Filomat 33:6 (2019), 1641–1648
https://doi.org/10.2298/FIL1906641W

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Lower Semicontinuity of Approximate Solution Mappings for a Parametric Generalized Strong Vector Equilibrium Problem

Qilin Wang[a], Liu He[a], Xiaobing Li[a]

[a]*College of Mathematics and Statistics, Chongqing Jiaotong University, Chongqing, 400074, China*

Abstract. In this paper, by using a property of convex mappings, one establishes the lower semicontinuity of the approximate solution mapping to a parametric generalized strong vector equilibrium problem without these assumptions about monotonicity and compactness. Our proof approach is different from the ones in the literature.

1. Introduction

Given a set E and a bifunction $f : E \times E \to R$, the scalar equilibrium problem for f is to find $x_0 \in E$ such that

$$f(x_0, y) \geq 0, \forall y \in E.$$

It is well known that the problem is closely related to the famous Ky Fan minimax inequality(see Refs. [6, 14]). According to [7], the following classical problems can be cast to this format:

(i) the generalized mathematical programming problem: $f(x, y) = g(y) - g(x)$;

(ii) the Gateaux differentiable convex mathematical programming problem: $f(x, y) = \langle Dg(x), y - x\rangle$;

(iii) the saddle point problem: $f(x, y) = h(y_1, x_2) - h(x_1, y_2)$, where $x = (x_1, x_2), y = (y_1, y_2)$;

(iv) the fixed point problem: $f(x, y) = \langle x - T(x), y - x\rangle$, where T is the operator of the fixed point problem;

(v) the variational inequality problem(in its simplest form): $f(x, y) = \langle g(x), y - x\rangle$, where g is a mapping;

(vi) the generalized variational inequality problem(in its simplest form):
$f(x, y) = max_{z \in G(x)}\langle z, y - x\rangle$, where G is a set-valued mapping;

2010 *Mathematics Subject Classification*. Primary 90C33; Secondary 90C31, 91B50
Keywords. Parametric generalized strong vector equilibrium problems, Approximate solution mappings, Lower semicontinuity.
Received: 14 December 2017; Revised: 03 September 2018; Accepted: 28 June 2019
Communicated by Dragan S. Djordjević
Research supported by the Programs of the National Natural Science Foundation of China (No.11571055), the Humanities and Social Sciences Project of Ministry of Education in China(No.17YJA630079), Chongqing Innovation Team Project in University (No.CXTDX201601022) Chongqing Natural Science Foundation Project of CQ CSTC(No.2017jcyjAX0382), postgraduate research innovation of CQJTU (No.2018S0152)
Email addresses: wangql97@126.com (Qilin Wang), heliu20170906@163.com (Liu He), xiaobinglicq@126.com (Xiaobing Li)

(vii) the Nash equilibrium problem in a non-cooperative game: $f(x, y) = \sum_{i \in I}(f_i(x^i, y_i) - f_i(x))$, where f_i is the loss function of the player i and x^i is the vector obtained from x by deleting component i.

Each of these classical problems has numerous applications, including but not limited to equilibrium problems in economics, game theory, traffic analysis and mechanics. Collectively, the equilibrium problem covers a vast range of applications. Hence the problem has been researched quite extensively. Oettli and Sch*ä*ger [36, 37] generalized the scalar equilibrium problem to vector equilibrium problems. They replaced the range R by a real topological vector space Z with an ordering cone P(meaning that $P \neq Z$ is a closed convex cone with nonempty interior) and considered a set-valued mapping $F : E \times E \to 2^Z$. Then, the inequality $f(x, y) \geq 0$ can be generalized in several possible ways, for instance as $F(x, y) \subset P$, $F(x, y) \bigcap P \neq \emptyset$, $F(x, y) \not\subset -\text{int}P$, $F(x, y) \bigcap(-\text{int}P) = \emptyset$.

One important problem of vector equilibrium problems is to study the existence of the solution, a number of papers have been devoted to this subject (see Refs [8, 13, 15–17, 35] and the references therein). Another important problem is to study the stability of the solution mapping to parametric vector variational inequalities and parametric vector equilibrium problems (see Refs[1, 2, 9–12, 18–22, 24–27, 29–31, 34, 39–41]).

Exact solutions may not exist in many practical problems because the data of these problems are not sufficiently regular. Moreover, these mathematical models are usually solved by numerical methods (iterative procedures or heuristic algorithms) which produce approximations to the exact solutions. So it is impossible to obtain an exact solution of many practical problems. Naturally, investigating approximate solutions of parametric equilibrium problems is of interest in both practical applications and computations. However, to the best of our knowledge, there are only a few results concerning the semicontinuity of approximate solution mappings for parametric variational inequality or parametric equilibrium problems. Kimura and Yao [28] have established the existence results for two types of approximate generalized vector equilibrium problems, and further obtained the semicontinuity of approximate solution mappings. Khanh and Luu [22] have discussed the semicontinuity of the approximate solution mappings of parametric multivalued quasivariational inequalities in topological vector spaces. Anh and Khanh [3] have considered two kinds of approximate solution mappings to parametric generalized vector quasiequilibrium problems and established the sufficient conditions for their Hausdorff semicontinuity (or Berge semicontinuity). By using a scalarization method, Li and Li [32] have investigated the Hausdorff continuity (or Berge continuity) of the approximate solution mapping for a parametric scalar equilibrium problem. By using a scalarization method, they obtained a sufficient condition of the lower semicontinuity of the approximate solution mapping for a parametric vector equilibrium problem. By using the monotonicity of the approximate solution mappings, Li et al. [33] established the Lipschitz continuity of the approximate solution mappings for a parametric scalar equilibrium problem.

Motivated and inspired by the work reported in [3, 20, 22, 32, 33, 38], the aim of this paper is to establish the lower semicontinuity of the approximate solution mapping to a parametric generalized strong vector equilibrium problem. By using a new proof method which is different from the ones used in the literature, we establish the lower semicontinuity of the approximate solution mapping to a parametric generalized strong vector equilibrium problem without these assumptions about monotonicity and compactness. Our results are new and different from the ones in the literature.

The rest of the paper is organized as follows. In Sect. 2, we introduce a parametric generalized strong vector equilibrium problem, and recall some basic concepts and some of their properties. In Sect. 3, we discuss the lower semicontinuity of the approximate solution mapping to a parametric generalized strong vector equilibrium problem.

2. Preliminaries and Notations

Throughout this paper, let m and n be two natural numbers. Let R^m and R^n be m-dimensional spaces and n-dimensional spaces, respectively. We also assume that C is a pointed closed convex cone in R^n with its interior $\text{int}C \neq \emptyset$. We denote by B_{R^n} the closed unit ball in R^n. We also denote by 0_{R^m} the origin of R^m . Let E be a nonempty subset of R^m and $F : E \times E \to 2^{R^n}$ be a nonempty set-valued mapping. We consider the following generalized strong vector equilibrium problem (in short, GSVEP) of finding $x \in E$ such that

$$F(x, y) \subseteq C, \forall y \in E.$$

Let l be a natural number. When the set E and the mapping F are perturbed by a parameter μ which varies over a subset Λ of R^l, we consider the following parametric generalized strong vector equilibrium problem (in short, PGSVEP) of finding $x \in E(\mu)$ such that

$$F(x, y, \mu) \subseteq C, \forall y \in E(\mu),$$

where $E : \Lambda \to 2^{R^m} \setminus \{\emptyset\}$ is a set-valued mapping, $F : B \times B \times \Lambda \subset R^m \times R^m \times R^l \to 2^{R^n} \setminus \{\emptyset\}$ is a set-valued mapping with $E(\Lambda) = \bigcup_{\mu\in\Lambda} E(\mu) \subset B$.

For each $\mu \in \Lambda$ and $e \in R^n$, let $S(\mu, e)$ denote the approximate solution set of PGSVEP corresponding to (μ, e), i.e.,

$$S(\mu, e) = \{x \in E(\mu) : F(x, y, \mu) + e \subseteq C, \forall y \in E(\mu)\}.$$

Remark 2.1. In general, the e of $S(\mu, e)$ belongs to int$C$ (C)(see [22, 32, 33, 42]). Since $C \subset R^n$ and we will establish the lower semicontinuity of $S(\mu, e)$ that has nothing to do with $e \in$ int$C$ (C), we set $e \in R^n$ and our result about the lower semicontinuity for $S(\mu, e)$ can deduce one in general case.

Remark 2.2. Let $\mu \in \Lambda, e_1, e_2 \in R^n$. If $e_1 - e_2 \in C$, then $S(\mu, e_2) \subseteq S(\mu, e_1)$.

Now, we recall some concepts and properties which will be useful in the sequel.

Definition 2.3. Let G be a set-valued mapping from R^m to R^n.

(i) (see [4]) G is said to be lower semicontinuous (in short, l.s.c.) at $x_0 \in R^m$ if for any sequence $\{x_n\}$ with $x_n \to x_0$ and $y_0 \in G(x_0)$, there exists a sequence $\{y_n\} \subseteq G(x_n)$ such that $y_n \to y_0$. It could be phrased as follows:
G is said to be lower semicontinuous at $x_0 \in R^m$ if for any $y_0 \in G(x_0)$ and any neighborhood $W(y_0)$ of y_0, there exists a neighborhood $V(x_0)$ of x_0 such that

$$G(x) \bigcap W(y_0) \neq \emptyset, \ \forall x \in V(x_0).$$

G is said to be lower semicontinuous if G is l.s.c. at every point $x \in R^m$.

(ii) (see [23]) G is said to be Hausdorff upper semicontinuous (in short, H-u.s.c.) at $x_0 \in R^m$ if for every neighborhood U of 0_{R^n}, there exists a neighborhood $N(x_0)$ of x_0 in R^m such that

$$G(x) \subset G(x_0) + U, \forall x \in N(x_0).$$

Definition 2.4. Let E be a convex subset of R^m and $G : E \to 2^{R^n}$ be a set-valued mapping with $G(x) \neq \emptyset$, for all $x \in E$. G is said to be

(i) convex on E, if for any $x_1, x_2 \in E$ and $\lambda \in (0, 1)$,

$$\lambda G(x_1) + (1 - \lambda)G(x_2) \subseteq G(\lambda x_1 + (1 - \lambda)x_2).$$

(ii) C-convex on E, if for any $x_1, x_2 \in E$ and $\lambda \in (0, 1)$,

$$\lambda G(x_1) + (1 - \lambda)G(x_2) \subseteq G(\lambda x_1 + (1 - \lambda)x_2) + C.$$

(iii) C-concave on E, if for any $x_1, x_2 \in E$ and $\lambda \in (0, 1)$,

$$G(\lambda x_1 + (1 - \lambda)x_2) \subseteq \lambda G(x_1) + (1 - \lambda)G(x_2) + C.$$

Definition 2.5. (see [20]) Let P and Q be two topological vector spaces. Let D be a nonempty subset of P. A set-valued mapping $H : P \to 2^Q$ is said to be uniformly continuous on D, if for any neighborhood V of $0_Q \in Q$, there exists a neighborhood U of $0_P \in P$ such that for any $x_1, x_2 \in D$ with $x_1 - x_2 \in U$,

$$H(x_1) \subseteq H(x_2) + V.$$

The following two lemmas play an important role in the proof of the lower semicontinuity of the solution mapping $S(\cdot,\cdot)$.

Lemma 2.6. (see [38]) Let E be a convex subset of R^m and $G : E \to 2^{R^n}$ be a set-valued mapping with $G(x) \neq \emptyset$, for all $x \in E$. If G is convex on E and $x_0 \in \mathrm{int}E$, then G is l.s.c. at x_0.

Lemma 2.7. (see [5]) For each neighborhood U of 0_{R^m}, there exists a balanced open neighborhood U_1 of 0_{R^m} such that

$$U_1 + U_1 \subset U.$$

3. Lower semicontinuity

In this section, we discuss the lower semicontinuity and upper semicontinuity of the approximate solution mapping of PGSVEP.

Let $\mu_0 \in \Lambda$. We define a set-valued mapping $L_{\mu_0} : R^n \to 2^{R^m}$ by

$$L_{\mu_0}(e) = S(\mu_0, e), \forall e \in R^n.$$

Firstly, we provide a few crucial lemmas to obtain the lower semicontinuity of the approximate solution mapping $S(\cdot,\cdot)$ of PGSVEP.

Lemma 3.1. Let $\mu_0 \in \Lambda$, $E(\mu_0)$ be a nonempty convex subset of R^m and $\mathrm{dom}L_{\mu_0} \neq \emptyset$. If, for any $y \in E(\mu_0)$, $F(\cdot, y, \mu_0)$ is C-concave on $E(\mu_0)$, then $\mathrm{dom}L_{\mu_0}$ is convex and $L_{\mu_0}(\cdot)$ is convex on $\mathrm{dom}L_{\mu_0}$.

Proof. Take any $e_1, e_2 \in \mathrm{dom}L_{\mu_0}$, $x_1 \in L_{\mu_0}(e_1)$, $x_2 \in L_{\mu_0}(e_2)$ and $\lambda \in [0, 1]$. Then, by the definition of L_{μ_0}, for any $y \in E(\mu_0)$, we have $F(x_1, y, \mu_0) + e_1 \subseteq C$ and $F(x_2, y, \mu_0) + e_2 \subseteq C$. Therefore, by the convexity of C, we have

$$\begin{aligned}&\lambda[F(x_1, y, \mu_0) + e_1] + (1 - \lambda)[F(x_2, y, \mu_0) + e_2]\\ &= [\lambda F(x_1, y, \mu_0) + (1 - \lambda)F(x_2, y, \mu_0)] + [\lambda e_1 + (1 - \lambda)e_2] \subseteq C, \forall y \in E(\mu_0). \qquad (1)\end{aligned}$$

Since, for any $y \in E(\mu_0)$, $F(\cdot, y, \mu_0)$ is C-concave on $E(\mu_0)$,

$$F(\lambda x_1 + (1 - \lambda)x_2, y, \mu_0) \subseteq \lambda F(x_1, y, \mu_0) + (1 - \lambda)F(x_2, y, \mu_0) + C.$$

Then it follows from (1) that

$$F(\lambda x_1 + (1 - \lambda)x_2, y, \mu_0) + [\lambda e_1 + (1 - \lambda)e_2] \subseteq C + C = C, \forall y \in E(\mu_0). \qquad (2)$$

Since $E(\mu_0)$ is convex, it follows from $x_1 \in L_{\mu_0}(e_1) \subset E(\mu_0)$ and $x_2 \in L_{\mu_0}(e_2) \subset E(\mu_0)$ that

$$\lambda x_1 + (1 - \lambda)x_2 \in E(\mu_0).$$

Combining this with (2), we have $\lambda x_1 + (1 - \lambda)x_2 \in L_{\mu_0}(\lambda e_1 + (1 - \lambda)e_2)$. Thus $\mathrm{dom}L_{\mu_0}$ is convex and

$$\lambda L_{\mu_0}(e_1) + (1 - \lambda)L_{\mu_0}(e_2) \subseteq L_{\mu_0}(\lambda e_1 + (1 - \lambda)e_2).$$

So $L_{\mu_0}(\cdot)$ is convex on $\mathrm{dom}L_{\mu_0}$, and the proof is complete. □

Remark 3.2. When F is a function and $E(\cdot) \equiv E$ in Lemma 3.1, it follows from Lemma 3.1 that [32, Lemma 3.3] holds.

Lemma 3.3. Let $\mu_0 \in \Lambda$ and $E(\mu_0)$ be a convex subset of R^m. Let $e_0 \in \mathrm{int}(\mathrm{dom}L_{\mu_0})$. If for any $y \in E(\mu_0)$, $F(\cdot, y, \mu_0)$ is C-concave on $E(\mu_0)$, then $L_{\mu_0}(\cdot)$ is l.s.c. at e_0.

Proof. By Lemma 3.1, $\mathrm{dom}L_{\mu_0}$ is convex and $L_{\mu_0}(\cdot)$ is convex on $\mathrm{dom}L_{\mu_0}$. So it follows from Lemma 2.6 that $L_{\mu_0}(\cdot)$ is l.s.c. at e_0, and the proof is complete. □

Now we establish the lower semicontinuity of $S(\cdot, \cdot)$.

Theorem 3.4. Let $\mu_0 \in \Lambda$ and $E(\mu_0)$ be a nonempty convex subset of R^m. Let $e_0 \in \mathrm{int}(\mathrm{dom}L_{\mu_0})$. Suppose that the following conditions are satisfied:

(i) for any $y \in E(\mu_0)$, $F(\cdot, y, \mu_0)$ is C-concave on $E(\mu_0)$;

(ii) $E(\cdot)$ is H-u.s.c. and l.s.c. at μ_0;

(iii) $F(\cdot, \cdot, \cdot)$ is uniformly continuous on $E(\Lambda) \times E(\Lambda) \times N(\mu_0)$, where $N(\mu_0)$ is a neighborhood of μ_0.

Then $S(\cdot, \cdot)$ is l.s.c. at (μ_0, e_0).

Proof. Suppose to the contrary that $S(\cdot, \cdot)$ is not l.s.c. at (μ_0, e_0). Then there exist $x_0 \in S(\mu_0, e_0)$ and a neighborhood W_0 of 0_{R^m}, for any neighborhoods $U(\mu_0)$ and $V(e_0)$ of μ_0 and e_0, respectively, there exist $\mu \in U(\mu_0)$ and $e \in V(e_0)$ such that

$$(\{x_0\} + W_0) \bigcap S(\mu, e) = \emptyset.$$

Hence, there exist sequences $\{\mu_n\}$ with $\mu_n \to \mu_0$ and $\{e_n\}$ with $e_n \to e_0$ such that

$$(\{x_0\} + W_0) \bigcap S(\mu_n, e_n) = \emptyset, \forall n. \tag{3}$$

For above W_0, it follows from Lemma 2.7 that there exists a balanced neighborhood W_1 of 0_{R^m} such that

$$W_1 + W_1 \subset W_0. \tag{4}$$

By condition (i) and Lemma 3.3, we get that $L_{\mu_0}(\cdot)$ is l.s.c. at e_0. Thus, for above $x_0 \in S(\mu_0, e_0) = L_{\mu_0}(e_0)$ and W_1, there exists a balanced neighborhood $O(0_{R^n})$ of 0_{R^n} such that

$$(\{x_0\} + W_1) \bigcap L_{\mu_0}(e_0 + e) = (\{x_0\} + W_1) \bigcap S(\mu_0, e_0 + e) \neq \emptyset, \forall e \in O(0_{R^n}).$$

We choose $e' \in O(0_{R^n}) \bigcap \mathrm{int}C$. Then

$$(\{x_0\} + W_1) \bigcap L_{\mu_0}(e_0 - e') = (\{x_0\} + W_1) \bigcap S(\mu_0, e_0 - e') \neq \emptyset.$$

Take

$$x_1 \in (\{x_0\} + W_1) \bigcap S(\mu_0, e_0 - e'). \tag{5}$$

Since $e' \in \mathrm{int}C$, there exists $\delta_0 > 0$ such that

$$\delta_0 B_{R^n} + \{e'\} \subset C. \tag{6}$$

For above δ_0, it follows from Lemma 2.7 that there exists $\delta_1 > 0$ such that

$$\delta_1 B_{R^n} + \delta_1 B_{R^n} \subset \delta_0 B_{R^n}. \tag{7}$$

Since $e_n \to e_0$, there exists a natural number N_0 such that

$$e_n \in \{e_0\} + \delta_1 B_{R^n}, \forall n > N_0.$$

Therefore, from (6) and (7), we get that

$$\delta_1 B_{R^n} + \{e_n - e_0\} + \{e^{'}\} \subset C, \forall n > N_0. \tag{8}$$

Since $F(\cdot,\cdot,\cdot)$ is uniformly continuous on $E(\Lambda) \times E(\Lambda) \times N(\mu_0)$, for above $\delta_1 B_{R^n}$, there exist a neighborhood $W_1(0_{R^m})$ of 0_{R^m}, a neighborhood $\bar{W}_1(0_{R^m})$ of 0_{R^m} and a neighborhood $V(0_{R^l})$ of 0_{R^l}, for any $(x_1, y_1, \mu_1), (x_2, y_2, \mu_2) \in E(\Lambda) \times E(\Lambda) \times N(\mu_0)$ with $x_1 - x_2 \in W_1(0_{R^m}), y_1 - y_2 \in \bar{W}_1(0_{R^m})$ and $\mu_1 - \mu_2 \in V(0_{R^l})$, we have

$$F(x_1, y_1, \mu_1) \subset \delta_1 B_{R^n} + F(x_2, y_2, \mu_2). \tag{9}$$

Since $E(\cdot)$ is H-u.s.c. at μ_0, for above $\bar{W}_1(0_{R^m})$, there exists a neighborhood $W(\mu_0)$ of μ_0 such that

$$E(\mu) \subset E(\mu_0) + \bar{W}_1(0_{R^m}), \forall \mu \in W(\mu_0). \tag{10}$$

By (5), we can see that $x_1 \in E(\mu_0)$. Since $E(\cdot)$ is l.s.c. at μ_0, for $W_1 \bigcap W_1(0_{R^n})$, there exists a neighborhood $U_2(\mu_0)$ of μ_0, such that

$$[\{x_1\} + W_1 \bigcap W_1(0_{R^m})] \bigcap E(\mu) \neq \emptyset, \forall \mu \in U_2(\mu_0). \tag{11}$$

It follows from $\mu_n \to \mu_0$ that there exists μ_{n_0} with $n_0 > N_0$ such that

$$\mu_{n_0} \in W(\mu_0) \bigcap U_2(\mu_0) \bigcap N(\mu_0) \bigcap (\{\mu_0\} + V(0_{R^l}))$$

Then, by (10) and (11), we have

$$E(\mu_{n_0}) \subset E(\mu_0) + \bar{W}_1(0_{R^m}) \tag{12}$$

and

$$[\{x_1\} + W_1 \bigcap W_1(0_{R^m})] \bigcap E(\mu_{n_0}) \neq \emptyset. \tag{13}$$

We take

$$x_2 \in [\{x_1\} + W_1 \bigcap W_1(0_{R^m})] \bigcap E(\mu_{n_0}). \tag{14}$$

We show that $x_2 \in S(\mu_{n_0}, e_{n_0})$. By (5), we have

$$F(x_1, y, \mu_0) + \{e_0 - e^{'}\} \subseteq C, \forall y \in E(\mu_0). \tag{15}$$

For any $y^{'} \in E(\mu_{n_0})$, it follows from (12) that there exists $y_0 \in E(\mu_0)$ such that $y^{'} - y_0 \in \bar{W}_1(0_{R^m})$. By (14), $x_2 - x_1 \in W_1(0_{R^m})$. Noting that $\mu_{n_0} \in N(\mu_0) \bigcap (\{\mu_0\} + V(0_{R^l}))$, it follows from (9) that

$$F(x_2, y^{'}, \mu_{n_0}) \subset \delta_1 B_{R^n} + F(x_1, y_0, \mu_0).$$

Thus, it follows from $n_0 > N_0$ and (8) that $F(x_2, y^{'}, \mu_{n_0}) \subset C - \{e_{n_0}\} + \{e_0 - e^{'}\} + F(x_1, y_0, \mu_0), \forall y^{'} \in E(\mu_{n_0})$. Combining this with (15), we have

$$F(x_2, y^{'}, \mu_{n_0}) + e_{n_0} \subset C, \forall y^{'} \in E(\mu_{n_0}).$$

So

$$x_2 \in S(\mu_{n_0}, e_{n_0}). \tag{16}$$

According to (4), (5) and (14), we get $x_2 \in \{x_0\} + W_0$. Thus it follows from (16) that

$$[\{x_0\} + W_0] \bigcap S(\mu_{n_0}, e_{n_0}) \neq \emptyset,$$

which contradicts (3). This completes the proof. □

Remark 3.5. Our proof approach on the lower semicontinuity of the solution mapping $S(\cdot,\cdot)$ is different from the ones used in [20, 39–42]. In our approach, Lemma 2.6 plays an essential role.

We give an example to illustrate Theorem 3.4.

Example 3.6. Let $C = R^2_+ = \{(y_1, y_2) : y_1 \geq 0, y_2 \geq 0\}, D = [0,2]\times[0,2], \Lambda = [-2,2] \subset R,\ E(\mu) = [\mu, 2],\ \forall \mu \in \Lambda$. $F : [-2,2] \times [-2,2] \times \Lambda \to 2^Y$ is defined by

$$F(x, y, \mu) = \{(-(x-y)^2 - \mu, -x + y - \mu^2)\} + D.$$

Take $\mu_0 = 0$, $N(\mu_0) = [-1,1]$ and $e_0 = (5,3) \in \mathrm{dom}L_{\mu_0}$, where $\mathrm{dom}L_{\mu_0} = \{(y_1, y_2) \in Y | y_1 \geq 4, y_2 \geq 2\}$. It is easy to see that all assumptions of Theorem 3.4 are fulfilled, by Theorem 3.4, $S(\cdot,\cdot)$ is l.s.c. at (μ_0, e_0).

4. Conclusions

In this paper, we use a new tool that is different from the ones used in the literature to establish the lower semicontinuity of the approximate solution mapping to a parametric generalized strong vector equilibrium problem. Simultaneously, these assumptions of monotonicity and compactness are deleted.

Acknowledgements

The authors are most grateful to anonymous referees for careful reading of the manuscript and valuable suggestions.

References

[1] L.Q. Anh and P.Q. Khanh: *Semicontinuity of the solution set of parametric multivalued vector quasiequilibrium problems*, J. Math. Anal. Appl., 294(2004) 699-711.
[2] L.Q. Anh and P.Q. Khanh: *On the stability of the solution sets of general multivalued vector quasiequilibrium problems*, J. Optim. Theory Appl.,135(2007) 271-284.
[3] L.Q. Anh and P.Q. Khanh: *Semicontinuity of the approximate solution sets of multivalued quasiequilibrium problems,* Numer. Funct. Anal. Optim., 29(2008) 24-42.
[4] J.P. Aubin and I. Ekeland: "Applied Nonlinear Analysis", Wiley, New York,1984.
[5] C. Berge: " Topological spaces", Oliver and Boyd, London,1963.
[6] H. Brézis, L.Nirenberg and G. Stampacchia: A Remark on Ky Fan's Minimax Principle, Bollettino Unione Matematica Italiana, (III), VI(1972) 129-132.
[7] E. Blum and W. Oettli: *From optimization and variational inequalities to equilibrium problems*. Math. Stud., 63(1994)123-145.
[8] B. Chen and N.J. Huang: *Continuity of the solution mapping to parametric generalized vector equilibrium problems,* J. Glob. Optim., 56(2012) 1515-1528.
[9] C.R. Chen and S.J. Li: *Semicontinuity of the solution set map to a set-valued weak vector variational inequality,* J. Ind. Manag. Optim., 3(2007) 519-528.
[10] C.R. Chen and S.J. Li: *On the solution continuity of parametric generalized systems,* Pac. J. Optim., 6(2010) 141-151.
[11] C.R. Chen, S.J. Li and K.L. Teo: *Solution semicontinuity of parametric generalized vector equilibrium problems,* J. Glob. Optim., 45(2009)309-318.
[12] Y.H. Cheng and D.L. Zhu: *Global stability results for the weak vector variational inequality,* J. Glob. Optim., 32(2005)543-550.
[13] C. Chiang, O. Chadli and J.C. Yao: *Genralized Vector equilibrium problems with trifunctions,* J. Glob. Optim., 30(2004)135-154.
[14] K. Fan : A Minimax Inequality and Applications, in Inequality III(ed. O. Shisha), Academic Press, New York, 103-113(1972).
[15] J.F. Fu: *Generalized Vector quasi-equilibrium problems,* Math.Methods Oper.Res., 52(2000)57-64.
[16] J.F. Fu: *Vector equilibrium problems, existence theorems and convexity of solution set,* J. Glob. Optim., 31(2005)109-119.
[17] F. Giannessi,(ed.): Vector Variational Inequalities and Vector Equilibria: Mathematical Theories, Kluwer Academic Publishers, Dordrecht,2000.
[18] X.H. Gong: *Continuity of the solution set to parametric weak vector equilibrium problems,* J. Optim. Theory Appl., 139(2008)35-46.
[19] X.H. Gong and J.C. Yao: *Lower semicontinuity of the set of efficient solutions for generalized systems,* J. Optim. Theory Appl., 138(2008)197-205.
[20] Y. Han and X.H. Gong: *Lower semicontinuity of solution mapping to parametric generalized strong vector equilibrium problems*, Appl. Math. Lett., 28(2014)38-41.
[21] N.J. Huang, J. Li and H.B. Thompson: *Stability for parametric implicit vector equilibrium problems,* Math. Comput. Model., 43(2006)1267-1274.

[22] P.Q. Khanh and L.M. Luu: *Lower and upper semicontinuity of the solution sets and approximate solution sets to parametric multivalued quasivariational inequalities,* J. Optim. Theory Appl., 133(2007)329-339.

[23] B.T. Kien: *On the lower semicontinuity of optimal solution sets*, Optimization, 54(2005)123-130.

[24] K. Kimura and J.C. Yao: *Semicontinuity of solution mappings of parametric generalized strong vector equilibrium problems,* J. Ind. Manag. Optim., 4(2008)167-181.

[25] K. Kimura and J.C. Yao: *Sensitivity analysis of solution mappings of parametric vector quasi-equilibrium problems,* J. Glob. Optim., 41(2008)187-202.

[26] K. Kimura and J.C. Yao: *Sensitivity analysis of vector equilibrium problems,* Taiwanese J. Math., 12(2008)649-669.

[27] K. Kimura and J.C. Yao: *Sensitivity analysis of solution mappings of parametric generalized quasivector equilibrium problems,* Taiwanese J. Math., 12(2008)2233-2268.

[28] K. Kimura and J.C. Yao: *Semicontinuity of solutiong mappings of parametric generalized vector equilibrium problems*, J. Optim. Theory Appl., 138(2008)429-443.

[29] S.J. Li, G.Y. Chen and K.L. Teo: *On the stability of generalized vector quasivariational inequality problems,* J. Optim. Theory Appl., 113(2002)297-323.

[30] S.J. Li and C.R. Chen: *Stability of weak vector variational inequality,* Nonlinear Anal., 70(2009)1528-1535.

[31] S.J. Li and Z.M. Fang: *Lower semicontinuity of the solution mappings to a parametric generalized Ky Fan inequality,* J. Optim. Theory Appl., 147(2010)507-515.

[32] X.B. Li and S.J. Li: *Continuity of approximate solution mappings for parametric equilibrium problems,* J. Glob. Optim., 51(2011)541-548.

[33] X.B. Li, S.J. Li and C.R. Chen: *Lipschitz continuity of an approximate solution mapping to equilibrium problems,* Taiwanese J. Math., 16(2012)1027-1040.

[34] S.J. Li, H.M. Liu, Y. Zhang and Z.M. Fang: *Continuity of solution mappings to parametric generalized strong vector equilibrium problems,* J. Glob. Optim., 55(2013)597-610.

[35] L.J. Lin, Q.H. Ansari and J.Y. Wu: *Geometric properties and coincidence theorems with applications to generalized vector equilibrium problems,* J. Optim. Theory Appl., 117(2003)121-137.

[36] W. Oettli and D. Schl*ä*ger: Generalized equilibria and generalized monotonicity, Functional Analysis with Current Applications in Science, Technology and Industry, Edited by M. Brokate and A.H.Siddiqi, Longman, London, England, 145-154(1998).

[37] W. Oettli and D. Schl*ä*ger: *Existence of equilibria for monotone multivalued mappings*, Math. Methods Oper. Res., 48(1998)219-228.

[38] T. Tanino: *Stability and sensitivity analysis in convex vector optimization*, SIAM J. Control. Optim., 26(1988)521-536.

[39] Q.L. Wang and S.J. Li: *Lower semicontinuity of the solution mapping to a parametric generalized vector equilibrium problem,* J. Ind. Manag. Optim., 10(2014)1225-1234.

[40] R. Wangkeeree, R. Wangkeeree and P. Preechasilp: *Continuity of the solution mappings to parametric generalized vector equilibrium problems*, Appl. Math. Lett., 29(2014)42-45.

[41] W.Y. Zhang, Z.M. Fang and Y. Zhang: *A note on the lower semicontinuity of efficient solutions for parametric vector equilibrium problems*, Appl. Math. Lett., 26(2013)469- 472.

[42] Y. Han and N. J. Huang: *Some characterizations of the approximate solutions to generalized vector equilibrium problems*, J. Ind. Manag. Optim., 12(2016)1135-1151.