Filomat 32:19 (2018), 6599–6614
https://doi.org/10.2298/FIL1819599M

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Applications of Rough Soft Sets to Krasner (m, n)-Hyperrings and Corresponding Decision Making Methods

Xueling Ma[a], Jianming Zhan[a], Bijan Davvaz[b]

[a]*Department of Mathematics, Hubei Minzu University, Enshi, Hubei Province 445000, China*
[b]*Department of Mathematics, Yazd University, Yazd, Iran*

Abstract. Let I be a normal hyperideal of a Krasner (m, n)-hyperring R, we define the relation $\equiv_I$ by $x \equiv_I y$ if and only if $f(x, -y, \overset{(m-2)}{0}) \cap I \neq \emptyset$, which is an equivalence relation on R. By means of this idea, we propose rough soft hyperrings (hyperideals) with respect to a normal hyperideal in a Krasner (m, n)-hyperring. Some lower and upper rough soft hyperideals with respect to a normal hyperideal are investigated, respectively. Further, we define the t-level set $U(\mu, t) = \{(x, y) \in R \times R | \bigwedge_{z \in f(x, -y, \overset{(m-2)}{0})} \mu(z) \geq t\}$ of a Krasner (m, n)-hyperring R and prove that it is an equivalence relation on R if μ is a fuzzy normal hyperideal of R. By means of this novel idea, we propose rough soft hyperideals by means of fuzzy normal hyperideals in Krasner (m, n)-hyperrings. Finally, two novel kinds of decision making methods to rough soft Krasner (m, n)-hyperrings are established.

1. Introduction

It is well known that algebraic hyperstructures (or hypersystems) introduced by Marty [32] in 1934 are generalizations of algebraic structures. In recent years, this theory has attracted wide attentions and many applications in automata, cryptography, combinatorics, artificial intelligence, and so on. For more details, see the books, [9, 10, 15, 39]. A well known type of hyperrings, called the Krasner hyperring [22], has been investigated by many researchers, for examples, see [15, 25, 42].

n-groups were introduced by Dönte [17]. Afterward, (m, n)-rings and their quotient structures were investigated by Crombez, Timm and Dudek, for examples, see [11, 12, 19]. Recently, the research on n-ary hyperstructures has been initiated by Davvaz and Vougiouklis [16]. Leoreanu-Fotea [24] applied rough set theory to n-ary hypergroups. At the same time, Leoreanu-Fotea [23] introduced canonical n-ary hypergroups and discussed their properties. In 2010, Mirvakili and Davvaz [34] firstly introduced the concept of (m, n)-hyperrings. Further, Anvariyeh [6] studied fundamental relations on (m, n)-ary hypermodules. In particular, Ameri [4, 5] invesstigated prime and primary hyperideals and subhypermodules of (m, n)-hyperrings and (m, n)-hypermodules, respectively. Davvaz [13, 14] characterized fuzzy (m, n)-ary hyperideals and (m, n)-ary subhypermodules of (m, n)-hyperrings and (m, n)-hypermodules, respectively.

2010 *Mathematics Subject Classification*. 16Y99; 13E05; 20N20.
Keywords. Krasner (m, n)-hyperring; (fuzzy) (normal) hyperideal; rough soft hyperring (hyperideal); decision making.
Received: 10 January 2017; Accepted: 02 September 2017
Communicated by Dragan S. Djordjević
Research supported by a grant of National Natural Science Foundation of China (11561023; 61866011).
Email addresses: zjmhbmy@126.com (Xueling Ma), zhanjianming@hotmail.com (Jianming Zhan), davvaz@yazd.ac.ir (Bijan Davvaz)

The theory of rough set, proposed by Pawlak [36], is an extension of set theory, in which a subset of a universe is described by a pair of ordinary sets called the lower and upper approximations. We know that a key concept in Pawlak rough set model is an equivalent relation. The equivalence classes are the building blocks for the construction of the lower and upper approximations. According to Yao [40], there are two main generalized methods for the Pawlak rough set models: the constructive and the algebric methods. Based on these two methods, rough set theory has been combined with other mathematical theories, for examples, see [18, 38, 45–47]. Nowadays, this theory has been applied to many research fields, such as machine learning, expert systems, intelligent decision, pattern recognition, image processing, knowledge discovering and others.

In 1999, Molodtsov [35] proposed soft set theory as a novel mathematical tool for dealing with uncertainties from the viewpoint of parametrization. In 2003, Maji et al. [30] gave some operations on soft sets. In particular, Ali [2, 3] proposed some novel operations and reduction of parameters on soft sets. Moreover, Li [26] studied soft covering and its parameter reduction. Recently, the algebraic structures of soft sets have been studied increasingly. At the same time, soft sets have potential applications in many fields, such as decision making, information systems, description logic, forecasting, data analysis, and so on (see [7, 8]). We know that uncertainties which arise from various domains have many different natures so that we can not capture them by a single mathematical tool. Some researchers proposed some hybrid soft computing models, such as fuzzy soft sets, fuzzy rough sets, rough soft sets and soft rough fuzzy sets, and so on. In 1990, Dubois and Prade [18] put forward rough fuzzy sets and fuzzy rough sets. In 2011, Maji [31] proposed fuzzy soft sets. In particular, Feng [21] put forth rough soft sets and soft rough fuzzy sets. Moreover, Feng [20] introduced soft rough sets. At the same time, Meng [33] modified soft rough fuzzy sets and soft fuzzy rough sets. In 2014, Li [27] investigated the relationships among soft sets, soft rough sets and topologies. Recently, Sun [38] proposed another soft fuzzy rough set, which is different from [33]. Further, he applied soft fuzzy rough sets to decision making. Recently, some hybrid soft set models were established by Zhan et al., see [28, 29, 37, 41, 44].

In 2015, Zhan [42] originally applied rough soft sets to algebraic structures and put forth rough soft hemirings based on Feng's idea in [21]. In the present paper, we put forth a novel rough soft algebraic structure–Krasner (m, n)-hyperrings by means of another idea. Let I be a normal hyperideal of a Krasner (m, n)-hyperring R. Then we define the relation $\equiv_I$ by $x \equiv_I y$ if and only if $f(x, -y, \overset{(m-2)}{0}) \cap I \neq \emptyset$. It is clear that the relation $\equiv_I$ is an equivalence relation on R. Based on this idea, we propose the concept of rough soft hyperideals with respect to a normal hyperideal of a Krasner (m, n)-hyperring, which is different from Zhan's idea in [42]. Further, we show that $U(\mu, t)$ is an equivalence relation if μ is a fuzzy normal hyperideal of a Krasner (m, n)-hyperring. Based on this novel idea, we propose rough soft hyperideals based on a fuzzy normal hyperideal in Krasner (m, n)-hyperrings.

This paper is organized as follows: We recall some concepts and results on (m, n)-hyperrings, rough sets and soft sets in section 2. In section 3, we propose the concept of rough soft hyperideals w.r.t. a normal hyperideal of a Krasner (m, n)-hyperring and investigate some rough strong approximation operations. Further, we propose rough soft hyperideals based on a fuzzy normal hyperideal in Krasner (m, n)-hyperrings in section 4. In section 5, we put forth two novel kinds of decision making methods to rough soft Krasner (m, n)-hyperrings.

2. Preliminaries

A mapping $f : H^n \to \mathcal{P}^*(H)$ is called an n-ary hyperoperation. An algebraic system (H, f), where f is an n-ary hyperoperation defined on H, is called an n-ary hypergroupoid.

We shall use the following abbreviated notation.

The sequence of elements $x_i, x_{i+1}, \cdots, x_j$ is denoted by x_i^j. In the case $j < i$, it is the empty symbol. If $x_{i+1} = x_{i+2} = \cdots = x_{i+t} = x$, then we write $\overset{(t)}{x}$ instead of x_{i+1}^{i+t}. In this convention, $f(x_1, x_2, \cdots, x_n) = f(x_1^n)$, and $f(x_1, \cdots, x_i, \underbrace{x, \cdots, x}_{t}, x_{i+t+1}, \cdots, x_n) = f(x_1^i, \overset{(t)}{x}, x_{i+t+1}^n)$.

For non-empty subsets $A_1, A_2, \cdots, A_n$ of H we define $f(A_1^n) = f(A_1, \cdots, A_n) = \bigcup\{f(x_1^n) | x_i \in A_i, i = 1, 2, \cdots, n\}$.

An n-ary hyperoperation f is called associative if

$$f(x_1^{i-1}, f(x_i^{n+i-1}), x_{n+i}^{2n-1}) = f(x_1^{j-1}, f(x_j^{n+j-1}), x_{n+j}^{2n-1})$$

hold for all $1 \le i < j \le n$ and $x_1, x_2, \cdots, x_{2n-1} \in H$. An n-ary hypergroupoid with the associative n-ary hyperoperation is called an n-ary semihypergroup.

An n-ary hypergroupoid (H, f) in which the equation

$$b \in f(a_1^{i-1}, x_i, a_{i+1}^n)$$

has a solution $x_i \in H$ for every $a_1^{i-1}, a_{i+1}^n, b \in H$ and $1 \le i \le n$ is called an n-ary quasihypergroup; when (H, f) is an n-ary semihypergroup, (H, f) is called an n-ary hypergroup.

An element $e \in H$ is called a scalar neutral element if

$$x = f(\overset{(i-1)}{e}, x, \overset{(n-i)}{e}),$$

for every $1 \le i \le n$ and for every $x \in H$.

An element 0 of an n-ary semihypergroup (H, g) is called a zero element if for every $x_2^n \in H$ we have

$$g(0, x_2^n) = g(x_2, 0, x_3^n) = \cdots = g(x_2^n, 0) = 0.$$

Definition 2.1. *[23] Let (H, f) be a commutative n-ary hypergroup. (H, f) is called a canonical n-ary hypergroup if*

(1) there exists a unique $e \in H$ such that for every $x \in H$, $f(x, \overset{(n-1)}{e}) = x$;

(2) for all $x \in H$ there exists a unique $x^{-1} \in H$ such that $e \in f(x, x^{-1}, \overset{(n-1)}{e})$;

(3) if $x \in f(x_1^n)$, then for all i, we have $x_i \in f(x, x^{-1}, \cdots, x_{i-1}^{-1}, x_{i+1}^{-1}, \cdots, x_n^{-1})$.

We say that e is the scalar identity of (H, f) and x^{-1} is the inverse of x. Notice that the inverse of e is e.

Definition 2.2. *[34] A Krasner (m, n)-hyperring is an algebraic hyperstructure (R, f, g) which satisfies the following axioms:*

(1) (R, f) is a canonical m-hypergroup;

(2) (R, g) is an n-ary semigroup;

(3) The n-ary operation g is distributive with respect to the m-ary hyperoperation f, that is, for every $a_1^{i-1}, a_{i+1}^n, x_1^m \in R$, $1 \le i \le n$,

$$g(a_1^{i-1}, f(x_1^m), a_{i+1}^n) = f(g(a_i^{i-1}, x_1, a_{i+1}^n), \cdots, g(a_1^{i-1}, x_m, a_{i+1}^n)).$$

(4) 0 is a zero element (absorbing element) of the n-ary operation g, i.e., for every $x_2^{n-1} \in R$, we have

$$g(0, x_2^n) = g(x_2, 0, x_3^n) = \cdots = g(x_2^n, 0) = 0.$$

Throughout this paper, (R, f, g) (briefly, R) is always a Krasner (m, n)-hyperring.

Let S be a non-empty subset of a Krasner (m, n)-hyperring (R, f, g). If (S, f, g) is a Krasner (m, n)-hyperring, then S is called a subhyperring of R.

Let I be a non-empty subset of R and $1 \le i \le n$; we call I an i-ideal of R if it satisfies:

(1) I is a subhypergroup of the canonical m-ary hypergroup (R, f), i.e., (I, f) is a canonical n-ary hypergroup;

(2) For every $x_1^n \in R$, $g(x_1^{i-1}, I, x_{i+1}^n) \subseteq I$.

Also, if every $1 \le i \le n$, I is an i-hyperideal, then I is called a hyperideal of R. Every hyperideal of R is a subhyperring of R.

A hyperideal I of R is called normal if for every $r \in R$,

$$f(-r, I, r, \overset{(m-3)}{0}) \subseteq I.$$

Definition 2.3. *[13] A fuzzy set μ of R is called a fuzzy hyperideal of R if it satisfies:*

(1) $\min\{\mu(x_1), \mu(x_2), \cdots, \mu(x_m)\} \leq \bigwedge_{z \in f(x_1^n)} \mu(z)$*, for all* $x_1^m \in R$*;*

(2) $\mu(x) \leq \mu(-x)$*, for all* $x \in R$*;*

(3) $\max\{\mu(x_1), \mu(x_2), \cdots, \mu(x_m)\} \leq \mu(g(x_1^n))$*, for all* $x_1^n \in R$.

Further, a fuzzy hyperideal μ of R is called a fuzzy normal hyperideal of R if for all $x, y \in R$, $\min\{\mu(-x), \mu(y), \mu(x), \mu(\overset{(m-3)}{0})\} \leq \bigwedge_{z \in f(-x, y, x, \overset{(m-3)}{0})} \mu(z)$.

Lemma 2.4. *[13] A fuzzy set μ of R is a fuzzy (normal) hyperideal of R if and only if non-empty subset μ_t is a (normal) hyperideal of R for all $t \in [0, 1]$.*

Definition 2.5. *[35] A pair $\mathfrak{S} = (F, A)$ is called a soft set over U, where $A \subseteq E$ and $F : A \to \mathscr{P}(U)$ is a set-valued mapping.*

Definition 2.6. *Let (F, A) be a non-null soft set over R. Then*

(1) (F, A) is called a soft Krasner (m, n)-hyperring over R if $F(x)$ is a subhyperring of R for all $x \in \mathrm{Supp}(F, A)$;

(2) (F, A) is called a soft (resp., left, right) hyperideal over R if $F(x)$ is a (resp., left, right) hyperideal of S for all $x \in \mathrm{Supp}(F, A)$.

Definition 2.7. *[3] Let (F, A) and (G, B) be two soft sets over U.*

(1) *The extended intersection of (F, A) and (G, B), denoted by $(F, A) \sqcap_\varepsilon (F, B)$, is defined as the soft set (H, C), where $C = A \cup B$, and $\forall e \in C$,*

$$H(e) = \begin{cases} F(e) & if\ e \in A \backslash B, \\ G(e) & if\ e \in B \backslash A, \\ F(e) \cap G(e) & if\ e \in A \cap B. \end{cases}$$

(2) *The restricted intersection of (F, A) and (G, B), denoted by $(F, A) \Cap (F, B)$, is defined as the soft set (H, C), where $C = A \cap B$ and $H(c) = F(c) \cap G(c)$ for all $c \in C$.*

(3) *The extended union of (F, A) and (G, B), denoted by $(F, A)\widetilde{\cup}(F, B)$, is defined as the soft set (H, C), where $C = A \cup B$, and $\forall e \in C$,*

$$H(e) = \begin{cases} F(e) & if\ e \in A \backslash B, \\ G(e) & if\ e \in B \backslash A, \\ F(e) \cup G(e) & if\ e \in A \cap B. \end{cases}$$

(4) *The restricted union of (F, A) and (G, B), denoted by $(F, A) \cup_{\mathcal{R}} (F, B)$, is defined as the soft set (H, C), where $C = A \cap B$ and $H(c) = F(c) \cup G(c)$ for all $c \in C$.*

Note that restricted intersection was also known as bi-intersection and extended union was at first introduced and called union by Maji et al. [30].

Definition 2.8. *[36] For an approximation space (U, ρ), by a rough approximation in (U, ρ) we mean a mapping $Apr : P(U) \to P(U) \times P(U)$ defined for every $X \in P(U)$ by $Apr(X) = (Apr(X), Apr(X))$, where*

$$\underline{Apr}(X) = \{x \in U : [x]_\rho \subseteq X\},$$

and

$$\overline{Apr}(X) = \{x \in U : [x]_\rho \cap X \neq \emptyset\}.$$

$\underline{Apr}(X)$ is called a lower rough approximation of X and $\overline{Apr}(X)$ is called an upper rough approximation of X in (U, ρ). Moreover, X is called definable if $\underline{Apr}(X) = \overline{Apr}(X)$.

By combining rough sets and soft sets, Feng [21] put forth rough soft sets as follows:

Definition 2.9. *[21] Let (U,ρ) a Pawlak approximation space and $\mathfrak{S}=(F,A)$ a soft set over U. The lower and upper rough approximations of $\mathfrak{S}=(F,A)$ w.r.t. (U,ρ) are denoted by $\underline{\rho}(\mathfrak{S})=(\underline{F},A)$ and $\overline{\rho}(\mathfrak{S})=(\overline{F},A)$, which are soft sets over U with*

$\underline{F}(x)=\underline{\rho}(F(x))=\{y\in U|[y]_\rho\subseteq F(x)\}$ *and* $\overline{F}(x)=\overline{\rho}(F(x))=\{y\in U|[y]_\rho\cap F(x)\neq\emptyset\}$,
for all $x\in A$.

If $\underline{\rho}(\mathfrak{S})=\overline{\rho}(\mathfrak{S})$, $\mathfrak{S}$ is called definable; otherwise $\mathfrak{S}$ is called a rough soft set.

3. Rough soft hyperideals w.r.t. a normal hyperideal

Let I be a normal hyperideal of a Krasner (m,n)-hyperring R. Then we define a relation $\equiv_I$ by
$x\equiv_I y$ if and only if $f(x,-y,\overset{(m-2)}{0})\cap I\neq\emptyset$.
It is clear that the relation $\equiv_I$ is an equivalence relation on R (see [34]).

Let $[x]_I$ denote the equivalence class of x w.r.t. I. Then $[x]_I=f(I,x,\overset{(m-2)}{0})$ for all $x\in R$ (see [34]).
From now on, we use the pair (R,I) instead of the approximation space (U,ρ).

Definition 3.1. *Let I be a normal hyperideal of a Krasner (m,n)-hyperring R, (R,I) a Pawlak approximation space and $\mathfrak{S}=(F,A)$ a soft set over R. The lower and upper rough approximations of $\mathfrak{S}=(F,A)$ w.r.t. (R,I) are denoted by:*
$\underline{Apr}_I(\mathfrak{S})=(\underline{F}_I,A)$ *and* $\overline{Apr}_I(\mathfrak{S})=(\overline{F}_I,A)$*, which are soft sets over R with*

$\underline{F}_I(x)=\underline{Apr}_I(F(x))=\{y\in R|f(I,y,\overset{(m-2)}{0})\subseteq F(x)\}$ *and* $\overline{F}_I(x)=\overline{Apr}_I(F(x))=\{y\in R|f(I,y,\overset{(m-2)}{0})\cap F(x)\neq\emptyset\}$*, for all $x\in A$.*

(i) $\underline{Apr}_I(\mathfrak{S})=\overline{Apr}_I(\mathfrak{S})$, the soft set $\mathfrak{S}$ is said to be definable;

(ii) $\underline{Apr}_I(\mathfrak{S})\neq\overline{Apr}_I(\mathfrak{S})$, $\underline{Apr}_I(\mathfrak{S})$ ($\overline{Apr}_I(\mathfrak{S})$) is called a lower (upper) rough soft hyperring (resp., hyperideal) w.r.t. I over R, if $\underline{F}_I(x)$ ($\overline{F}_I(x)$) is a subhyperring (resp., hyperideal) of R, for all $x\in \mathrm{Supp}(F,A)$. Moreover, $\mathfrak{S}$ is called a rough soft hyperring (resp., hyperideal) w.r.t. I over R, if $\underline{F}_I(x)$ and $\overline{F}_I(x)$ are subhyperrings (resp., hyperideals) of R, for all $x\in \mathrm{Supp}(F,A)$.

Example 3.2. *Let $R=\{0,a,b,c\}$ and define a 2-ary hyperoperation $(+)$ as follows:*

$+$	0	a	b	c
0	0	a	b	c
a	a	$\{0,a\}$	c	$\{b,c\}$
b	b	c	0	a
c	c	$\{b,c\}$	a	$\{0,a\}$

Then $(R,+)$ is a canonical 2-ary hypergroup. Let g be an n-ary operation on R such that

$$g(x_1^n)=\begin{cases} b & if\ x_1^n\in\{b,c\},\\ 0 & else.\end{cases}$$

Then (R,f,g) is a Krasner $(2,n)$-hyperring.
Let $I=\{0,a\}$, then it is a normal hyperideal of R and $[0]_I=[a]_I=\{0,a\}$ and $[b]_I=[c]_I=\{b,c\}$.
Define a soft set $\mathfrak{S}=(F,A)$ over R, where $A=\{0,a\}$, by $F(0)=\{0,a,b\}$ and $F(a)=\{0,b\}$.
By calculations, $\underline{F}_I(0)=\{0,a\}$, $\overline{F}_I(0)=R$, $\underline{F}_I(a)=\emptyset$ and $\overline{F}_I(a)=R$. Thus, $\mathfrak{S}$ is a rough soft hyperideal w.r.t. I over R.

Example 3.3. *Consider $R=\{0,1,2\}$ with 3-ary hyperoperation f and 3-ary operation g as follows:*
$f(0,0,0)=0, f(1,1,1)=1, f(0,0,2)=0, f(1,2,2)=S,$
$f(0,0,1)=1, f(1,1,2)=S, f(0,2,2)=2, f(2,2,2)=2,$

$f(0,1,1)=1, f(0,1,2)=S$.
and $g(0,x_1,x_2)=0$ *for all* $x_1,x_2\in S$,
$g(1,1,1)=1, g(1,1,2)=g(1,2,2)=g(2,2,2)=2$.
such that f *and* g *are commutative. Then* (R,f,g) *is a Krasner (3,3)-hyperring. Let* $I=\{0,2\}$, *then it is a normal hyperideal of* R *and* $[0]_I=[2]_I=\{0,2\}$ *and* $[1]_I=\{1\}$.
Define a soft set $\mathfrak{S}=(F,A)$ *over* R, *where* $A=\{0\}$, *by* $F(0)=\{0,2\}$.
By calculations, $\underline{F}_I(0)=\{0,2\}$ *and* $\overline{F}_I(0)=R$. *Thus,* $\mathfrak{S}$ *is a rough soft hyperideal w.r.t.* I *over* R.

Denote $\hat{f}(I,\mathfrak{S})=f(I,F(x),\overset{(m-2)}{0})$ for all $x\in A$.

Lemma 3.4. *Let* I *be a normal hyperideal of* R *and* $\mathfrak{S}=(F,A)$ *a soft set over* R. *Then* $\overline{Apr_I}(\mathfrak{S})=\hat{f}(I,\mathfrak{S})$.

Proof. Let y be any element of $\overline{F}_I(x)$. Then $f(I,y,\overset{(m-2)}{0})\cap F(x)\neq\emptyset$, $\forall x\in A$, and so there exists $a\in R$ such that $a\in f(I,y,\overset{(m-2)}{0})\cap F(x)$, that is, $y\in f(I,a,\overset{(m-2)}{0})$ and $a\in F(x)$, hence $y\in f(I,F(x),\overset{(m-2)}{0})$. This means that $\overline{Apr_I}(\mathfrak{S})\subseteq\hat{f}(I,\mathfrak{S})$.

Conversely, if $y\in f(I,F(x),\overset{(m-2)}{0})$, then there exist $a\in I$ and $b\in F(x)$ such that $y\in f(a,b,\overset{(m-2)}{0})$, that is, $b\in f(-a,y,\overset{(m-2)}{0})\subseteq f(I,y,\overset{(m-2)}{0})$, and so, $b\in f(-a,y,\overset{(m-2)}{0})\cap F(x)$, that is, $y\in\overline{F}_I(x)$. This means that $\hat{f}(I,\mathfrak{S})\subseteq\overline{Apr_I}(\mathfrak{S})$. Thus, $\overline{Apr_I}(\mathfrak{S})=\hat{f}(I,\mathfrak{S})$.

Definition 3.5. *Let* $\mathfrak{S}=(F,A)$ *and* $\mathfrak{T}=(G,B)$ *be any two non-null soft sets over* R. *Then*
(1) $\dot{f}$*-hyperoperation of* $\mathfrak{S}$ *and* $\mathfrak{T}$, *denoted by* $f(\mathfrak{S},\mathfrak{T})$ *is defined by*
$\dot{f}(\mathfrak{S},\mathfrak{T})=\dot{f}((F,A),(G,B))=(H,A\times B)$,
where $H(x,y)=f(F(x),G(y),\overset{(m-2)}{0})$, *for all* $(x,y)\in(A,B)$.
(2) $\dot{g}$*-operation of* $\mathfrak{S}$ *and* $\mathfrak{T}$, *denoted by* $g(\mathfrak{S},\mathfrak{T})$ *is defined by*
$\dot{g}(\mathfrak{S},\mathfrak{T})=\dot{g}((F,A),(G,b))=(H,A\times B)$,
where $H(x,y)=g(F(x),G(y),\overset{(n-2)}{e})$, *for all* $(x,y)\in(A,B)$.

Now, we discuss some properties of lower and upper approximations in Krasner (m,n)-hyperrings.

Theorem 3.6. *Let* I *be a normal hyperideal of* R, $\mathfrak{S}=(F,A)$ *and* $\mathfrak{T}=(G,B)$ *any two non-null soft sets over* R. *Then*
(1) $\dot{f}(\overline{Apr_I}(\mathfrak{S}),\overline{Apr_I}(\mathfrak{T}))=\overline{Apr_I}(\dot{f}(\mathfrak{S},\mathfrak{T}))$;
(2) $\dot{f}(\underline{Apr}_I(\mathfrak{S}),\underline{Apr}_I(\mathfrak{T}))\subseteq\underline{Apr}_I(\dot{f}(\mathfrak{S},\mathfrak{T}))$.

Proof. (1) By Lemma 3.4, $\dot{f}(\overline{Apr_I}(\mathfrak{S}),\overline{Apr_I}(\mathfrak{T}))=\dot{f}(\hat{f}(I,\mathfrak{S}),\hat{f}(I,\mathfrak{T}))=\hat{f}(I,\dot{f}(\mathfrak{S},\mathfrak{T}))=\overline{Apr_I}(\dot{f}(\mathfrak{S},\mathfrak{T}))$.

(2) $\forall x\in\mathrm{Supp}(F,A), y\in\mathrm{Supp}(F,B)$, let $c\in f(\underline{F}_I(x),\underline{G}_I(y),\overset{(m-2)}{0})$, then $c\in f(a,b,\overset{(m-2)}{0})$ for some $a\in\underline{F}_I(x)$ and $b\in\underline{G}_I(y)$. Hence $f(I,a,\overset{(m-2)}{0})\subseteq F(x)$ and $f(I,b,\overset{(m-2)}{0})\subseteq G(y)$, and so $f(I,c,\overset{(m-2)}{0})\subseteq f(I,f(a,b,\overset{(m-2)}{0}),\overset{(m-2)}{0})=f(I,a,b,\overset{(m-3)}{0})=f(f(I,a,\overset{(m-2)}{0}),f(I,b,\overset{(m-2)}{0}),\overset{(m-2)}{0})\subseteq f(F(x),G(y),\overset{(m-2)}{0})=\dot{f}(\mathfrak{S},\mathfrak{T})$, which implies, $c\in\overline{Apr_I}(\dot{f}(\mathfrak{S},\mathfrak{T}))$. Thus, $\dot{f}(\underline{Apr}_I(\mathfrak{S}),\underline{Apr}_I(\mathfrak{T}))\subseteq\underline{Apr}_I(\dot{f}(\mathfrak{S},\mathfrak{T}))$.

The following example shows that inclusion symbol " $\subseteq$ " in above Theorem 3.6(2) may not be replaced an equal sign.

Example 3.7. *Let* $R=\{0,a,b,c\}$ *be a set with the a 2-ary hyperoperation* $(+)$ *and a 2-ary operation* $(\cdot)$ *as follows:*

$+$	0	a	b	c
0	0	a	b	c
a	a	$\{0,c\}$	c	$\{a,b\}$
b	b	c	$\{0,c\}$	$\{a,b\}$
c	c	c	$\{a,b\}$	$\{0,c\}$

$\cdot$	0	a	b	c
0	0	0	0	0
a	0	a	a	0
b	0	a	a	0
c	0	0	0	0

Then R is a (2,2)-hyperring. Let $I = \{0, c\}$, then it is a normal hyperideal of R and $[0]_I = [c]_I = \{0, c\}$ and $[a]_I = [b]_I = \{a, b\}$.

Define two soft sets $\mathfrak{S} = (F, A)$ and $\mathfrak{T} = (G, B)$ over R, where $A = \{0\}$, $B = \{a\}$, by $F(0) = \{0, a, c\}$, $G(a) = \{0, b, c\}$.

By calculations, $\underline{F}_I(0) = \{0, c\}$ and $\underline{G}_I(a) = \{0, c\}$. Hence, $\underline{F}_I(0) + \overline{G}_I(a) = \{0, c\}$, but $\underline{Apr}_I(F(0) + G(a)) = R$. This means that $\dot{f}(\underline{Apr}_I(\mathfrak{S}), \underline{Apr}_I(\mathfrak{T})) \subsetneq \underline{Apr}_I(\dot{f}(\mathfrak{S}, \mathfrak{T}))$.

Theorem 3.8. *Let I be a normal hyperideal of R with a scalar identity e, $\mathfrak{S} = (F, A)$ and $\mathfrak{T} = (G, B)$ any two non-null soft sets over R, then*

$$\dot{g}(\overline{Apr}_I(\mathfrak{S}), \overline{Apr}_I(\mathfrak{T})) \subseteq \overline{Apr}_I(\dot{g}(\mathfrak{S}, \mathfrak{T})).$$

Proof. For any $x \in \mathrm{Supp}(F, A)$, $y \in \mathrm{Supp}(F, B)$, we have

$$\begin{aligned}
\dot{g}(\overline{Apr}_I(\mathfrak{S}), \overline{Apr}_I(\mathfrak{T})) &= \dot{g}(\hat{f}(I, \mathfrak{S}), \hat{f}(I, \mathfrak{T}))\\
&= g(f(I, F(x), \overset{(m-2)}{0}), f(I, G(y), \overset{(m-2)}{0}), \overset{(m-2)}{e})\\
&= f(g(\overset{(2)}{I}, \overset{(n-2)}{e}), g(I, G(y), \overset{(n-2)}{e}), g(I, F(x), \overset{(n-2)}{e}),\\
&\quad g(F(x), G(y), \overset{(n-2)}{e}), \overset{(m-4)}{0})\\
&\subseteq f(I, g(F(x), G(y), \overset{(n-2)}{e}), \overset{(m-2)}{0})\\
&= f(I, \dot{g}(\mathfrak{S}, \mathfrak{T}), \overset{(m-2)}{0})\\
&= \hat{f}(I, \dot{g}(\mathfrak{S}, \mathfrak{T}))\\
&= \overline{Apr}_I(\dot{g}(\mathfrak{S}, \mathfrak{T})).
\end{aligned}$$

This completes the proof. □

If we strength the condition, we can obtain the following result:

Theorem 3.9. *Let I be an idempotent normal hyperideal of R with a scalar identity e, $\mathfrak{S} = (F, A)$ and $\mathfrak{T} = (G, B)$ any two non-null soft sets over R, then*

(1) $\dot{g}(\overline{Apr}_I(\mathfrak{S}), \overline{Apr}_I(\mathfrak{T})) = \overline{Apr}_I(\dot{g}(\mathfrak{S}, \mathfrak{T}))$;

(2) $\dot{g}(\underline{Apr}_I(\mathfrak{S}), \underline{Apr}_I(\mathfrak{T})) \subseteq \underline{Apr}_I(\dot{g}(\mathfrak{S}, \mathfrak{T}))$.

Proof. (1) It is similar to the proof of Theorem 3.8.

(2) For all $x \in \mathrm{Supp}(F, A)$, $y \in \mathrm{Supp}(F, B)$. Let $c \in g(\underline{F}_I(x), \underline{G}_I(y), \overset{(n-2)}{e})$, then $c = g(a, b, \overset{(n-2)}{e})$ for some $a \in \underline{F}_I(x)$ and $b \in \underline{G}_I(y)$, which implies $f(I, a, \overset{(m-2)}{0}) \subseteq F(x)$ and $f(I, b, \overset{(m-2)}{0}) \subseteq G(y)$. Hence $g(f(I, a, \overset{(m-2)}{0}), f(I, b, \overset{(m-2)}{0}), \overset{(m-2)}{e}) \subseteq g(F(x), G(y), \overset{(n-2)}{e})$.

Thus,

$$\begin{aligned}
g(f(I, a, \overset{(m-2)}{0}), f(I, b, \overset{(m-2)}{0}), \overset{(m-2)}{e}) &= f(g(\overset{(2)}{I}, \overset{(n-2)}{e}), g(I, b, \overset{(n-2)}{e}), g(a, I, \overset{(n-2)}{e}),\\
&\quad g(a, b, \overset{(n-2)}{e}), \overset{(m-4)}{0})\\
&= f(I, g(a, b, \overset{(n-2)}{e}), \overset{(m-2)}{0})\\
&= f(I, c, \overset{(m-2)}{0})\\
&\subseteq g(F(x), G(y), \overset{(n-2)}{e}).
\end{aligned}$$

This means that $c \in \underline{Apr}_I(\dot{g}(\mathfrak{S}, \mathfrak{T}))$. Therefore, $\dot{g}(\underline{Apr}_I(\mathfrak{S}), \underline{Apr}_I(\mathfrak{T})) \subseteq \underline{Apr}_I(\dot{g}(\mathfrak{S}, \mathfrak{T}))$. □

Definition 3.10. *A Krasner (m, n)-hyperring R is called idempotent if $g(\overset{(n)}{x}) = x$ for all $x \in R$.*

Example 3.11. *Let $R = \{0, a, b, c\}$ be a set with the 2-ary hyperoperation $(+)$ and 2-ary operation $(\cdot)$ as follows:*

$+$	0	a	b	c
0	0	a	b	c
a	a	$\{0, c\}$	b	a
b	b	b	$\{0, a, c\}$	b
c	c	a	b	0

$\cdot$	0	a	b	c
0	0	0	0	0
a	0	a	0	0
b	0	0	b	c
c	0	0	c	c

Then R is an idempotent Krasner $(2, 2)$-hyperring.

Theorem 3.12. *Let R be an idempotent Krasner (m, n)-hyperring R with a scalar identity e, I a normal hyperideal of R, $\mathfrak{S} = (F, A)$ and $\mathfrak{T} = (G, B)$ any two non-null soft sets over R. Then*
(1) $\overline{Apr}_I(\mathfrak{S}) \Cap \overline{Apr}_I(\mathfrak{T}) \subseteq \overline{Apr}_I(\dot{g}(\mathfrak{S}, \mathfrak{T}))$;
(2) $\underline{Apr}_I(\mathfrak{S}) \Cap \underline{Apr}_I(\mathfrak{T}) \subseteq \underline{Apr}_I(\dot{g}(\mathfrak{S}, \mathfrak{T}))$.

Proof. (1) For all $x \in \mathrm{Supp}(F, A), y \in \mathrm{Supp}(F, B)$, let $c \in \overline{F}_I(x) \cap \overline{G}_I(y)$. Then $f(I, c, \overset{(m-2)}{0}) \cap F(x) \neq \emptyset$ and $f(I, c, \overset{(m-2)}{0}) \cap G(y) \neq \emptyset$, and so there exist $a \in f(I, c, \overset{(m-2)}{0}) \cap F(x)$ and $b \in f(I, c, \overset{(m-2)}{0}) \cap G(y)$. That is $a \in f(d, c, \overset{(m-2)}{0})$, $a \in F(x)$, $b \in f(k, c, \overset{(m-2)}{0})$ and $b \in G(y)$ for some $d, k \in I$.

Thus

$$\begin{aligned} g(a, b, \overset{(n-2)}{e}) &\in g(f(d, c, \overset{(m-2)}{0}), f(k, c, \overset{(m-2)}{0}), \overset{(n-2)}{e}) \\ &= f(g(d, k, \overset{(n-2)}{e}), g(d, c, \overset{(n-2)}{e}), g(k, c, \overset{(n-2)}{e}), g(\overset{(2)}{c}, \overset{(n-2)}{e}), \overset{(m-4)}{0}) \\ &\subseteq f(c, I, \overset{(m-2)}{0}), \end{aligned}$$

which implies,

$$\begin{aligned} c &\in f(g(a, b, \overset{(n-2)}{e}), I, \overset{(m-2)}{0}) \\ &\subseteq f(g(F(x), G(y), \overset{(n-2)}{e}), I, \overset{(m-2)}{0}) \\ &\subseteq f(g(F(x), G(y), \overset{(n-2)}{e}), I, \overset{(m-2)}{0}) \\ &\subseteq f(\dot{g}(\mathfrak{S}, \mathfrak{T}), I, \overset{(m-2)}{0}) \\ &= \underline{\hat{f}(I, \dot{g}(\mathfrak{S}, \mathfrak{T}))} \\ &= \overline{Apr}_I(\dot{g}(\mathfrak{S}, \mathfrak{T})). \end{aligned}$$

This means that $\overline{Apr}_I(\mathfrak{S}) \Cap \overline{Apr}_I(\mathfrak{T}) \subseteq \overline{Apr}_I(\dot{g}(\mathfrak{S}, \mathfrak{T}))$.

(2) For all $x \in \mathrm{Supp}(F, A), y \in \mathrm{Supp}(F, B)$, let $c \in \underline{F}_I(x) \cap \underline{G}_I(y)$. Then $f(I, c, \overset{(m-2)}{0}) \subseteq F(x)$ and $f(I, c, \overset{(m-2)}{0}) \subseteq G(y)$.

Hence

$$\begin{aligned} g(f(I, c, \overset{(m-2)}{0}), f(I, c, \overset{(m-2)}{0}), \overset{(n-2)}{e}) &= f(g(\overset{(2)}{c}, \overset{(n-2)}{e}), g(c, I, \overset{(n-2)}{e}), g(I, c, \overset{(n-2)}{e}), \overset{(m-4)}{0}) \\ &= f(c, I, \overset{(m-2)}{0}) \\ &\subseteq g(F(x), G(y), \overset{(n-2)}{e}), \end{aligned}$$

which implies, $c \in \underline{Apr}_I(\dot{g}(\mathfrak{S}, \mathfrak{T}))$. This means that $\underline{Apr}_I(\mathfrak{S}) \Cap \underline{Apr}_I(\mathfrak{T}) \subseteq \underline{Apr}_I(\dot{g}(\mathfrak{S}, \mathfrak{T}))$.

Corollary 3.13. *Let R be an idempotent Krasner (m, n)-hyperring R with a scalar identity e and I a normal hyperideal of R. If $\mathfrak{S} = (F, A)$ and $\mathfrak{T} = (G, B)$ are a soft right hyperideal and a soft left hyperideal of R, respectively. Then*
(1) $\overline{Apr}_I(\mathfrak{S}) \Cap \overline{Apr}_I(\mathfrak{T}) = \overline{Apr}_I(\dot{g}(\mathfrak{S}, \mathfrak{T}))$;
(2) $\underline{Apr}_I(\mathfrak{S}) \Cap \underline{Apr}_I(\mathfrak{T}) = \underline{Apr}_I(\dot{g}(\mathfrak{S}, \mathfrak{T}))$.

Theorem 3.14. *Let I be a normal hyperideal of R and $\mathfrak{S} = (F, A)$ a soft hyperideal over R. Then $\mathfrak{S}$ is an upper rough soft hyperideal w.r.t. I over R.*

Proof. Let $y, z \in \overline{F}_I(x)$, then $f(I, y, \overset{(m-2)}{0}) \cap F(x) \neq \emptyset$ and $f(I, z, \overset{(m-2)}{0}) \cap F(x) \neq \emptyset$, and so there exist $a, b \in R$ such that $a \in f(I, y, \overset{(m-2)}{0}) \cap F(x)$ and $b \in f(I, z, \overset{(m-2)}{0}) \cap F(x)$, that is, $y \in f(I, a, \overset{(m-2)}{0})$, $z \in f(I, b, \overset{(m-2)}{0})$ and $a, b \in F(x)$. Hence $y, z \in f(I, F(x), \overset{(m-2)}{0})$. This means that $f(I, y, z, \overset{(m-3)}{0}) \subseteq f(I, F(x), \overset{(m-2)}{0}) = \hat{f}(I, \mathfrak{S})$. By Lemma 3.4, $\hat{f}(I, \mathfrak{S}) = \overline{Apr_I}(\mathfrak{S})$, and so, $f(I, y, z, \overset{(m-3)}{0}) \subseteq \overline{Apr_I}(\mathfrak{S})$. This means that $f(I, y, z, \overset{(m-3)}{0}) \subseteq \overline{F}_I(x)$. We can easily show that $-z \in \overline{F}_I(x)$. This proves that $\overline{F}_I(x)$ is a canonical m-ary hypergroup.

For any $x_1^n \in R$ and $y \in \overline{F}_I(x)$, then by the above discussion, $y \in f(I, F(x), \overset{(m-2)}{0})$. Hence, $g(x_1^{i-1}, y, x_{i+1}^n) \subseteq g(x_1^{i-1}, f(I, F(x), \overset{(m-2)}{0}), x_{i+1}^n) = f(g(x_1^{i-1}, I, x_{i-1}^n), g(x_1^{i-1}, F(x), x_{i+1}^n), \overset{(m-2)}{0})$. Since I and $F(x)$ are hyperideals of R, $g(x_1^{i-1}, I, x_{i+1}^n) \subseteq I$ and $g(x_1^{i-1}, F(x), x_{i+1}^n) \subseteq F(x)$. Thus, $g(x_1^{i-1}, y, x_{i+1}^n) \subseteq f(I, F(x), \overset{(m-2)}{0}) = \overline{Apr_I}(\mathfrak{S})$. This means that $g(x_1^{i-1}, y, x_{i+1}^n) \in \overline{F}_I(x)$.

This shows that for all $x \in \mathrm{Supp}(F, A)$, $\overline{F}_I(x)$ is a hyperideal of R. Thus, $\overline{Apr_I}(\mathfrak{S})$ is an upper rough soft hyperideal over R.

Theorem 3.15. *Let I be a normal hyperideal of R and $\mathfrak{S} = (F, A)$ any soft hyperideal over R. Then $\underline{Apr}_I(\mathfrak{S}) \neq \emptyset \Leftrightarrow \underline{Apr}_I(\mathfrak{S}) = \mathfrak{S}$.*

Proof. If $\underline{Apr}_I(\mathfrak{S}) \neq \emptyset$, then for all $x \in \mathrm{Supp}(F, A)$, there exists $a \in \underline{F}_I(x) \subseteq F(x)$, that is, $f(I, y, \overset{(m-2)}{0}) \subseteq F(x)$.

Hence

$$\begin{aligned} I &\subseteq f(-y, I, y, \overset{(m-3)}{0}) \\ &= f(-y, f(I, y, \overset{(m-2)}{0}), \overset{(m-2)}{0}) \\ &\subseteq f(-y, F(x), \overset{(m-2)}{0}) \\ &\subseteq F(x). \end{aligned}$$

For any $a \in F(x)$, then $f(a, I, \overset{(m-2)}{0}) \subseteq f(a, F(x), \overset{(m-2)}{0}) \subseteq F(x)$, this implies that $a \in \underline{F}_I(x)$, so $F(x) \subseteq \underline{F}_I(x)$, hence $\underline{F}_I(x) = F(x)$, $\forall x \in \mathrm{Supp}(F, A)$. This means that $\underline{Apr}_I(\mathfrak{S}) = \mathfrak{S}$.

Conversely, if $\underline{Apr}_I(\mathfrak{S}) = \mathfrak{S}$, it is clear that $\underline{Apr}_I(\mathfrak{S}) \neq \emptyset$. This completes the proof.

4. Rough soft hyperideals w.r.t. a fuzzy normal hyperideal

In this section, we show that $U(\mu, t)$ is an equivalence relation if μ is a fuzzy normal hyperideal of a Krasner (m, n)-hyperring. Based on this novel idea, we propose rough soft hyperideals w.r.t. a fuzzy normal hyperideal in Krasner (m, n)-hyperrings.

Definition 4.1. *Let μ be a fuzzy set of R. For each $t \in [0, \mu(0)]$, the set $U(\mu, t) = \{(x, y) \in R \times R \mid \bigwedge_{z \in f(x, -y, \overset{(m-2)}{0})} \mu(z) \geq t\}$ is called a t-level relation of μ.*

Lemma 4.2. *Let μ be a fuzzy normal hyperideal of R and $t \in [0, \mu(0)]$, the set $U(\mu, t)$ is an equivalence relation on R.*

Proof. (1) For any $x \in R$, $0 \in f(x,-x,\overset{(m-2)}{0})$, then $\mu(0) \geq t$, which implies, $(x,x) \in U(\mu,t)$.

(2) If $(x,y) \in U(\mu,t)$, then $\bigwedge\limits_{z \in f(x,-y,\overset{(m-2)}{0})} \mu(z) \geq t$, and so

$$\begin{aligned}\bigwedge_{z \in f(y,-x,\overset{(m-2)}{0})} \mu(z) &= \bigwedge_{-z \in f(y,-x,\overset{(m-2)}{0})} \mu(-z)\\ &= \bigwedge_{z \in f(x,-y,\overset{(m-2)}{0})} \mu(-z)\\ &= \bigwedge_{z \in f(x,-y,\overset{(m-2)}{0})} \mu(z)\\ &\geq t,\end{aligned}$$

which implies, $(y,x) \in U(\mu,t)$.

(3) If $(x,y) \in U(\mu,t)$ and $(y,z) \in U(\mu,t)$, then $\bigwedge\limits_{w \in f(x,-y,\overset{(m-2)}{0})} \mu(w) \geq t$ and $\bigwedge\limits_{w \in f(y,-z,\overset{(m-2)}{0})} \mu(w) \geq t$.

Hence,

$$\begin{aligned}\bigwedge_{w \in f(x,-z,\overset{(m-2)}{0})} \mu(w) &= \bigwedge_{w \in f(f(x,-y,\overset{(m-2)}{0}),f(y,-z,\overset{(m-2)}{0}),\overset{(m-2)}{0})} \mu(w)\\ &\geq \bigwedge_{w \in f(x,-y,\overset{(m-2)}{0})} \mu(w) \wedge \bigwedge_{w \in f(y,-z,\overset{(m-2)}{0})} \mu(w) \wedge \mu(0)\\ &\geq t \wedge t\\ &= t,\end{aligned}$$

which implies, $(x,z) \in U(\mu,t)$.

This proves that $U(\mu,t)$ is an equivalence relation on R.

Let μ be a fuzzy normal hyperideal of R and $t \in [0,\mu(0)]$. By the above lemma, $U(\mu,t)$ is an equivalence relation on R. Hence, when $U = R$ and ρ is the above equivalence relation, then we use (R,μ,t) instead of approximation space (U,ρ).

Definition 4.3. *Let (R,μ,t) be a Pawlak approximation space and $\mathfrak{S} = (F,A)$ a soft set over R. The lower and upper rough approximations of $\mathfrak{S} = (F,A)$ w.r.t. $U(\mu,t)$ are denoted by*

$\underline{U}(\mu,t,\mathfrak{S}) = (\underline{F}_\mu, A)$ and $\overline{U}(\mu,t,\mathfrak{S}) = (\overline{F}_\mu, A)$, which are soft sets over R with

$\underline{F}_\mu(x) = \underline{U}(\mu,t,F(x)) = \{y \in R | f(\mu_t, y, \overset{(m-2)}{0}) \subseteq F(x)\}$ and $\overline{F}_\mu(x) = \overline{U}(\mu,t,F(x)) = \{y \in R | f(\mu_t, y, \overset{(m-2)}{0}) \cap F(x) \neq \emptyset\}$, for all $x \in A$.

(i) $\underline{U}(\mu,t,\mathfrak{S}) = \overline{U}(\mu,t,\mathfrak{S})$, the soft set $\mathfrak{S}$ is said to be definable;

(ii) $\underline{U}(\mu,t,\mathfrak{S}) \neq \overline{U}(\mu,t,\mathfrak{S})$, $\underline{U}(\mu,t,\mathfrak{S})$ ($\overline{U}(\mu,t,\mathfrak{S})$) is called a lower (upper) rough soft hyperideal w.r.t. $U(\mu,t)$ over R, if $\underline{F}_\mu(x)$ ($\overline{F}_\mu(x)$) is a hyperideal of R, for all $x \in \mathrm{Supp}(F,A)$. Moreover, $\mathfrak{S}$ is called a rough soft hyperideal w.r.t. $U(\mu,t)$ over R, if $\underline{F}_\mu(x)$ and $\overline{F}_\mu(x)$ are hyperideals of R, for all $x \in \mathrm{Supp}(F,A)$.

Example 4.4. *Consider the Krasner $(2,n)$-hyperring R as in Example 3.2. Define a fuzzy set μ of R by*

$$\mu(x) = \begin{cases} 0.6 & if\ x \in \{0,a\},\\ 0.2 & otherwise. \end{cases}$$

Let $t = 0.5$, then $\mu_t = \{0,a\}$ is a normal hyperideal of R. Define a soft set $\mathfrak{S} = (F,A)$ over R, where $A = \{e\}$, by $F(e) = \{0,a,b\}$.
By calculations, $F_\mu(e) = \{0,a\}$ and $F_\mu(e) = R$. This shows that $\mathfrak{S}$ is a rough soft hyperideal w.r.t. $U(\mu,t)$ over R.

Combining Lemma 3.4 and Definition 4.3, we can easily obtain the following result.

Lemma 4.5. *Let μ be a fuzzy normal hyperideal of R, $t \in [0, \mu(0)]$ and $\mathfrak{S} = (F, A)$ a soft set over R. Then $\overline{U}(\mu, t, \mathfrak{S}) = f(\mu_t, \mathfrak{S}, \overset{(m-2)}{0})$, that is, for all $x \in \mathrm{Supp}(F, A)$, $\overline{U}(\mu, t, F(x)) = f(\mu_t, F(x), \overset{(m-2)}{0})$.*

Theorem 4.6. *Let μ be a fuzzy normal hyperideal of R and $t \in [0, \mu(0)]$. If $\mathfrak{S} = (F, A)$ is a soft hyperideal over R. Then*

(1) $\overline{U}(\mu, t, \mathfrak{S})$ is an upper rough soft hyperideal w.r.t. $U(\mu, t)$ over R;

(2) If $\underline{U}(\mu, t, \mathfrak{S}) \neq \emptyset$, $\underline{U}(\mu, t, \mathfrak{S})$ is a lower rough soft hyperideal w.r.t. $U(\mu, t)$ over R.

Proof. It is similar to the proof of Theorems 3.14 and 3.15.

Definition 4.7. *Let μ and ν be two fuzzy sets of R. The sum $\mu + \nu$ of μ and ν is defined as follows:*

$$(\mu + \nu)(x) = \bigvee_{x \in f(a, b, \overset{(m-2)}{0})} \mu(a) \wedge \nu(b).$$

Lemma 4.8. *Let μ and ν be two fuzzy sets of R and $t \in [0, \mu(0)]$. Then*

$$f(\mu_t, \nu_t, \overset{(m-2)}{0}) = (\mu + \nu)_t.$$

Proof. Let $x \in f(\mu_t, \nu_t, \overset{(m-2)}{0})$, then there exist $a \in \mu_t$ and $b \in \nu_t$ such that $x \in f(a, b, \overset{(m-2)}{0})$. Hence $(\mu + \nu)(x) = \bigvee_{x \in f(a', b', \overset{(m-2)}{0})} \mu(a') \wedge \nu(b') \geq \mu(a) \wedge \nu(b) \geq t$, which implies, $x \in (\mu + \nu)_t$, that is, $f(\mu_t, \nu_t, \overset{(m-2)}{0}) \subseteq (\mu + \nu)_t$.

Conversely, let $x \in (\mu + \nu)_t$, then $(\mu + \nu)(x) \geq t$, and so, $\bigvee_{x \in f(a, b, \overset{(m-2)}{0})} \mu(a) \wedge \nu(b) \geq t$, then there exist $c, d \in R$ such that $x \in f(c, d, \overset{(m-2)}{0})$ and $\mu(c) \wedge \nu(d) \geq t$, that is, $\mu(c) \geq t$ and $\nu(d) \geq t$, and so, $c \in \mu_t$ and $d \in \nu_t$. Hence $x \in f(\mu_t, \nu_t, \overset{(m-2)}{0})$, that is, $(\mu + \nu)_t \subseteq f(\mu_t, \nu_t, \overset{(m-2)}{0})$. This completes the proof.

Remark 4.9. *Let μ and ν be two fuzzy sets of R with a scalar identity e, $t \in [0, \mu(0)]$ and $\mathfrak{S} = (F, A)$ a soft set over R. For all $x, y \in \mathrm{Supp}(F, A)$;*

(1) Denote $\overline{f}(\overline{U}(\mu, t, \mathfrak{S}), \overline{U}(\nu, t, \mathfrak{S}))$ by $f(\overline{U}(\mu, t, F(x)), \overline{U}(\nu, t, F(y)), \overset{(m-2)}{0})$;

(2) Denote $\underline{f}(\underline{U}(\mu, t, \mathfrak{S}), \underline{U}(\nu, t, \mathfrak{S}))$ by $f(\underline{U}(\mu, t, F(x)), \underline{U}(\nu, t, F(y)), \overset{(m-2)}{0})$;

(3) Denote $\overline{g}(\overline{U}(\mu, t, \mathfrak{S}), \overline{U}(\nu, t, \mathfrak{S}))$ by $g(\overline{U}(\mu, t, F(x)), \overline{U}(\nu, t, F(y)), \overset{(n-2)}{e})$;

(4) Denote $\underline{g}(\underline{U}(\mu, t, \mathfrak{S}), \underline{U}(\nu, t, \mathfrak{S}))$ by $g(\underline{U}(\mu, t, F(x)), \underline{U}(\nu, t, F(y)), \overset{(n-2)}{e})$.

Definition 4.10. *Let $\mathfrak{S} = (F, A)$ be a soft set over R. Then*

(1) $\mathfrak{S}$ is called f-closed if for all $x, y \in A$, then there exists $z \in A$ such that $f(F(x), F(y), \overset{(m-2)}{0}) \subseteq F(z)$;

(2) $\mathfrak{S}$ is called g-closed if for all $x, y \in A$, then there exists $z \in A$ such that $g(F(x), F(y), \overset{(n-2)}{e}) \subseteq F(z)$.

Theorem 4.11. *Let μ and ν be two fuzzy normal hyperideals of R with a scalar identity e. If $\mathfrak{S} = (F, A)$ is a g-closed soft set over R. Then*

$$\overline{g}(\overline{U}(\mu, t, \mathfrak{S}), \overline{U}(\nu, t, \mathfrak{S})) \subseteq \overline{U}(\mu + \nu, t, \mathfrak{S}).$$

Proof. Let μ and ν be two fuzzy normal hyperideals of R, then by Lemma 2.4, μ_t and ν_t are normal hyperideals of R for all $t \in [0, \mu(0)]$.

For any $x, y \in \mathrm{Supp}(F, A)$, then by Lemmas 4.5 and 4.8, we have

$$
\begin{aligned}
g(\overline{U}(\mu,t,F(x)),\overline{U}(\nu,t,F(y)),\overset{(n-2)}{e}) &= g(f(\mu_t,F(x),\overset{(m-2)}{0}),f(\nu_t,F(y),\overset{(m-2)}{0}),\overset{(n-2)}{e})\\
&= f(g(\mu_t,\nu_t,\overset{(n-2)}{e}),g(\mu_t,F(y),\overset{(n-2)}{e}),\\
&\quad g(F(x),\mu_t,\overset{(n-2)}{e}),g(F(x),F(y),\overset{(n-2)}{e}),\overset{(m-4)}{0})\\
&\subseteq f(\mu_t,\nu_t,g(F(x),F(y),\overset{(n-2)}{e}),\overset{(m-3)}{0})\\
&\subseteq f(\mu_t,\nu_t,F(z),\overset{(m-3)}{0}) \quad (\text{since } \mathfrak{S} \text{ is g} - \text{closed})\\
&= f(f(\mu_t,\nu_t,\overset{(m-2)}{0}),F(z),\overset{(m-2)}{0})\\
&= \underline{f}((\mu+\nu)_t,F(z),\overset{(m-2)}{0})\\
&= \overline{U}(\mu+\nu,t,F(z))\\
&\subseteq \overline{U}(\mu+\nu,t,\mathfrak{S}).
\end{aligned}
$$

This completes the proof.

Theorem 4.12. *Let μ and ν be two fuzzy normal hyperideals of R. If $\mathfrak{S}=(F,A)$ is an f-closed soft set over R. Then*
(1) $\overline{f}(\overline{U}(\mu,t,\mathfrak{S}),\overline{U}(\nu,t,\mathfrak{S}))\subseteq\overline{U}(\mu+\nu,t,\mathfrak{S})$;
(2) $\underline{f}(\underline{U}(\mu,t,\mathfrak{S}),\underline{U}(\nu,t,\mathfrak{S}))\subseteq\underline{U}(\mu+\nu,t,\mathfrak{S})$.

Proof. Let μ and ν be two fuzzy normal hyperideals of R, then by Lemma 2.4, μ_t and ν_t are normal hyperideals of R for all $t\in[0,\mu(0)]$.

(1) For any $x,y\in \mathrm{Supp}(F,A)$, then by Lemmas 4.5 and 4.8, we have

$$
\begin{aligned}
f(\overline{U}(\mu,t,F(x)),\overline{U}(\nu,t,F(y)),\overset{(m-2)}{0}) &= f(f(\mu_t,F(x),\overset{(m-2)}{0}),f(\nu_t,F(y),\overset{(m-2)}{0}),\overset{(m-2)}{0})\\
&= f(\mu_t,\nu_t,F(x),F(y),\overset{(m-4)}{0})\\
&= f(f(\mu_t,\nu_t,\overset{(m-2)}{0}),f(F(x),F(y),\overset{(m-2)}{0}),\overset{(m-2)}{0})\\
&= f((\mu+\nu)_t,f(F(x),F(y),\overset{(m-2)}{0}),\overset{(m-2)}{0})\\
&\subseteq \underline{f}((\mu+\nu)_t,F(z),\overset{(m-2)}{0}) \quad (\text{since } \mathfrak{S} \text{ is f} - \text{closed})\\
&= \overline{U}(\mu+\nu,t,F(z))\\
&\subseteq \overline{U}(\mu+\nu,t,\mathfrak{S}).
\end{aligned}
$$

(2) Let $x\in \underline{f}(\underline{U}(\mu,t,\mathfrak{S}),\underline{U}(\nu,t,\mathfrak{S}))$, then $x\in f(a,b,\overset{(m-2)}{0})$ for some $a\in\underline{U}(\mu,t,\mathfrak{S})$ and $b\in\underline{U}(\nu,t,\mathfrak{S})$, and so, $f(\mu_t,a,\overset{(m-2)}{0})\subseteq F(x)$ and $f(\nu_t,b,\overset{(m-2)}{0})\subseteq F(x)$.

Hence,

$$
\begin{aligned}
f(x,(\mu+\nu)_t,\overset{(m-2)}{0}) &= f(x,\mu_t,\nu_t,\overset{(m-2)}{0})\\
&\subseteq f(f(a,b,\overset{(m-2)}{0}),\mu_t,\nu_t,\overset{(m-3)}{0})\\
&= f(f(a,\mu_t,\overset{(m-2)}{0}),f(b,\nu_t,\overset{(m-2)}{0}),\overset{(m-3)}{0})\\
&\subseteq f(F(x),F(x),\overset{(m-2)}{0})\\
&\subseteq F(z), (\text{since } \mathfrak{S} \text{ is f} - \text{closed})
\end{aligned}
$$

which implies, $x\in\underline{U}(\mu+\nu,t,\mathfrak{S})$. This completes the proof.

5. Applications of rough soft Krasner (m, n)-hyperrings in decision making

In this section, we illustrate two novel kinds of decision making methods for rough soft Krasner (m, n)-hyperrings, and provide some relevant algebraic and applied examples, respectively. Maybe it would be served as a foundation of rough soft set theory and other decision making methods in different areas, such as theoretical computer sciences, information sciences and intelligent systems, and so on.

Decision making method I:

Let R be a Krasner (m, n)-hyperring and E a set of related parameters. Let $A = \{e_1, e_2, \cdots, e_m\} \subseteq E$ and $\mathfrak{S} = (F, A)$ be an original description soft set over R. Let I be a normal hyperideal of R and (R, I) be a Pawlak approximation space. Then we present the decision algorithm for rough soft hyperrings as follows:

Step 1 Input the original description Krasner (m, n)-hyperring R, soft set $\mathfrak{S}$ and Pawlak approximation space (R, I), where I is a normal hyperideal of R.

Step 2 Compute the lower and upper rough soft approximation operators $\underline{Apr}_I(\mathfrak{S})$ and $\overline{Apr}_I(\mathfrak{S})$ on $\mathfrak{S}$, respectively.

Step 3 Compute the different values of $\|F(e_i)\|$, where $\|F(e_i)\| = \frac{|\overline{F}_I(e_i)|-|\underline{F}_I(e_i)|}{|F(e_i)|}$.

Step 4 Find the minimum value $\|F(e_k)\|$ of $\|F(e_i)\|$, where $\|F(e_k)\| = \min\limits_i \|F(e_i)\|$.

Step 5 The decision is $F(e_k)$.

Example 5.1. Assume that we want to find the nearest accurate Krasner (m, n)-hyperring on a soft set $\mathfrak{S}$. Let $R = \{0, a, b, c\}$ be a set with the a 2-ary hyperoperation $(+)$ and a 2-ary operation $(\cdot)$ as follows:

$+$	0	a	b	c
0	0	a	b	c
a	a	$\{0, c\}$	b	$\{a, c\}$
b	b	b	$\{0, a, c\}$	b
c	c	$\{0, c\}$	b	$\{0, a\}$

$\cdot$	0	a	b	c
0	0	0	0	0
a	0	a	0	c
b	0	0	b	0
c	0	c	0	c

Then R is a (2,2)-hyperring. Let $I = \{0, c\}$ be a normal hyperideal of R. Define a soft set $\mathfrak{S} = (F, A)$ over R, where $A = \{e_1, e_2, e_3, e_4\}$. The tabular representation of the soft set $\mathfrak{S}$ is given in Table 1.

Table 1 Table for soft set $\mathfrak{S}$

	0	a	b	c
e_1	1	1	1	0
e_2	1	0	1	1
e_3	0	1	1	0
e_4	1	0	0	0

Now, the tabular representations of two soft sets $\underline{Apr}_I(\mathfrak{S})$ and $\overline{Apr}_I(\mathfrak{S})$ over R are given by Tables 2 and 3, respectively.

Table 2 Table for soft set $\underline{Apr}_I(\mathfrak{S})$

	0	a	b	c
e_1	1	1	0	0
e_2	1	0	0	1
e_3	0	1	0	0
e_4	1	0	0	0

Table 3 Table for soft set $\overline{Apr}_I(\mathfrak{S})$

	0	a	b	c
e_1	1	1	1	1
e_2	1	1	1	1
e_3	1	1	1	1
e_4	1	1	0	1

Then, we can calculate $||F(e_1)|| = 0.67$, $||F(e_2)|| = 0.67$, $||F(e_3)|| = 1.5$, $||F(e_4)|| = 2$. *This means the minimum value for* $||F(e_i)||$ *is* $||F(e_1)|| = ||F(e_2)|| = 0.67$. *That is,* $F(e_1)$ *and* $F(e_2)$ *are the closest accurate (2,2)-hyperrings on* $\mathfrak{S}$.

Finally, we revise the above algorithm according to weighted choice values of objects.

Decision making method II:

Let R be a Krasner (m,n)-hyperring and E a set of related parameters. Let $A = \{e_1, e_2, \cdots, e_m\} \subseteq E$, denote by w_i the weight value of e_i $(i = 1, 2 \cdots, m)$, where $\sum_{i=1}^{m} e_i = 1$, and $\mathfrak{S} = (F, A)$ be an original description soft set over R. Let I be a normal hyperideal of R and (R, I) be a Pawlak approximation space. Then we present the decision algorithm for rough soft Krasner (m,n)-hyperrings as follows:

Step 1 Input the original description Krasner (m,n)-hyperring R, soft set $\mathfrak{S}$ and Pawlak approximation space (R, I), where I is a normal hyperideal of R.

Step 2 Compute the lower and upper rough soft approximation operators $\underline{Apr}_I(\mathfrak{S})$ and $\overline{Apr}_I(\mathfrak{S})$ on $\mathfrak{S}$, respectively.

Step 3 Compute the different values of $||F(e_i)||$, where $||F(e_i)|| = \frac{|\overline{F}_I(e_i)| - |\underline{F}_I(e_i)|}{|F(e_i)|} \times w_i$.

Step 4 Find the minimum value $||F(e_k)||$ of $||F(e_i)||$, where $||F(e_k)|| = \min_i ||F(e_i)||$.

Step 5 The decision is $F(e_k)$.

Example 5.2. *Assume that we want to find the nearest accurate Krasner* (m,n)*-hyperring on a soft set* $\mathfrak{S}$. *Consider the Krasner* (m,n)*-hyperring* R *as in Example 5.1. Let* $I = \{0, c\}$ *be a normal hyperideal of* R. *Define a soft set* $\mathfrak{S} = (F, A)$ *over* R, *where* $A = \{e_1, e_2, e_3, e_4\}$, *where* w_i *is the weight value of* $e_i (i = 1, 2, 3, 4)$. *The tabular representation of the soft set* $\mathfrak{S}$ *is given in Table 4.*

Table 4 Table for soft set $\mathfrak{S}$

	0	a	b	c
$e_1, w_1 = 0.3$	1	1	1	0
$e_2, w_2 = 0.2$	1	0	1	1
$e_3, w_3 = 0.4$	0	1	1	0
$e_4, w_4 = 0.1$	1	0	0	0

Now, the tabular representations of two soft sets $\underline{Apr}_I(\mathfrak{S})$ *and* $\overline{Apr}_I(\mathfrak{S})$ *over* R *are given by Tables 5 and 6, respectively.*

Table 5 Table for soft set $\underline{Apr}_I(\mathfrak{S})$

	0	a	b	c
$e_1, w_1 = 0.3$	1	1	0	0
$e_2, w_2 = 0.2$	1	0	1	0
$e_3, w_3 = 0.4$	0	1	0	0
$e_4, w_4 = 0.1$	1	0	0	0

Table 6 Table for soft set $\overline{Apr}_I(\mathfrak{S})$

	0	a	b	c
$e_1, w_1 = 0.3$	1	1	1	1
$e_2, w_2 = 0.2$	1	1	1	1
$e_3, w_3 = 0.4$	1	1	1	1
$e_4, w_4 = 0.1$	1	1	0	1

Then, we can calculate $||F(e_1)|| = 0.2$, $||F(e_2)|| = 0.13$, $||F(e_3)|| = 0.6$, $||F(e_4)|| = 0.2$. *This means the minimum value for* $||F(e_i)||$ *is* $||F(e_2)|| = 0.13$. *That is,* $F(e_2) = \{0, b, c\}$ *is the closest accurate* $(2, 2)$*-hyperring on* $\mathfrak{S}$.

6. Conclusions

In 2010, Feng [21] proposed the concept of rough soft sets by combining rough sets and soft sets. By means of Feng's idea in [21], Zhan [43] originally applied rough soft sets to algebraic structures–hemirings, and gave some characterizations of rough soft hemirings. In the present paper, we put forward a novel rough soft algebraic hyperstructure–Krasner (m,n)-hyperrings by another kind of method. Let I be a normal hyperideal of a Krasner (m,n)-hyperring R, then we define the relation $\equiv_I$ by $x \equiv_I y$ if and only if $f(x,-y,\overset{(m-2)}{0}) \cap I \neq \emptyset$. It is clear that the relation $\equiv_I$ is an equivalence relation on R. Based on this novel idea, we propose the concept of rough soft hyperrings (hyperideals) w.r.t. a normal hyperideal of a Krasner (m,n)-hyperring, which is different from Zhan's idea in [43].

Besides, we define the t-level set $U(\mu,t) = \{(x,y) \in R \times R | \bigwedge_{z \in f(x,-y,\overset{(m-2)}{0})} \mu(z) \geq t\}$ of a Krasner (m,n)-hyperrings R and prove that it is an equivalence relation on R if μ is a fuzzy normal hyperideal of R. Based on this novel idea, we propose rough soft hyperideals w.r.t. a fuzzy normal hyperideal in Krasner (m,n)-hyperrings and investigate some characterizations.

In recent years, the problem of decision making in an imprecise environment has been found very important. Based on the above conditions, we first try to put forth a kind of decision making approaches based on rough soft Krasner (m,n)-hyperrings. It is pointed out that the primary motivation for decision algorithm is to found which is the best parameter of a given soft set. In other words, we focus on finding which is the nearest accurate Krasner (m,n)-hyperring on a soft set w.r.t. a normal hyperideal of Krasner (m,n)-hyperrings. By Examples 5.1 and 5.2, the approach is proved to be flexible, effective, practical and accurate.

We hope it would be served as a foundation of rough soft set theory and other decision making methods in different areas, such as theoretical computer sciences, information sciences and intelligent systems, and so on.

As an extension of this work, maybe the following topics can be considered:

(1) Establishing decision making methods based on rough soft hyperideals with respect to a fuzzy normal hyperideal in Krasner (m,n)-hyperrings;

(2) Applying soft rough sets to Krasner (m,n)-hyperrings;

(3) Investigating soft rough fuzzy sets to Krasner (m,n)-hyperrings;

(4) Studying soft fuzzy rough Krasner (m,n)-hyperrings.

References

[1] M.I. Ali, A note on soft sets, rough sets and fuzzy soft sets, Appl. Soft Comput. 11 (2011) 3329-3332.
[2] M.I. Ali, Another view on reduction of parameters in soft sets, Appl. Soft Comput. 12 (2012) 1814-1821.
[3] M.I. Ali, F. Feng, X.Y. Liu, W.K. Min, M. Shabir, On some new operations in soft set theory, Comput. Math. Appl. 57 (9) (2009) 1547-1553.
[4] R. Ameri, M. Norouzi, Prime and primary hyperideals in Krasner (m,n)-hyperrings, European J. Combin. 34 (2013) 379-390.
[5] R. Ameri, M. Norouzi, Prime and primary subhypermodules in (m,n)-hypermodules, European J. Combin. 44 (2013) 179-190.
[6] S.M. Anvariyeh, S. Mirvakili, B. Davvaz, Fundamental relations on (m,n)-ary hypermodules over (m,n)-ary hyperrings, Ars. Combin. 94(2010) 273-288.
[7] N. Çağman, S. Enginoglu, Soft matrix theory and its decision making, Comput. Math. Appl. 59 (2010) 3308-3314.
[8] N. Çağman, S. Enginoglu, Soft set theory and uni-int decision making, Eur. J. Oper. Res. 207 (2010) 848-855.
[9] P. Corsini, Prolegomena of Hypergroup Theory, Aviani Editore, 1993.
[10] P. Corsini, V. Leoreanu, Applications of hyperstructure theory, Advance in Mathematics, Kluwer Academic Publishers, 2003.
[11] G. Grombez, J. Timm, On (n,m)-rings, Abh. Math. Sem. Univ. Hamburg 37 (1972) 180-199.
[12] G. Grombez, J. Timm, On (n,m)-quotient rings, Abh. Math. Sem. Univ. Hamburg 37 (1972)200-203.
[13] B. Davvaz, Fuzzy Krasner (m,n)-hyperrings, Comput. Math. Appl. 59 (2010) 3879-3891.
[14] B. Davvaz, P. Corsini, Fuzzy (m,n)-ary subhypermodules (with thresholds), J. Intell. Fuzzy Systems 23 (2012) 1-8.
[15] B. Davvaz, V. Leoreanu-Fotea, Hyperring Theory snd Applications, Academic Press, 2007.
[16] B. Davvaz, T. Vougiouklis, n-ary hypergroups, Iran. J. Sci. Techonl. Trans. A. Sci. 30 (2006) 165-174.
[17] W. Dörnte, Untersuchungen uber einen verallgemeinerten Gruppenbegriff, Math. Z. 29 (1928) 1-19.
[18] D. Dubois, H. Prade, Rough fuzzy sets and fuzzy rough sets, Int. J. Gen. Syst. 17 (1990) 191-209.
[19] W.A. Dudek, On the divisibility theory in (m,n)-rings, Demonstratio Math. 14 (1981) 19-32.
[20] F. Feng, X.Y. Liu, V. Leoreanu-Fotea, Y.B. Jun, Soft sets and soft rough sets, Inform. Sci. 181 (6) (2011) 1125-1137.

[21] F. Feng, C. Li, B. Davvaz, M.I. Ali, Soft sets combined with fuzzy sets and rough sets: a tentative approach, Soft Comput. 14 (9) (2010) 899-911.
[22] M. Krasner, A class of hyperrings and hyperfield, Int. J. Math. Math. Sci. 2 (1983) 307-312.
[23] V. Leoreanu-Fotea, n-ary canonical hypergroups, Ital. J. Pure Appl. Math. 24 (2008) 247–254.
[24] V. Leoreanu-Fotea, B. Davvaz, Roughness in n-ary hypergroups, Inform. Sci. 178 (2008) 4114-4124.
[25] V. Leoreanu-Fotea, B. Davvaz, Fuzzy hyperrings, Fuzzy Sets and Systems 160 (2009) 2366-2378.
[26] Z. Li, N. Xie, G. Wen, Soft coverings and their parameter reductions, Appl. Soft Comput. 31 (2015) 48-60.
[27] Z. Li, T. Xie, The relationship among soft sets, soft rough sets and topologies, Soft Comput. 18 (2014) 717-728. 2594-2600.
[28] X. Ma, Q. Liu, J. Zhan, A survey of decision making methods based on certain hybrid soft set models, Artif. Intell. Rev. 47 (2017) 507-530.
[29] X. Ma, J. Zhan, M.I. Ali, Applications of a kind of novel Z-soft fuzzy rough ideals to hemirings, J. Intell. Fuzzy Systems 32 (2017) 2071-2082.
[30] P.K. Maji, R. Biswas, A.R. Roy, Soft set theory, Comput. Math. Appl. 45 (4) (2003) 555-562.
[31] P.K. Maji, R. Biswas, A.R. Roy, Fuzzy soft sets, J. Fuzzy Math. 9 (2001) 589-602.
[32] F. Marty, Sur une generalization de la notion de group 8th Congress Math Scandenaves, Stockholm, 1934, pp. 45-49.
[33] D. Meng, X. Zhang, K. Qin, Soft rough fuzzy sets and soft fuzzy rough sets, Comput. Math. Appl. 62 (12) (2011) 4635-4645.
[34] S. Mirvakili, B. Davvaz, Relation on Krasner (m, n)-hyperrings, European J. Combin. 31 (2010)790-802.
[35] D. Molodtsov, Soft set theory-first results, Comput. Math. Appl. 37 (4) (1999) 19-31.
[36] Z. Pawlak, Rough sets, Internat. J. Comput. Inform. Sci. 11 (5) (1982) 341-356.
[37] X. Peng, Y. Yang, Algorithms for interval-valued fuzzy soft sets in stochastic multi-criteria decision making based on regret theory and prospect theory with combined weight, Appl. Soft Comput. 54 (2017) 415-430.
[38] B. Sun, W. Ma, Soft fuzzy rough sets and its application in decision making, Artif. Intell. Rev. 41 (1) (2014) 67-80.
[39] T. Vougiouklis, Hyperstructures and their Representations, Hadronic Press, Inc 115, Palm Harber, USA, 1994.
[40] Y.Y. Yao, Constructive and algebraic methods of theory of rough sets, Inform. Sci. 109 (1998) 21-47.
[41] J. Zhan, M.I. Ali, N. Mehmood, On a novel uncertain soft set model: Z-soft fuzzy rough set model and corresponding decision making methods, Appl. Soft Comput. 56 (2017) 446-457.
[42] J. Zhan, B. Davvaz, K.P. Shum, Generalized fuzzy hyperideals of hyperrings, Comput. Math. Appl. 56 (2008) 1732-1740.
[43] J. Zhan, Q. Liu, B. Davvaz, A new rough set theory: rough soft hemirings, J. Intell. Fuzzy Systems 28 (2015) 1687-1697.
[44] J. Zhan, Q. Liu, T. Herawan, A novel soft rough set: Soft rough hemirings and corresponding multicriteria group decision making, Appl. Soft. Comput. 54 (2017) 393-402.
[45] X.H. Zhang, Fuzzy anti-grouped filters and fuzzy normal filters in pseudo-BCI algebras, J. Intell. Fuzzy Systems 33 (2017) 1767-1774.
[46] X.H. Zhang, J. Dai, Y. Yu, On the union and intersection operations of rough sets based on various approximation spaces, Inform. Sci. 292 (2015) 214-229.
[47] W. Zhu, Generalized rough sets based on realtions, Inform. Sci. 177 (22) (2007) 4997-5011.