\prod\limits_{j=1}^{s}\beta_j(\beta_j+1)}{2(1+2\lambda)\prod\limits_{i=1}^{q}\alpha_i(\alpha_i+1)}.$$

From (12) and (13), we have

$$\begin{aligned} a_3-\mu a_2^2 &= \mathfrak{U}_{\alpha_i}^{\beta_j}(h,\lambda)\Big(c_2-\frac{1}{2}c_1^2\Big)-\mu\frac{(h'(0))^2\prod\limits_{j=1}^{s}\beta_j^2}{4(1+\lambda)^2\prod\limits_{i=1}^{q}\alpha_i^2}c_1^2+\frac{\Big(\frac{1}{4}(h'(0))^2+\frac{1}{8}h''(0)\Big)\prod\limits_{j=1}^{s}\beta_j(\beta_j+1)}{(1+2\lambda)\prod\limits_{i=1}^{q}\alpha_i(\alpha_i+1)}c_1^2 \\ &= \mathfrak{U}_{\alpha_i}^{\beta_j}(h,\lambda)\Big(c_2-\frac{1}{2}c_1^2\Big)+\frac{\mathfrak{P}_\mu(\lambda)}{4(1+2\lambda)(1+\lambda)^2\prod\limits_{i=1}^{q}\alpha_i^2(\alpha_i+1)}c_1^2, \end{aligned} \tag{16}$$

where

$$\mathfrak{P}_\mu(\lambda)=(1+\lambda)^2\Big((h'(0))^2+\frac{1}{2}h''(0)\Big)\prod_{i=1}^{q}\prod_{j=1}^{s}\alpha_i\beta_j(\beta_j+1)-\mu(1+2\lambda)(h'(0))^2\prod_{i=1}^{q}\prod_{j=1}^{s}(\alpha_i+1)\beta_j^2.$$

Therefore, (16) gives us

$$|a_3-\mu a_2^2|\leqslant|\mathfrak{U}_{\alpha_i}^{\beta_j}(h,\lambda)|\Big|c_2-\frac{1}{2}c_1^2\Big|+\frac{\Big|\mathfrak{P}_\mu(\lambda)\Big|}{4(1+2\lambda)(1+\lambda)^2\prod\limits_{i=1}^{q}\alpha_i^2(\alpha_i+1)}|c_1|^2. \tag{17}$$

In view of Lemma 2.1 and (17), we obtain

$$\begin{aligned} |a_3-\mu a_2^2| &\leqslant|\mathfrak{U}_{\alpha_i}^{\beta_j}(h,\lambda)|\Big(2-\frac{1}{2}|c_1|^2\Big)+\frac{\Big|\mathfrak{P}_\mu(\lambda)\Big|}{4(1+2\lambda)(1+\lambda)^2\prod\limits_{i=1}^{q}|\alpha_i^2(\alpha_i+1)|}|c_1|^2| \\ &= 2|\mathfrak{U}_{\alpha_i}^{\beta_j}(h,\lambda)+\frac{\Big|\mathfrak{P}_\mu(\lambda)\Big|-(1+\lambda)^2|h'(0)|\prod\limits_{i=1}^{q}\prod\limits_{j=1}^{s}\alpha_i\beta_j(\beta_j+1)}{4(1+2\lambda)(1+\lambda)^2\prod\limits_{i=1}^{q}\alpha_i^2(\alpha_i+1)}|c_1|^2. \end{aligned} \tag{18}$$

Here, for later convenience , we define $\mathcal{J}_1$, $\mathcal{J}_2$, and $\mathcal{N}$ as follows:

$$\begin{aligned} &M_1=(h'(0))^2+\frac{1}{2}h''(0)-|h'(0)|,\ M_2=(h'(0))^2+\frac{1}{2}h''(0)+|h'(0)|, \\ &\mathcal{J}_1=(1+\lambda)^2M_1\prod_{i=1}^{q}\prod_{j=1}^{s}\alpha_i\beta_j(\beta_j+1)-\mu(1+2\lambda)(h'(0))^2\prod_{i=1}^{q}\prod_{j=1}^{s}(\alpha_i+1)\beta_j^2, \\ &\mathcal{J}_2=\mu(1+2\lambda)(h'(0))^2\prod_{i=1}^{q}\prod_{j=1}^{s}(\alpha_i+1)\beta_j^2-(1+\lambda)^2M_2\prod_{i=1}^{q}\prod_{j=1}^{s}\alpha_i\beta_j(\beta_j+1), \end{aligned}$$

$$\mathcal{N}=\frac{(1+\lambda)^2\prod\limits_{i=1}^{q}\prod\limits_{j=1}^{s}\alpha_i\beta_j(\beta_j+1)}{(1+2\lambda)(h'(0))^2\prod\limits_{i=1}^{q}\prod\limits_{j=1}^{s}(\alpha_i+1)\beta_j^2}. \tag{19}$$

Now, we have to consider the following four cases.

Case 1. If $\mu \leqslant [(h'(0))^2 + \frac{1}{2}h''(0) - |h'(0)|]\mathcal{N}$, by (18), then

$$|a_3 - \mu a_2^2| \leqslant 2|\mathfrak{U}_{\alpha_i}^{\beta_j}(h,\lambda)| + \frac{\mathcal{J}_1}{4(1+2\lambda)(1+\lambda)^2 \prod_{i=1}^{q} \alpha_i^2(\alpha_i+1)}|c_1|^2 \leqslant 2|\mathfrak{U}_{\alpha_i}^{\beta_j}(h,\lambda)| + \frac{\mathcal{J}_1}{(1+2\lambda)(1+\lambda)^2 \prod_{i=1}^{q} \alpha_i^2(\alpha_i+1)}$$

$$= \frac{(1+\lambda)^2|h'(0)| \prod_{i=1}^{q}\prod_{j=1}^{s} \alpha_i\beta_j(\beta_j+1) + \mathcal{J}_1}{(1+2\lambda)(1+\lambda)^2 \prod_{i=1}^{q} \alpha_i^2(\alpha_i+1)}. \tag{20}$$

Case 2. If $[(h'(0))^2 + \frac{1}{2}h''(0) - |h'(0)|]\mathcal{N} \leqslant \mu \leqslant [(h'(0))^2 + \frac{1}{2}h''(0)]\mathcal{N}$, from (18), then we have

$$|a_3 - \mu a_2^2| \leqslant 2|\mathfrak{U}_{\alpha_i}^{\beta_j}(h,\lambda)| + \frac{\mathcal{J}_1}{4(1+2\lambda)(1+\lambda)^2 \prod_{i=1}^{q} \alpha_i^2(\alpha_i+1)}|c_1|^2 \leqslant \frac{|h'(0)| \prod_{j=1}^{s} \beta_j(\beta_j+1)}{(1+2\lambda) \prod_{i=1}^{q} \alpha_i(\alpha_i+1)}. \tag{21}$$

Case 3. If $[(h'(0))^2 + \frac{1}{2}h''(0)]\mathcal{N} \leqslant \mu \leqslant [(h'(0))^2 + \frac{1}{2}h''(0) + |h'(0)|]\mathcal{N}$, from (18), then we have

$$|a_3 - \mu a_2^2| \leqslant 2|\mathfrak{U}_{\alpha_i}^{\beta_j}(h,\lambda)| + \frac{\mathcal{J}_2}{4(1+2\lambda)(1+\lambda)^2 \prod_{i=1}^{q} \alpha_i^2(\alpha_i+1)}|c_1|^2 \leqslant \frac{|h'(0)| \prod_{j=1}^{s} \beta_j(\beta_j+1)}{(1+2\lambda) \prod_{i=1}^{q} \alpha_i(\alpha_i+1)}. \tag{22}$$

Case 4. If $\mu \geqslant [(h'(0))^2 + \frac{1}{2}h''(0) + |h'(0)|]\mathcal{N}$, from (18), then we have

$$|a_3 - \mu a_2^2| \leqslant 2|\mathfrak{U}_{\alpha_i}^{\beta_j}(h,\lambda)| + \frac{\mathcal{J}_2}{4(1+2\lambda)(1+\lambda)^2 \prod_{i=1}^{q} \alpha_i^2(\alpha_i+1)}|c_1|^2 \leqslant 2|\mathfrak{U}_{\alpha_i}^{\beta_j}(h,\lambda)| + \frac{\mathcal{J}_2}{(1+2\lambda)(1+\lambda)^2 \prod_{i=1}^{q} \alpha_i^2(\alpha_i+1)}$$

$$= \frac{(1+\lambda)^2|h'(0)| \prod_{i=1}^{q}\prod_{j=1}^{s} \alpha_i\beta_j(\beta_j+1) + \mathcal{J}_2}{(1+2\lambda)(1+\lambda)^2 \prod_{i=1}^{q} \alpha_i^2(\alpha_i+1)}. \tag{23}$$

An examination of proof shows the equalities in (20) and (23) hold if $c_1 = 2$, thus, we have $\mathbb{p}_1(z) = \frac{1+z}{1-z}$ by Lemma 2.1. The extremal function in $\mathcal{P}_{\alpha_1,\dots\alpha_q;\beta_1,\dots,\beta_s}(h,\lambda)$ is given by

$$\frac{z(\mathbb{H}f)'(z) + \lambda z^2(\mathbb{H}f)''(z)}{(1-\lambda)(\mathbb{H}f)(z) + \lambda z(\mathbb{H}f)'(z)} = h\Big(\frac{\mathbb{p}_1(z)-1}{\mathbb{p}_1(z)+1}\Big) = h(z),\ z \in \mathbb{U}.$$

The equalities in (21) and (22) hold if $c_1 = 0$ and $c_2 = 2$, thus, we have $\mathbb{p}_2(z) = \frac{1+z^2}{1-z^2}$ by Lemma 2.1. The extremal function in $\mathcal{P}_{\alpha_1,\dots\alpha_q;\beta_1,\dots,\beta_s}(h,\lambda)$ is given by

$$\frac{z(\mathbb{H}f)'(z) + \lambda z^2(\mathbb{H}f)''(z)}{(1-\lambda)(\mathbb{H}f)(z) + \lambda z(\mathbb{H}f)'(z)} = h\Big(\frac{\mathbb{p}_2(z)-1}{\mathbb{p}_2(z)+1}\Big) = h(z^2),\ z \in \mathbb{U}.$$

This completes the proof of Theorem 3.2. □

4. Some applications related to conic domain and almost starlike functions

Kanas-Wiśniowska [11] were the first who defined the conic domain $\Omega_k (k \geqslant 0)$ as

$$\Omega_k = \{u + iv : u > k\sqrt{(u-1)^2 + v^2}\},$$

and then, several authors studied the domain for different purposes (see, e.g. [6, 10]). With the conic domain Ω_k, following extremal functions $p_k(z)$ defined by

$$p_k(z) = \begin{cases} \frac{1+z}{1-z}, & k = 0, \\ 1 + \frac{2}{\pi^2}(\log \frac{1+\sqrt{z}}{1-\sqrt{z}})^2, & k = 1, \\ \frac{1}{1-k^2}\cos\left(\left(\frac{2}{\pi}\arccos k\right) \mathrm{i} \log \frac{1+\sqrt{z}}{1-\sqrt{z}}\right) - \frac{k^2}{1-k^2}, & 0 < k < 1, \\ \frac{1}{k^2-1}\sin\left(\frac{\pi}{2K(t)}\int_0^{\frac{u(z)}{\sqrt{t}}} \frac{dt}{\sqrt{1-x^2}\sqrt{1-x^2t^2}}\right) + \frac{k^2}{k^2-1}, & k > 1, \end{cases}$$

where $t \in (0, 1)$ is chose such that $k = \cosh(\frac{\pi K'(t)}{4K(t)})$ and

$$u(z) = \frac{z - \sqrt{t}}{1 - \sqrt{t}z}, \ z \in \mathbb{U}.$$

Here $K(t)$ is Legendre's complete elliptic integral of first kind and $K'(t) = K(\sqrt{1-t^2})$. In [10], Kanas proved that

$$p_1(z) = 1 + \frac{8}{\pi^2}z + \frac{16}{3\pi^2}z^2 + ..., z \in \mathbb{U}. \tag{24}$$

Using (24) in Example 1.3 and Example 1.4, then Theorem 3.1 and Theorem 3.2 reduce to the following Corollary 4.1 and Corollary 4.2, respectively. We omit the proof.

Corollary 4.1. *Let* $f = z + \sum_{n=2}^{\infty} a_n z^n \in \mathcal{P}_p(\alpha_1, ..., \alpha_q; \beta_1, ..., \beta_s)$.
(i) *If* $\{\alpha_i\}_{i=1}^{i=q}, \{\beta_j\}_{j=1}^{j=s} \in \mathbb{C} - \{0, -1, -2, ...\}$ *, then*

$$|a_n| \leqslant \begin{cases} \dfrac{8\prod_{j=1}^{s}|\beta_j|}{\pi^2 \prod_{i=1}^{q}|\alpha_i|}, & n = 2, \\ \dfrac{(192 + 16\pi^2)\prod_{j=1}^{s}|\beta_j||\beta_j + 1|}{3\pi^4 \prod_{i=1}^{q}|\alpha_i||\alpha_i + 1|}, & n = 3. \end{cases}$$

(ii) *If* $\{\alpha_i\}_{i=1}^{i=q}, \{\beta_j\}_{j=1}^{j=s} \in \mathbb{R}^+ - \{0\}$, *then for any* $\mu \in \mathbb{R}$, *we have*

$$|a_3 - \mu a_2^2| \leqslant \begin{cases} \dfrac{8\prod_{i=1}^{q}\prod_{j=1}^{s}\alpha_i\beta_j(\beta_j + 1) + \pi^2\mathcal{J}_1}{\pi^2\prod_{i=1}^{q}\alpha_i^2(\alpha_i + 1)}, & \mu \leqslant \frac{192-8\pi^2}{3\pi^4}\mathcal{N}, \\ \dfrac{8\prod_{j=1}^{s}\beta_j(\beta_j + 1)}{\pi^2\prod_{i=1}^{q}\alpha_i(\alpha_i + 1)}, & \frac{192-8\pi^2}{3\pi^4}\mathcal{N} \leqslant \mu \leqslant \frac{192+40\pi^2}{3\pi^4}\mathcal{N}, \\ \dfrac{8\prod_{i=1}^{q}\prod_{j=1}^{s}\alpha_i\beta_j(\beta_j + 1) + \pi^2\mathcal{J}_2}{\pi^2\prod_{i=1}^{q}\alpha_i^2(\alpha_i + 1)}, & \mu \geqslant \frac{192+40\pi^2}{3\pi^4}\mathcal{N}, \end{cases}$$

where

$$\mathcal{J}_1 = \frac{192-8\pi^2}{3\pi^4}\prod_{i=1}^{q}\prod_{j=1}^{s}\alpha_i\beta_j(\beta_j+1) - \frac{64}{\pi^4}\mu\prod_{i=1}^{q}\prod_{j=1}^{s}(\alpha_i+1)\beta_j^2,$$

$$\mathcal{J}_2 = \frac{64}{\pi^4}\mu\prod_{i=1}^{q}\prod_{j=1}^{s}(\alpha_i+1)\beta_j^2 - \frac{192+40\pi^2}{\pi^4}\prod_{i=1}^{q}\prod_{j=1}^{s}\alpha_i\beta_j(\beta_j+1),\ \ \mathcal{N} = \frac{\pi^4\prod_{i=1}^{q}\prod_{j=1}^{s}\alpha_i\beta_j(\beta_j+1)}{64\prod_{i=1}^{q}\prod_{j=1}^{s}(\alpha_i+1)\beta_j^2}.$$

The above estimate in **(i)** *and* **(ii)** *are sharp.*

Corollary 4.2. *Let* $f = z + \sum_{n=2}^{\infty} a_n z^n \in \mathcal{CP}(\alpha_1, ..., \alpha_q; \beta_1, ..., \beta_s)$.
(i) *If* $\{\alpha_i\}_{i=1}^{i=q}, \{\beta_j\}_{j=1}^{j=s} \in \mathbb{C} - \{0, -1, -2, ...\}$ *, then*

$$|a_n| \leqslant \begin{cases} \dfrac{4\prod_{j=1}^{s}|\beta_j|}{\pi^2\prod_{i=1}^{q}|\alpha_i|}, & n=2, \\[2ex] \dfrac{(192+16\pi^2)\prod_{j=1}^{s}|\beta_j||\beta_j+1|}{9\pi^4\prod_{i=1}^{q}|\alpha_i||\alpha_i+1|}, & n=3. \end{cases}$$

(ii) *If* $\{\alpha_i\}_{i=1}^{i=q}, \{\beta_j\}_{j=1}^{j=s} \in \mathbb{R}^+ - \{0\}$*, then for any* $\mu \in \mathbb{R}$*, we have*

$$|a_3 - \mu a_2^2| \leqslant \begin{cases} \dfrac{32\prod_{i=1}^{q}\prod_{j=1}^{s}\alpha_i\beta_j(\beta_j+1) + \pi^2\mathcal{J}_1}{12\pi^2\prod_{i=1}^{q}\alpha_i^2(\alpha_i+1)}, & \mu \leqslant \frac{192-8\pi^2}{3\pi^4}\mathcal{N}, \\[2ex] \dfrac{8\prod_{j=1}^{s}\beta_j(\beta_j+1)}{3\pi^2\prod_{i=1}^{q}\alpha_i(\alpha_i+1)}, & \frac{192-8\pi^2}{3\pi^4}\mathcal{N} \leqslant \mu \leqslant \frac{192+40\pi^2}{3\pi^4}\mathcal{N}, \\[2ex] \dfrac{32\prod_{i=1}^{q}\prod_{j=1}^{s}\alpha_i\beta_j(\beta_j+1) + \pi^2\mathcal{J}_2}{12\pi^2\prod_{i=1}^{q}\alpha_i^2(\alpha_i+1)}, & \mu \geqslant \frac{192+40\pi^2}{3\pi^4}\mathcal{N}, \end{cases}$$

where

$$\mathcal{J}_1 = \frac{768-32\pi^2}{3\pi^4}\prod_{i=1}^{q}\prod_{j=1}^{s}\alpha_i\beta_j(\beta_j+1) - \frac{192}{\pi^4}\mu\prod_{i=1}^{q}\prod_{j=1}^{s}(\alpha_i+1)\beta_j^2,$$

$$\mathcal{J}_2 = \frac{192}{\pi^4}\mu\prod_{i=1}^{q}\prod_{j=1}^{s}(\alpha_i+1)\beta_j^2 - \frac{768+160\pi^2}{3\pi^4}\prod_{i=1}^{q}\prod_{j=1}^{s}\alpha_i\beta_j(\beta_j+1),\ \ \mathcal{N} = \frac{\pi^4\prod_{i=1}^{q}\prod_{j=1}^{s}\alpha_i\beta_j(\beta_j+1)}{48\prod_{i=1}^{q}\prod_{j=1}^{s}(\alpha_i+1)\beta_j^2}.$$

The above estimate in **(i)** *and* **(ii)** *are sharp.*

Corollary 4.3. *Let* $f = z + \sum_{n=2}^{\infty} a_n z^n \in \mathcal{AP}_\gamma(\alpha_1, ..., \alpha_q; \beta_1, ..., \beta_s)$.
(i) *If* $\{\alpha_i\}_{i=1}^{i=q}, \{\beta_j\}_{j=1}^{j=s} \in \mathbb{C} - \{0, -1, -2, ...\}$ *, then*

$$|a_n| \leqslant \begin{cases} \dfrac{2(1-\gamma)\prod_{j=1}^{s}|\beta_j|}{\prod_{i=1}^{q}|\alpha_i|}, & n = 2, \\ \dfrac{2(1-\gamma)(3-4\gamma)\prod_{j=1}^{s}|\beta_j||\beta_j+1|}{\prod_{i=1}^{q}|\alpha_i||\alpha_i+1|}, & n = 3, \gamma \in [0, \frac{1}{2}], \\ \dfrac{2(1-\gamma)\prod_{j=1}^{s}|\beta_j||\beta_j+1|}{\prod_{i=1}^{q}|\alpha_i||\alpha_i+1|}, & n = 3, \gamma \in [\frac{1}{2}, 1). \end{cases}$$

(ii) *If* $\{\alpha_i\}_{i=1}^{i=q}, \{\beta_j\}_{j=1}^{j=s} \in \mathbb{R}^+ - \{0\}$*, then for any* $\mu \in \mathbb{R}$*, we have*

$$|a_3 - \mu a_2^2| \leqslant \begin{cases} \dfrac{4(1-\gamma)^2\prod_{i=1}^{q}\prod_{j=1}^{s}\alpha_i\beta_j(\beta_j+1) + \mathcal{J}_1}{\prod_{i=1}^{q}\alpha_i^2(\alpha_i+1)}, & \mu \leqslant M_1\mathcal{N}, \\ \dfrac{4(1-\gamma)^2\prod_{j=1}^{s}\beta_j(\beta_j+1)}{\prod_{i=1}^{q}\alpha_i(\alpha_i+1)}, & M_1\mathcal{N} \leqslant \mu \leqslant M_2\mathcal{N}, \\ \dfrac{4(1-\gamma)^2\prod_{i=1}^{q}\prod_{j=1}^{s}\alpha_i\beta_j(\beta_j+1) + \mathcal{J}_2}{\prod_{i=1}^{q}\alpha_i^2(\alpha_i+1)}, & \mu \geqslant M_2\mathcal{N}, \end{cases}$$

where

$$\mathcal{J}_1 = 4(1-\gamma)(1-2\gamma)\prod_{i=1}^{q}\prod_{j=1}^{s}\alpha_i\beta_j(\beta_j+1) - 4(1-\gamma)^2\mu\prod_{i=1}^{q}\prod_{j=1}^{s}(\alpha_i+1)\beta_j^2,$$

$$\mathcal{J}_2 = 4(1-\gamma)^2\mu\prod_{i=1}^{q}\prod_{j=1}^{s}(\alpha_i+1)\beta_j^2 - 8(1-\gamma)^2\prod_{i=1}^{q}\prod_{j=1}^{s}\alpha_i\beta_j(\beta_j+1),\ \ \mathcal{N} = \frac{\prod_{i=1}^{q}\prod_{j=1}^{s}\alpha_i\beta_j(\beta_j+1)}{4(1-\gamma)^2\prod_{i=1}^{q}\prod_{j=1}^{s}(\alpha_i+1)\beta_j^2}.$$

The above estimate in **(i)** *and* **(ii)** *are sharp.*

Proof. Since $f \in \mathcal{AP}_\gamma(\alpha_1, ..., \alpha_q; \beta_1, ..., \beta_s)$, following example 1.5, then

$$h(z) = \frac{1+z}{1+(2\gamma-1)z} = 1 + 2(1-\gamma)z - 2(1-\gamma)(2\gamma-1)z^2 + ...,\ \ z \in \mathbb{U}. \tag{25}$$

(25) shows that

$$h'(0) = 2(1-\gamma),\ \ h''(0) = 4(1-\gamma)(1-2\gamma). \tag{26}$$

Using (26) in Theorem 3.1 and Theorem 3.2, we can obtain the Corollary 4.3. □

Remark 4.4. *If we take* $\lambda = 0$ *and* $h(z) = \left(\frac{1+z}{1-z}\right)^\beta$ *in Theorem 3.1 and Theorem 3.2, where* $0 < \beta \leqslant 1$, *then the results related to strongly starlike functions were proved by Orhan-Răducanu [18].*

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