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Soft Bigeneralized Topological Spaces

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Abstract. In this paper, soft bigeneralized topological spaces are introduced. Besides, we defined soft open set, soft closed set, soft closure set, soft interior set, soft basis and soft neighborhood on the soft bigeneralized topological spaces. Furthermore, some important theorems are proved and interesting examples are given.

1. Introduction

Russian researcher D. Molodtsov [17] introduced the concept of a soft set in order to solve complicated problems in the economics, engineering and enviromental areas because no mathematical tools can succesfully deal with the various kinds of uncertainties in these problems. Since soft set theory has a rich potential, researches on soft set theory and its applications in various fields are progressing rapidly in [13, 14].

The novel of generalized topology goes to back to 1963. Then, N. Levine [12] tried to generalize a topology by replacing open sets with semi-open sets. Later, similiar studies have been done. In the long run in 1997, Á. Császár [4] generalized these new open sets by introduced the concept of $\gamma-$open sets. The theory of generalized topological spaces (briefly GT), introduced by Á. Császár [3], is one of the most important developments of general topology in recent years. Let X be a nonempty set and g be a collection of subsets of X. g is called a generalized topology on X and (X, g) is called a generalized topological space, if g satisfies the following two conditions:

(1) $\emptyset \in g$,

(2) $G_i \in g$ for $i \in I \neq \emptyset$ implies $\underset{i \in I}{\cup} G_i \in g$.

Undoubtedly, generalized topological spaces are an important generalization of topological spaces. Á. Császár defined some basic operators on generalized topological spaces. It is observed in the last few years that a large number of papers is devoted to the study of generalized topological spaces. Many topologist have faced generalized topologies. Á. Császár was actively working on this area, despite being approximately at the age of 87. Later, W. K. Min and Y. K. Kim [16] introduced the notion of bigeneralized topological spaces and quasi generalized open sets. They studied some basic properties for the sets. Let X be a nonempty set and g_1, g_2 be two generalized topologies on X. A triple (X, g_1, g_2) is called a bigeneralized topological space (briefly $BGTS$). In addition to, P. Torton et al. [22] studied some separation axioms in bigeneralized topological spaces.

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Topological structures of soft set have been studied by some authors in recent years. M. Shabir and M. Naz [19] have initiated the study of soft topological spaces which are defined over an initial universe with a fixed set of parameters and showed that a soft topological space gives a parameterized family of topological spaces. Theoretical studies of soft topological spaces have also been researched by some authors in [2, 5–7, 15, 18, 20, 23]. As a generalized of soft topological spaces, S.A. El-Sheikh and A. M. Abd-El-Latif [8] introduced the notion of supra soft topological spaces by neglecting only the soft intersection condition. After the concept of bitopological spaces was introduced by J.C. Kelly [11] as an extension of topological spaces in 1963, B.M. Ittanagi [9] defined the notion of soft bitopological space and gave some of types of soft separation axioms. A study of soft bitopological spaces is a generalization of the study of soft topological spaces as every soft bitopological space (X, τ_1, τ_2, E) can be regarded as a soft topological space (X, τ, E) if $\tau_1 = \tau_2 = \tau$.

In later years, many researchers studied bitopological spaces and pair-wise open (closed) sets. Therefore, handling of these spaces in soft concept is important and actual (e. g. [9, 10]).

In 2014, J. Thomas and S.J. John [21] defined basic notions and concepts of soft generalized topological spaces such as soft basis, subspace of soft generalized topology. They showed that a soft generalized topology gave a parameterized family of generalized topologies on the initial universe.

The main purpose of the present paper is to generalize the notions of soft generalized topological spaces. Besides, we define soft open set, soft closed set, soft closure set, soft interior set, soft basis and soft neighborhood on the soft bigeneralized topological spaces. Furthermore, their important theorems are proved and interesting examples are solved.

2. Preliminaries

In this section we will introduce necessary definitions and theorems for soft sets. Throughout this paper X denotes initial universe, E denotes the set of all parameters, $P(X)$ denotes the power set of X.

Definition 2.1. ([17]) A pair (F, E) is called a soft set over X, where F is a mapping given by $F : E \to P(X)$.

In other words, the soft set is a parameterized family of subsets of the set X. For $e \in E$, $F(e)$ may be considered as the set of $e-$elements of the soft set (F, E), or as the set of $e-$approximate elements of the soft set, i.e.,

$$(F, E) = \{(e, F(e)) : e \in E, F : E \to P(X)\}.$$

After this, $SS(X)_E$ denotes the family of all soft sets over X with a fixed set of parameters E.

Definition 2.2. ([14]) For two soft sets (F, E) and (G, E) over X, (F, E) is called a soft subset of (G, E) if $\forall e \in E$, $F(e) \subseteq G(e)$. This relationship is denoted by $(F, E)\widetilde{\subseteq}(G, E)$.

Similarly, (F, E) is called a soft superset of (G, E) if (G, E) is a soft subset of (F, E). This relationship is denoted by $(F, E)\widetilde{\supseteq}(G, E)$. Two soft sets (F, E) and (G, E) over X are called soft equal if (F, E) is a soft subset of (G, E) and (G, E) is a soft subset of (F, E).

Definition 2.3. ([1]) The intersection of two soft sets (F, E) and (G, E) over X is the soft set (H, E), where $\forall e \in E$, $H(e) = F(e) \cap G(e)$. This is denoted by $(F, E)\widetilde{\cap}(G, E) = (H, E)$.

Definition 2.4. ([1]) The union of two soft sets (F, E) and (G, E) over X is the soft set (H, E), where $\forall e \in E$, $H(e) = F(e) \cup G(e)$. This is denoted by $(F, E)\widetilde{\cup}(G, E) = (H, E)$.

Definition 2.5. ([19]) A soft set (F, E) over X is said to be a null soft set denoted by $(\widetilde{\phi}, E)$ if for all $e \in E$, $F(e) = \emptyset$.

Definition 2.6. ([19]) A soft set (F, E) over X is said to be an absolute soft set denoted by $(\widetilde{X}, E)$ if for all $e \in E$, $F(e) = X$.

Definition 2.7. ([19]) The difference (H, E) of two soft sets (F, E) and (G, E) over X, denoted by $(F, E)\backslash(G, E)$, is defined as $H(e) = F(e)\backslash G(e)$ for all $e \in E$.

Definition 2.8. ([19]) The complement of a soft set (F, E), denoted by $(F, E)^c$, is defined as $(F, E)^c = (F^c, E)$, where $F^c : E \to P(X)$ is a mapping given by $F^c(e) = X\backslash F(e)$, $\forall e \in E$ and F^c is called the soft complement function of F.

Definition 2.9. ([19]) Let $\widetilde{\tau}$ be the collection of soft sets over X, then $\widetilde{\tau}$ is said to be a soft topology on X if
1) $(\widetilde{\phi}, E), (\widetilde{X}, E)$ belong to $\widetilde{\tau}$;
2) the union of any number of soft sets in $\widetilde{\tau}$ belongs to $\widetilde{\tau}$;
3) the intersection of any two soft sets in $\widetilde{\tau}$ belongs to $\widetilde{\tau}$.
The triplet $(X, \widetilde{\tau}, E)$ is called a soft topological space over X.

Definition 2.10. ([19]) Let $(X, \widetilde{\tau}, E)$ be a soft topological space over X, then members of $\widetilde{\tau}$ are said to be soft open sets in X.

Definition 2.11. ([19]) Let $(X, \widetilde{\tau}, E)$ be a soft topological space over X. A soft set (F, E) over X is said to be a soft closed in X, if its complement $(F, E)^c$ belongs to $\widetilde{\tau}$.

Proposition 2.12. ([19]) *Let $(X, \widetilde{\tau}, E)$ be a soft topological space over X. Then the collection $\widetilde{\tau}_e = \{F(e) : (F, E) \in \widetilde{\tau}\}$ for each $e \in E$, defines a topology on X.*

Definition 2.13. ([19]) Let $(X, \widetilde{\tau}, E)$ be a soft topological space over X and (F, E) be a soft set over X. Then the soft closure of (F, E), denoted by $cl^s_{\widetilde{\tau}}(F, E)$ is the intersection of all soft closed super sets of (F, E). Clearly $cl^s_{\widetilde{\tau}}(F, E)$ is the smallest soft closed set over X which contains (F, E).

Definition 2.14. ([7]) Let $(X, \widetilde{\tau}, E)$ be a soft topological space over X and (F, E) be a soft set over X. Then the soft interior of (F, E), denoted by $int^s_{\widetilde{\tau}}(F, E)$ is the union of all soft open subsets of (F, E)

Definition 2.15. ([2]) Let (F, E) be a soft set over X. The soft set (F, E) is called a soft point, denoted by (x_e, E), if for the element $e \in E$, $F(e) = \{x\}$ and $F(e^c) = \varnothing$ for all $e^c \in E - \{e\}$ (briefly denoted by x_e).

It is obvious that each soft set can be expressed as a union of soft points. For this reason, to give the family of all soft sets on X it is sufficient to give only soft points on X.

Definition 2.16. ([2]) Let x_e and $y_{e'}$ be two soft points over a common universe X, we say that the soft points are different if $x \neq y$ or $e \neq e'$.

Definition 2.17. ([2]) The soft point x_e is said to be belonging to the soft set (F, E), denoted by $x_e \widetilde{\in} (F, E)$, if $x_e(e) \in F(e)$,i.e., $\{x\} \subseteq F(e)$.

Definition 2.18. ([2]) Let $(X, \widetilde{\tau}, E)$ be a soft topological space over X. A soft set $(F, E) \widetilde{\subseteq} (X, E)$ is called a soft neighborhood of the soft point $x_e \widetilde{\in} (F, E)$ if there exists a soft open set (G, E) such that $x_e \widetilde{\in} (G, E) \widetilde{\subset} (F, E)$.

Definition 2.19. ([21]) Let $\widetilde{\mu}$ be the collection of soft set over X. Then $\widetilde{\mu}$ is said to be a soft generalized topology on X if

1. $(\widetilde{\phi}, E)$ belongs to $\widetilde{\mu}$;
2. the union of any number of soft sets in $\widetilde{\mu}$ belongs to $\widetilde{\mu}$.

The triplet $(X, \widetilde{\mu}, E)$ is called a soft generalized topological space (briefly $SGT-$space) over X.

Definition 2.20. ([21]) A soft generalized topology $\widetilde{\mu}$ on (X, E) is called strong if $\widetilde{X} \in \widetilde{\mu}$.

Definition 2.21. ([21]) Let $(X, \widetilde{\mu}, E)$ be a $SGT-$space over (X, E). Then every element of $\widetilde{\mu}$ is called a soft $g_{\widetilde{\mu}}-$open set.

Definition 2.22. ([21]) Let $(X, \widetilde{\mu}, E)$ be a $SGT-$space over (X, E) and $(F, E)\widetilde{\subseteq}(X, E)$. Then (F, E) is called a soft $g_{\widetilde{\mu}}-$closed set if its soft complement $(F, E)^c$ is a soft $g_{\widetilde{\mu}}-$open set.

Definition 2.23. ([21]) Let $(X, \widetilde{\mu}, E)$ be a $SGT-$space over (X, E) and $(F, E)\widetilde{\subseteq}(X, E)$. Then the soft $g_{\widetilde{\mu}}-$interior of (F, E) denoted by $s\imath nt_{\widetilde{\mu}}((F, E))$ is defined as the soft union of all soft $g_{\widetilde{\mu}}-$open subsets of (F, E). That is, $s\imath nt_{\widetilde{\mu}}((F, E))$ is the largest soft $g_{\widetilde{\mu}}-$open set that is contained in (F, E).

Definition 2.24. ([21]) Let $(X, \widetilde{\mu}, E)$ be a $SGT-$space over (X, E) and $(F, E)\widetilde{\subseteq}(X, E)$. Then the soft $g_{\widetilde{\mu}}-$closure of (F, E) is denoted by $scl_{\widetilde{\mu}}((F, E))$ is defined as the soft intersection of all soft $g_{\widetilde{\mu}}-$closed super sets of (F, E). That is, $scl_{\widetilde{\mu}}((F, E))$ is the smallest soft $g_{\widetilde{\mu}}-$closed set containing (F, E).

For $x_e\widetilde{\in}SS(X)_E$, let $(U, E)_{x_e}$ be a soft $g_{\widetilde{\mu}}-$open subset containing x_e and

$$\widetilde{\mu}_{x_e} = \left((U, E) : x_e\widetilde{\in}(U, E) \in \widetilde{\mu}\right).$$

Proposition 2.25. ([21]) *Let $(X, \widetilde{\mu}, E)$ be a SGT−space over (X, E) and $(F, E)\widetilde{\subseteq}(X, E)$.Then the following statements hold.*

1. $s\imath nt_{\widetilde{\mu}}((F, E))\widetilde{\subseteq}(F, E)\widetilde{\subseteq}scl_{\widetilde{\mu}}((F, E))$,
2. $s\imath nt_{\widetilde{\mu}}\left(s\imath nt_{\widetilde{\mu}}((F, E))\right) = s\imath nt_{\widetilde{\mu}}((F, E))$ *and* $scl_{\widetilde{\mu}}\left(scl_{\widetilde{\mu}}((F, E))\right) = scl_{\widetilde{\mu}}((F, E))$,
3. *If* $(G, E)\widetilde{\subseteq}(F, E)$, *then* $s\imath nt_{\widetilde{\mu}}((G, E))\widetilde{\subseteq}s\imath nt_{\widetilde{\mu}}((F, E))$, $scl_{\widetilde{\mu}}((G, E))\widetilde{\subseteq}scl_{\widetilde{\mu}}((F, E))$,
4. $s\imath nt_{\widetilde{\mu}}((F, E)) = (F, E) \Leftrightarrow (F, E)$ *is soft* $g_{\widetilde{\mu}}-$*open set in* $(X, E) \Leftrightarrow$ *for each* $x_e\widetilde{\in}(F, E)$, $(U, E)_{x_e}\widetilde{\subseteq}(F, E)$ *for some* $(U, E)_{x_e} \in \widetilde{\mu}_{x_e}$,
5. $scl_{\widetilde{\mu}}((F, E)) = (F, E) \Leftrightarrow (F, E)$ *is soft* $g_{\widetilde{\mu}}-$*closed set in* $(X, E) \Leftrightarrow$ *for each* $x_e\widetilde{\in}(X, E)\backslash(F, E)$, $(U, E)_{x_e}\widetilde{\cap}(F, E) = (\widetilde{\phi}, E)$ *for some* $(U, E)_{x_e} \in \widetilde{\mu}_{x_e}$,
6. $scl_{\widetilde{\mu}}((F, E)) = (X, E)\backslash s\imath nt_{\widetilde{\mu}}((X, E)\backslash(F, E))$ *and* $s\imath nt_{\widetilde{\mu}}((F, E)) = (X, E)\backslash scl_{\widetilde{\mu}}((X, E)\backslash(F, E))$,
7. $x_e\widetilde{\in}scl_{\widetilde{\mu}}((F, E)) \Leftrightarrow (U, E)_{x_e}\widetilde{\cap}(F, E) \neq (\widetilde{\phi}, E)$ *for each* $(U, E)_{x_e} \in \widetilde{\mu}_{x_e}$,
8. $x_e\widetilde{\in}s\imath nt_{\widetilde{\mu}}((F, E)) \Leftrightarrow (U, E)_{x_e}\widetilde{\subseteq}(F, E)$ *for some* $(U, E)_{x_e} \in \widetilde{\mu}_{x_e}$.

3. Soft Bigeneralized Topological Spaces

Definition 3.1. Let $\widetilde{g}_1$ and $\widetilde{g}_2$ be two soft generalized topologies on X and E be a set of parameters. Then the quadruple system $(X, \widetilde{g}_1, \widetilde{g}_2, E)$ is called a soft bigeneralized topological space (briefly $SBGT-$space).

Proposition 3.2. *Let $(X, \widetilde{g}_1, \widetilde{g}_2, E)$ be a SBGT−space. We define*

$$\begin{aligned} \widetilde{g}_{1e} &= \{F(e) : (F, E) \in \widetilde{g}_1\} \\ \widetilde{g}_{2e} &= \{G(e) : (G, E) \in \widetilde{g}_2\} \end{aligned}$$

for each $e \in E$. Then $(X, \widetilde{g}_{1e}, \widetilde{g}_{2e})$ is a bigeneralized topological space.

Proof. Straightforward. □

Definition 3.3. Let $(X, \widetilde{g}_1, \widetilde{g}_2, E)$ be a $SBGT-$space. A soft set $(F, E)\widetilde{\subseteq}(X, E)$ is called soft $\widetilde{g}_{1,2} - open$ set if $(F, E) = (F_1, E)\widetilde{\cup}(F_2, E)$ where $(F_1, E) \in \widetilde{g}_1$ and $(F_2, E) \in \widetilde{g}_2$.

The complement of soft $\widetilde{g}_{1,2} - open$ set is called soft $\widetilde{g}_{1,2} - closed$ set. Clearly, a soft set (G, E) over X is a soft $\widetilde{g}_{1,2} - closed$ set in $(X, \widetilde{g}_1, \widetilde{g}_2, E)$ if $(G, E) = (G_1, E)\widetilde{\cap}(G_2, E)$ such that $(G_1, E) \in \widetilde{g}_1^c$ and $(G_2, E) \in \widetilde{g}_2^c$, where

$$\widetilde{g}_i^c = \{(G, E)^c \in SS(X)_E : (G, E) \in \widetilde{g}_i\},\ i = 1, 2.$$

Proposition 3.4. *Let $(X, \widetilde{g}_1, \widetilde{g}_2, E)$ be a SBGT−space and $(F, E) \in SS(X)_E$. (F, E) is a soft $\widetilde{g}_{1,2}$ − open set iff for every $x_e \widetilde{\in} (F, E)$, there exists a soft $\widetilde{g}_1$ − open set (F_1, E) such that $x_e \widetilde{\in} (F_1, E) \widetilde{\subseteq} (F, E)$, or a soft $\widetilde{g}_2$ − open set (F_2, E) such that $x_e \widetilde{\in} (F_2, E) \widetilde{\subseteq} (F, E)$.*

Proof. Let (F, E) be a soft $\widetilde{g}_{1,2}$ − *open* set in $(X, \widetilde{g}_1, \widetilde{g}_2, E)$ and $x_e \widetilde{\in} (F, E)$. Then there exists $(F_1, E) \in \widetilde{g}_1$ and $(F_2, E) \in \widetilde{g}_2$ such that $(F, E) = (F_1, E) \widetilde{\cup} (F_2, E)$. Since $x_e \widetilde{\in} (F, E) = (F_1, E) \widetilde{\cup} (F_2, E)$, then $x_e \widetilde{\in} (F_1, E)$ or $x_e \widetilde{\in} (F_2, E)$. We get $x_e \widetilde{\in} (F_1, E) \widetilde{\subseteq} (F, E)$, or $x_e \widetilde{\in} (F_2, E) \widetilde{\subseteq} (F, E)$.

Conversely, it can be easily obtained. □

Theorem 3.5. *Let $(X, \widetilde{g}_1, \widetilde{g}_2, E)$ be a SBGT−space. Then*

1. $(\widetilde{\phi}, E)$ *is a soft* $\widetilde{g}_{1,2}$ − *open set,*
2. *An arbitrary union of soft* $\widetilde{g}_{1,2}$ − *open sets is a soft* $\widetilde{g}_{1,2}$ − *open set,*
3. *An arbitrary intersection of soft* $\widetilde{g}_{1,2}$ − *closed sets is a soft* $\widetilde{g}_{1,2}$ − *closed set.*

Proof. (1) Since $(\widetilde{\phi}, E) \in \widetilde{g}_1$ and $(\widetilde{\phi}, E) \in \widetilde{g}_2$, $(\widetilde{\phi}, E) = (\widetilde{\phi}, E) \widetilde{\cup} (\widetilde{\phi}, E)$, then $(\widetilde{\phi}, E)$ is a soft $\widetilde{g}_{1,2}$ − *open* set.

(2) Let $\{(F_i, E)\}_{i \in I}$ be a family of soft $\widetilde{g}_{1,2}$ − *open* sets in $(X, \widetilde{g}_1, \widetilde{g}_2, E)$. Then there exist $(F_i^1, E) \in \widetilde{g}_1$ and $(F_i^2, E) \in \widetilde{g}_2$ such that $(F_i, E) = (F_i^1, E) \widetilde{\cup} (F_i^2, E)$ for all $i \in I$ which implies that

$$\widetilde{\bigcup_{i \in I}} (F_i, E) = \widetilde{\bigcup_{i \in I}} \left[(F_i^1, E) \widetilde{\cup} (F_i^2, E) \right] = \left[\widetilde{\bigcup_{i \in I}} (F_i^1, E) \right] \widetilde{\cup} \left[\widetilde{\bigcup_{i \in I}} (F_i^2, E) \right].$$

Now, since $\widetilde{g}_1$ and $\widetilde{g}_2$ are soft generalized topologies, then $\left[\widetilde{\bigcup_{i \in I}} (F_i^1, E) \right] \in \widetilde{g}_1$ and $\left[\widetilde{\bigcup_{i \in I}} (F_i^2, E) \right] \in \widetilde{g}_2$. Therefore, $\widetilde{\bigcup_{i \in I}} (F_i, E)$ is a soft $\widetilde{g}_{1,2}$ − *open* set.

(3) It is similar to (2). □

Corollary 3.6. *Let $(X, \widetilde{g}_1, \widetilde{g}_2, E)$ be a SBGT−space. Then the family of all soft $\widetilde{g}_{1,2}$ − open sets is a soft generalized topological space on X. This soft generalized topology is denoted by $\widetilde{g}_{1,2}$, i.e.,*

$$\widetilde{g}_{1,2} = \left\{ (F, E) = (F_1, E) \widetilde{\cup} (F_2, E) : (F_i, E) \in \widetilde{g}_i, \ i = 1, 2 \right\}.$$

Example 3.7. Let $X = \{a, b, c, d\}$, $E = \{e_1, e_2\}$ and let

$$\begin{aligned} \widetilde{g}_1 &= \left\{ (\widetilde{\phi}, E), \ (F_1, E), \ (F_2, E), \ (F_3, E) \right\} \text{ and} \\ \widetilde{g}_2 &= \left\{ (\widetilde{\phi}, E), \ (G_1, E), \ (G_2, E), \ (G_3, E) \right\} \end{aligned}$$

where

$$\begin{aligned} (F_1, E) &= \{(e_1, \{a, c\}), (e_2, \{d\})\}, \\ (F_2, E) &= \{(e_1, \{a, b\}), (e_2, \{c, d\})\}, \\ (F_3, E) &= \{(e_1, \{a, b, c\}), (e_2, \{c, d\})\}, \\ \\ (G_1, E) &= \{(e_1, \{b, c\}), (e_2, \{a\})\}, \\ (G_2, E) &= \{(e_1, \{a, d\}), (e_2, \{b, c\})\}, \\ (G_3, E) &= \{(e_1, X), (e_2, \{a, b, c\})\}. \end{aligned}$$

Then, $(X, \widetilde{g}_1, \widetilde{g}_2, E)$ is a *SBGT*−space. It is clear that

$$\widetilde{g}_{1,2} = \left\{ \begin{array}{c} (\widetilde{\phi}, E), \ (F_1, E), \ (F_2, E), \ (F_3, E), \ (G_1, E), \ (G_2, E), \ (G_3, E), \\ (H_1, E), \ (H_2, E), \ (H_3, E), \ (H_4, E), \ (H_5, E), \ (H_6, E) \end{array} \right\},$$

where

$$\begin{aligned}
(H_1,E) &= (F_1,E)\widetilde{\cup}(G_1,E) = \{(e_1,\ \{a,\ b,\ c\}),\ (e_2,\ \{a,\ d\})\},\\
(H_2,E) &= (F_1,E)\widetilde{\cup}(G_2,E) = \{(e_1,\ \{a,\ c,\ d\}),\ (e_2,\ \{b,\ c,\ d\})\},\\
(H_3,E) &= (F_1,E)\widetilde{\cup}(G_3,E) = \{(e_1,\ X),\ (e_2,\ X)\} = (\widetilde{X},E),\\
(H_4,E) &= (F_2,E)\widetilde{\cup}(G_1,E) = \{(e_1,\ \{a,\ b,\ c\}),\ (e_2,\ \{a,\ c,\ d\})\},\\
(H_5,E) &= (F_2,E)\widetilde{\cup}(G_2,E) = \{(e_1,\ \{a,\ b,\ d\}),\ (e_2,\ \{b,c,d\})\},\\
(H_6,E) &= (F_3,E)\widetilde{\cup}(G_2,E) = \{(e_1,\ X),\ (e_2,\ \{b,c,\ d\})\}.
\end{aligned}$$

Consequently,

$$\widetilde{g}^c_{1,2} = \left\{ \begin{array}{c} (\widetilde{X},E),\ (F_1,E)^c,\ (F_2,E)^c,\ (F_3,E)^c,\ (G_1,E)^c,\ (G_2,E)^c,\ (G_3,E)^c, \\ (H_1,E)^c,\ (H_2,E)^c,\ (H_3,E)^c,\ (H_4,E)^c,\ (H_5,E)^c,\ (H_6,E)^c \end{array} \right\},$$

where

$$\begin{aligned}
(F_1,E)^c &= \{(e_1,\ \{b,\ d\}),\ (e_2,\ \{a,\ b,\ c\})\},\\
(F_2,E)^c &= \{(e_1,\ \{c,\ d\}),\ (e_2,\ \{a,\ b\})\},\\
(F_3,E)^c &= \{(e_1,\ \{d\}),\ (e_2,\ \{a,\ b\})\},\\
\\
(G_1,E)^c &= \{(e_1,\ \{a,\ d\}),\ (e_2,\ \{b,\ c,\ d\})\},\\
(G_2,E)^c &= \{(e_1,\ \{b,\ c\}),\ (e_2,\ \{a,\ d\})\},\\
(G_3,E)^c &= \{(e_1,\ \emptyset),\ (e_2,\ \{d\})\}.\\
\\
(H_1,E)^c &= \{(e_1,\ \{d\}),\ (e_2,\ \{b,\ c\})\},\\
(H_2,E)^c &= \{(e_1,\ \{b\}),\ (e_2,\ \{a\})\},\\
(H_3,E)^c &= \{(e_1,\ \emptyset),\ (e_2,\ \emptyset)\} = (\widetilde{\phi},E)\\
(H_4,E)^c &= \{(e_1,\ \{d\}),\ (e_2,\ \{b\})\},\\
(H_5,E)^c &= \{(e_1,\ \{c\}),\ (e_2,\ \{a\})\},\\
(H_6,E)^c &= \{(e_1,\ \emptyset),\ (e_2,\ \{a\})\}.
\end{aligned}$$

Theorem 3.8. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a SBGT−space. Then,*

1. *every soft $\widetilde{g}_i$ − open set is a soft $\widetilde{g}_{1,2}$ − open set, $i = 1,2$,*
2. *every soft $\widetilde{g}_i$ − closed set is a soft $\widetilde{g}_{1,2}$ − closed set, $i = 1,2$,*
3. *if $\widetilde{g}_1 \subseteq \widetilde{g}_2$, then $\widetilde{g}_{1,2} = \widetilde{g}_2$ and $\widetilde{g}^c_{1,2} = \widetilde{g}^c_2$.*

Proof. Straightforward. □

Remark 3.9. In a given *SBGT*−space $(X,\widetilde{g}_1,\widetilde{g}_2,E)$, a soft $\widetilde{g}_{1,2}$ − *open* set need not be a soft $\widetilde{g}_1$ − *open* set and a soft $\widetilde{g}_2$ − *open* set as shown in the following example.

Example 3.10. According to Example 3.7., $(H_i,E)_{i=\overline{1,6}}$ is a soft $\widetilde{g}_{1,2}$ − *open* set but it is not a soft $\widetilde{g}_1$ − *open* set and a soft $\widetilde{g}_2$ − *open* set.

Definition 3.11. Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a *SBGT*−space and $(F,E) \in SS(X)_E$. Then, soft $\widetilde{g}_{1,2}$ − *closure* set of (F,E) is denoted by $scl_{\widetilde{g}_{1,2}}(F,E)$, defined by

$$scl_{\widetilde{g}_{1,2}}(F,E) = \widetilde{\cap}\left\{(G,E) \in \widetilde{g}^c_{1,2} : (F,E)\widetilde{\subseteq}(G,E)\right\}.$$

Note that, $scl_{\widetilde{g}_{1,2}}(F,E)$ is the smallest soft $\widetilde{g}_{1,2}$ − *closed* set that containing (F,E).

Example 3.12. According to Example 3.7., let $(K, E) = \{(e_1, \{b\}), (e_2, \{a, b\})\}$. Then, the soft $\widetilde{g}_{1,2}-closed$ sets which contain (K, E) are $(F_1, E)^c$ and $(\widetilde{X}, E)$. Therefore,

$$scl_{\widetilde{g}_{1,2}}(K, E) = (F_1, E)^c \widetilde{\cap} (\widetilde{X}, E) = \{(e_1, \{b, d\}), (e_2, \{a, b, c\})\}.$$

Theorem 3.13. *Let $(X, \widetilde{g}_1, \widetilde{g}_2, E)$ be a SBGT−space and $(F, E), (G, E) \in SS(X)_E$. Then,*

1. $scl_{\widetilde{g}_{1,2}}(\widetilde{X}, E) = (\widetilde{X}, E)$,
2. $(F, E) \widetilde{\subseteq} scl_{\widetilde{g}_{1,2}}(F, E)$,
3. *(F, E) is a soft $\widetilde{g}_{1,2} - closed$ set if and only if $scl_{\widetilde{g}_{1,2}}(F, E) = (F, E)$,*
4. *if $(F, E) \widetilde{\subseteq} (G, E)$, then $scl_{\widetilde{g}_{1,2}}(F, E) \widetilde{\subseteq} scl_{\widetilde{g}_{1,2}}(G, E)$,*
5. *$scl_{\widetilde{g}_{1,2}}(F, E) \widetilde{\cup} scl_{\widetilde{g}_{1,2}}(G, E) \widetilde{\subseteq} scl_{\widetilde{g}_{1,2}}\left[(F, E) \widetilde{\cup} (G, E)\right]$ and $scl_{\widetilde{g}_{1,2}}\left[(F, E) \widetilde{\cap} (G, E)\right] \widetilde{\subseteq} scl_{\widetilde{g}_{1,2}}(F, E) \widetilde{\cap} scl_{\widetilde{g}_{1,2}}(G, E)$,*
6. *$scl_{\widetilde{g}_{1,2}}\left[scl_{\widetilde{g}_{1,2}}(F, E)\right] = scl_{\widetilde{g}_{1,2}}(F, E)$, i.e., $scl_{\widetilde{g}_{1,2}}(F, E)$ is a soft $\widetilde{g}_{1,2} - closed$ set.*

Proof. Straightforward. □

Remark 3.14. In Theorem 3.13(5), "$\widetilde{\supseteq}$" can not be satisfied.

Example 3.15. According to Example 3.7., let $(K, E) = \{(e_1, \{b\}), (e_2, \{a, b\})\}$ and $(S, E) = \{(e_1, \{b, c\}), (e_2, \{a, d\})\}$. Then,

$$scl_{\widetilde{g}_{1,2}}(K, E) \widetilde{\cup} scl_{\widetilde{g}_{1,2}}(S, E) = \{(e_1, \{b, c, d\}), (e_2, X)\}$$

$$scl_{\widetilde{g}_{1,2}}\left[(K, E) \widetilde{\cup} (S, E)\right] = (\widetilde{X}, E).$$

Therefore, $scl_{\widetilde{g}_{1,2}}(K, E) \widetilde{\cup} scl_{\widetilde{g}_{1,2}}(S, E) \widetilde{\not\supseteq} scl_{\widetilde{g}_{1,2}}\left[(K, E) \widetilde{\cup} (S, E)\right]$. Similarly, let $(K, E) = \{(e_1, \{b\}), (e_2, \{a, b\})\}$ and $(C, E) = \{(e_1, \{b, d\}), (e_2, \{a, c\})\}$. Then,

$$scl_{\widetilde{g}_{1,2}}(K, E) \widetilde{\cap} scl_{\widetilde{g}_{1,2}}(C, E) = \{(e_1, \{b, d\}), (e_2, \{a, b, c\}\}$$

$$scl_{\widetilde{g}_{1,2}}\left[(K, E) \widetilde{\cap} (C, E)\right] = \{(e_1, \{b\}), (e_2, \{a\})\}.$$

Therefore, $scl_{\widetilde{g}_{1,2}}\left[(K, E) \widetilde{\cap} (C, E)\right] \widetilde{\not\supseteq} scl_{\widetilde{g}_{1,2}}(K, E) \widetilde{\cap} scl_{\widetilde{g}_{1,2}}(C, E)$.

Theorem 3.16. *Let $(X, \widetilde{g}_1, \widetilde{g}_2, E)$ be a SBGT−space and $(F, E) \in SS(X)_E$. Then,*

$$x_e \widetilde{\in} scl_{\widetilde{g}_{1,2}}(F, E) \Leftrightarrow (U_{x_e}, E) \widetilde{\cap} (F, E) \neq (\widetilde{\phi}, E), \ \forall (U_{x_e}, E) \in \widetilde{g}_{1,2}(x_e)$$

where (U_{x_e}, E) is any soft $\widetilde{g}_{1,2} - open$ set contains x_e and $\widetilde{g}_{1,2}(x_e)$ is the family of all soft $\widetilde{g}_{1,2} - open$ sets contains x_e.

Proof. ⇒ Let $x_e \widetilde{\in} scl_{\widetilde{g}_{1,2}}(F, E)$ and $(U_{x_e}, E) \in \widetilde{g}_{1,2}(x_e)$. Suppose that $(U_{x_e}, E) \widetilde{\cap} (F, E) = (\widetilde{\phi}, E)$. Then $(F, E) \widetilde{\subseteq} (U_{x_e}, E)^c$, so $scl_{\widetilde{g}_{1,2}}(F, E) \widetilde{\subseteq} scl_{\widetilde{g}_{1,2}}(U_{x_e}, E)^c = (U_{x_e}, E)^c$ which implies $scl_{\widetilde{g}_{1,2}}(F, E) \widetilde{\cap} (U_{x_e}, E) = (\widetilde{\phi}, E)$, a contradiction.

⇐ Assume that $x_e \widetilde{\notin} scl_{\widetilde{g}_{1,2}}(F, E)$, then $x_e \widetilde{\in} \left[scl_{\widetilde{g}_{1,2}}(F, E)\right]^c$. Thus, $\left[scl_{\widetilde{g}_{1,2}}(F, E)\right]^c \in \widetilde{g}_{1,2}(x_e)$, so from the hypothesis, $\left[scl_{\widetilde{g}_{1,2}}(F, E)\right]^c \widetilde{\cap} (F, E) = (\widetilde{\phi}, E)$, a contradiction. □

Theorem 3.17. *Let $(X, \widetilde{g}_1, \widetilde{g}_2, E)$ be a SBGT−space and $(F, E) \in SS(X)_E$. Then,*

$$scl_{\widetilde{g}_{1,2}}(F, E) = scl_{\widetilde{g}_1}(F, E) \widetilde{\cap} scl_{\widetilde{g}_2}(F, E).$$

Proof.

$$
\begin{aligned}
scl_{\widetilde{g}_{1,2}}(F,E) &= \widetilde{\cap}\left\{(G,E) : (F,E)\widetilde{\subseteq}(G,E) \text{ for a soft } \widetilde{g}_{1,2} - closed \text{ set } (G,E)\right\} \\
&= \widetilde{\cap}\left\{\begin{array}{c} (G,E) : (F,E)\widetilde{\subseteq}(G,E),\ (G,E) = (G_1,E)\widetilde{\cap}(G_2,E), \\ \text{for a } \widetilde{g}_i - closed \text{ soft set } (G_i,E),\ i = 1,2 \end{array}\right\} \\
&= \widetilde{\cap}\left\{\begin{array}{l} (G_1,E)\widetilde{\cap}(G_2,E) : (F,E)\widetilde{\subseteq}(G_1,E)\widetilde{\cap}(G_2,E), \\ \text{for a } \widetilde{g}_i - closed \text{ soft set } (G_i,E),\ i = 1,2 \end{array}\right\} \\
&= \left[\widetilde{\cap}\left\{(G_1,E) : (F,E)\widetilde{\subseteq}(G_1,E) \text{ for a } \widetilde{g}_1 - closed \text{ soft set } (G_1,E)\right\}\right] \\
&\quad \widetilde{\cap}\left[\widetilde{\cap}\left\{(G_2,E) : (F,E)\widetilde{\subseteq}(G_2,E) \text{ for a } \widetilde{g}_2 - closed \text{ soft set } (G_2,E)\right\}\right] \\
&= scl_{\widetilde{g}_1}(F,E)\widetilde{\cap}scl_{\widetilde{g}_2}(F,E).
\end{aligned}
$$

□

Definition 3.18. Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a $SBGT-$space and $(F,E) \in SS(X)_E$. Then, soft $\widetilde{g}_{1,2} - interior$ set of (F,E) is denoted by $s\imath nt_{\widetilde{g}_{1,2}}(F,E)$, defined by

$$s\imath nt_{\widetilde{g}_{1,2}}(F,E) = \widetilde{\cup}\left\{(U,E) \in \widetilde{g}_{1,2} : (U,E)\widetilde{\subseteq}(F,E)\right\}.$$

Note that, $s\imath nt_{\widetilde{g}_{1,2}}(F,E)$ is the largest soft $\widetilde{g}_{1,2}-$open sets which are contained by (F,E).

Example 3.19. According to Example 3.7., let $(K,E) = \{(e_1,\ \{a,\ b,\ c\}),\ (e_2,\ \{a,\ c,\ d\})\}$. Then, the soft $\widetilde{g}_{1,2} - open$ set which containing (K,E) are (F_1,E), (F_2,E), (F_3,E), (G_1,E), (H_1,E), (H_4,E) and $(\widetilde{\phi},E)$. Therefore,

$$s\imath nt_{\widetilde{g}_{1,2}}(K,E) = \{(e_1,\ \{a,\ b,\ c\}),\ (e_2,\ \{a,\ c,\ d\})\}.$$

Theorem 3.20. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a SBGT−space and (F,E), $(G,E) \in SS(X)_E$. Then,*

1. $s\imath nt_{\widetilde{g}_{1,2}}(\widetilde{\phi},E) = (\widetilde{\phi},E)$,
2. $s\imath nt_{\widetilde{g}_{1,2}}(F,E)\widetilde{\subseteq}(F,E)$,
3. *(F,E) is a soft $\widetilde{g}_{1,2}-$open set iff $s\imath nt_{\widetilde{g}_{1,2}}(F,E) = (F,E)$,*
4. $(F,E)\widetilde{\subseteq}(G,E) \Rightarrow s\imath nt_{\widetilde{g}_{1,2}}(F,E)\widetilde{\subseteq}s\imath nt_{\widetilde{g}_{1,2}}(G,E)$,
5. $s\imath nt_{\widetilde{g}_{1,2}}\left[(F,E)\widetilde{\cap}(G,E)\right]\widetilde{\subseteq}s\imath nt_{\widetilde{g}_{1,2}}(F,E)\widetilde{\cap}s\imath nt_{\widetilde{g}_{1,2}}(G,E)$ *and*
   $s\imath nt_{\widetilde{g}_{1,2}}(F,E)\widetilde{\cup}s\imath nt_{\widetilde{g}_{1,2}}(G,E)\widetilde{\subseteq}s\imath nt_{\widetilde{g}_{1,2}}\left[(F,E)\widetilde{\cup}(G,E)\right]$,
6. $s\imath nt_{\widetilde{g}_{1,2}}\left[s\imath nt_{\widetilde{g}_{1,2}}(F,E)\right] = s\imath nt_{\widetilde{g}_{1,2}}(F,E)$, *i.e., $s\imath nt_{\widetilde{g}_{1,2}}(F,E)$ is a soft $\widetilde{g}_{1,2} - open$ set.*

Proof. Straightforward. □

Theorem 3.21. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a SBGT−space and $(F,E) \in SS(X)_E$. Then,*

$$x_e\widetilde{\in}s\imath nt_{\widetilde{g}_{1,2}}(F,E) \Leftrightarrow \exists\,(U_{x_e},E) \in \widetilde{g}_{1,2}(x_e) \text{ such that } (U_{x_e},E)\,\widetilde{\subseteq}(F,E).$$

Proof. Straightforward. □

Theorem 3.22. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a SBGT−space and $(F,E) \in SS(X)_E$. Then,*

$$s\imath nt_{\widetilde{g}_{1,2}}(F,E) = s\imath nt_{\widetilde{g}_1}(F,E)\widetilde{\cup}s\imath nt_{\widetilde{g}_2}(F,E).$$

Proof.

$$\begin{aligned}
sint_{\widetilde{g}_{1,2}}(F,E) &= \widetilde{\cup}\left\{(G,E):(G,E)\widetilde{\subseteq}(F,E) \text{ for a soft } \widetilde{g}_{1,2}-\text{open set } (G,E)\right\}\\
&= \widetilde{\cup}\left\{\begin{array}{c}(G,E):(G,E)\widetilde{\subseteq}(F,E),\ (G,E)=(G_1,E)\widetilde{\cup}(G_2,E),\\ \text{for a soft } \widetilde{g}_i-\text{open set } (G_i,E),\ i=1,2\end{array}\right\}\\
&= \widetilde{\cup}\left\{\begin{array}{c}(G_1,E)\widetilde{\cup}(G_2,E):(G_1,E)\widetilde{\cup}(G_2,E)\widetilde{\subseteq}(F,E),\\ \text{for a soft } \widetilde{g}_i-\text{open set } (G_i,E),\ i=1,2\end{array}\right\}\\
&= \left[\widetilde{\cup}\left\{(G_1,E):(G_1,E)\widetilde{\subseteq}(F,E) \text{ for a soft } \widetilde{g}_1-\text{open set } (G_1,E)\right\}\right]\\
&\quad \widetilde{\cup}\left[\widetilde{\cup}\left\{(G_2,E):(G_2,E)\widetilde{\subseteq}(F,E) \text{ for a soft } \widetilde{g}_2-\text{open set } (G_2,E)\right\}\right]\\
&= sint_{\widetilde{g}_1}(F,E)\widetilde{\cup}sint_{\widetilde{g}_2}(F,E).
\end{aligned}$$

□

Theorem 3.23. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a SBGT−space and $(F,E)\in SS(X)_E$. Then,*

1. $scl_{\widetilde{g}_{1,2}}(F,E)=\left[sint_{\widetilde{g}_{1,2}}(F,E)^c\right]^c$,
2. $sint_{\widetilde{g}_{1,2}}(F,E)=\left[scl_{\widetilde{g}_{1,2}}(F,E)^c\right]^c$.

Proof. 1.

$$\begin{aligned}
scl_{\widetilde{g}_{1,2}}(F,E) &= scl_{\widetilde{g}_1}(F,E)\widetilde{\cap}scl_{\widetilde{g}_2}(F,E)\\
&= \left((\widetilde{X},E)\backslash sint_{\widetilde{g}_1}\left((\widetilde{X},E)\backslash(F,E)\right)\right)\widetilde{\cap}\left((\widetilde{X},E)\backslash sint_{\widetilde{g}_2}\left((\widetilde{X},E)\backslash(F,E)\right)\right)\\
&= (\widetilde{X},E)\backslash\left[sint_{\widetilde{g}_1}\left((\widetilde{X},E)\backslash(F,E)\right)\widetilde{\cup}sint_{\widetilde{g}_2}\left((\widetilde{X},E)\backslash(F,E)\right)\right]\\
&= (\widetilde{X},E)\backslash sint_{\widetilde{g}_{1,2}}\left((\widetilde{X},E)\backslash(F,E)\right)\\
&= \left[sint_{\widetilde{g}_{1,2}}(F,E)^c\right]^c
\end{aligned}$$

2. Similarly, we can show that (2) is satisfied. □

Definition 3.24. Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a $SBGT$−space and $\widetilde{B}_{\widetilde{g}_{1,2}}\subseteq\widetilde{g}_{1,2}$. Then, $\widetilde{B}_{\widetilde{g}_{1,2}}$ is called soft $\widetilde{g}_{1,2}$−basis for $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ if every element of $\widetilde{g}_{1,2}$ can be written as a union of elements of $\widetilde{B}_{\widetilde{g}_{1,2}}$.

Example 3.25. According to Example 3.7., $\widetilde{B}_{\widetilde{g}_{1,2}}=\{(F_1,E),\ (F_2,E),\ (F_3,E),\ (G_1,E),\ (G_2,E),\ (G_3,E)\}$ is a soft $\widetilde{g}_{1,2}$−basis for $(X,\widetilde{g}_1,\widetilde{g}_2,E)$.

Theorem 3.26. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a SBGT−space and $\widetilde{B}_{\widetilde{g}_{1,2}}$ be a soft $\widetilde{g}_{1,2}$−basis for $(X,\widetilde{g}_1,\widetilde{g}_2,E)$. Then, $\widetilde{g}_{1,2}$ equals the collection of all soft unions of elements $\widetilde{B}_{\widetilde{g}_{1,2}}$.*

Proof. It is easily obtained from the Definition 3.24. □

Proposition 3.27. *$\widetilde{B}_{\widetilde{g}_{1,2}}$ is a soft $\widetilde{g}_{1,2}$−basis for $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ iff whenever (U,E) is a soft $\widetilde{g}_{1,2}$ − open set and $x_e\widetilde{\in}(U,E)$, then there exists $(\beta_{x_e},E)\in\widetilde{B}_{\widetilde{g}_{1,2}}$ such that $x_e\widetilde{\in}(\beta_{x_e},E)\widetilde{\subseteq}(U,E)$.*

Proof. ⇒ Let $\widetilde{B}_{\widetilde{g}_{1,2}}$ be a soft $\widetilde{g}_{1,2}$−basis for $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ and $x_e\widetilde{\in}(U,E)\in\widetilde{g}_{1,2}$. Then $(U,E)=\underset{(\beta,E)\in B'_{\widetilde{g}_{1,2}}\widetilde{\subseteq}\widetilde{B}_{\widetilde{g}_{1,2}}}{\widetilde{\cup}}(\beta,E)$.
Therefore, there exists $(\beta_{x_e},E)\in B'$ such that $x_e\widetilde{\in}(\beta_{x_e},E)\widetilde{\subseteq}(U,E)$.

$\Leftarrow$ Suppose that $x_e \widetilde{\in} (\beta_{x_e}, E) \widetilde{\subseteq} (U, E)$ is satisfied for every $(U, E) \in \widetilde{g}_{1,2}$ and every $x_e \widetilde{\in} (U, E)$. Then,

$$(U, E) = \widetilde{\bigcup}_{x_e \widetilde{\in} (U,E)} \{x_e\} \widetilde{\subseteq} \widetilde{\bigcup}_{x_e \widetilde{\in} (U,E)} (\beta_{x_e}, E) \widetilde{\subseteq} \widetilde{\bigcup} (U, E) \Rightarrow (U, E) = \widetilde{\bigcup}_{x_e \widetilde{\in} (U,E)} (\beta_{x_e}, E).$$

That is, $\widetilde{B}_{\widetilde{g}_{1,2}}$ is a soft $\widetilde{g}_{1,2}-$basis for $(X, \widetilde{g}_1, \widetilde{g}_2, E)$. □

Theorem 3.28. *Let $(X, \widetilde{g}_1, \widetilde{g}_2, E)$ be a SBGT−space and $\widetilde{B}_{\widetilde{g}_{1,2}}$ be a soft $\widetilde{g}_{1,2}-$basis for $(X, \widetilde{g}_1, \widetilde{g}_2, E)$. Then,*

$$\widetilde{B}_{\widetilde{g}_{1,2}} = \widetilde{B}_{\widetilde{g}_1} \widetilde{\cup} \widetilde{B}_{\widetilde{g}_2}.$$

Proof. Suppose that $\widetilde{B}_{\widetilde{g}_{1,2}}$ is a soft $\widetilde{g}_{1,2}-$basis for $(X, \widetilde{g}_1, \widetilde{g}_2, E)$ and $(U, E) \in \widetilde{g}_{1,2}$. Then,

$$\widetilde{\bigcup}_{(\beta,E) \in B'_{\widetilde{g}_{1,2}} \widetilde{\subseteq} B_{\widetilde{g}_{1,2}}} (\beta, E) = (U, E) = (U_1, E) \widetilde{\cup} (U_2, E)$$

such that $(U_1, E) \in \widetilde{g}_1$ and $(U_2, E) \in \widetilde{g}_2$. Therefore,

$$(U_1, E) = \widetilde{\bigcup}_{(\beta_1,E) \in B'_{\widetilde{g}_1} \widetilde{\subseteq} B_{\widetilde{g}_1}} (\beta_1, E) \text{ and } (U_2, E) = \widetilde{\bigcup}_{(\beta_2,E) \in B'_{\widetilde{g}_2} \widetilde{\subseteq} B_{\widetilde{g}_2}} (\beta_2, E).$$

In this case,

$$\widetilde{\bigcup}_{(\beta,E) \in B'_{\widetilde{g}_{1,2}} \widetilde{\subseteq} \widetilde{B}_{\widetilde{g}_{1,2}}} (\beta, E) = \widetilde{\bigcup}_{(\beta_1,E) \in B'_{\widetilde{g}_1} \widetilde{\subseteq} \widetilde{B}_{\widetilde{g}_1}} (\beta_1, E) \; \widetilde{\cup} \; \widetilde{\bigcup}_{(\beta_2,E) \in B'_{\widetilde{g}_2} \widetilde{\subseteq} \widetilde{B}_{\widetilde{g}_2}} (\beta_2, E).$$

That is, $\widetilde{B}_{\widetilde{g}_{1,2}} = \widetilde{B}_{\widetilde{g}_1} \widetilde{\cup} \widetilde{B}_{\widetilde{g}_2}$. □

Theorem 3.29. *Let $\widetilde{B}_{\widetilde{g}_{1,2}}$ and $\widetilde{B}'_{\widetilde{g}'_{1,2}}$ be two soft bases for $(X, \widetilde{g}_1, \widetilde{g}_2, E)$ and $(X, \widetilde{g}'_1, \widetilde{g}'_2, E)$, respectively. If $\widetilde{B}'_{\widetilde{g}'_{1,2}} \subseteq \widetilde{B}_{\widetilde{g}_{1,2}}$, then $\widetilde{g}_{1,2} \subseteq \widetilde{g}'_{1,2}$.*

Proof. Straightforward. □

Definition 3.30. Let $(X, \widetilde{g}_1, \widetilde{g}_2, E)$ be a $SBGT-$space, (F, E) be a soft set over X and $x_e \widetilde{\in} SS(X)_E$. Then, (F, E) is said to be a soft $\widetilde{g}_{1,2}-$neighborhood (briefly soft $\widetilde{g}_{1,2} - nbd$) of x_e if there exists a soft $\widetilde{g}_{1,2}-$open set (G, E) such that $x_e \widetilde{\in} (G, E) \widetilde{\subseteq} (F, E)$. The set of all soft $\widetilde{g}_{1,2} - nbd$ of x_e is denoted by $\widetilde{N}_{\widetilde{g}_{1,2}}(x_e)$, is called family of soft $\widetilde{g}_{1,2} - nbd$ of x_e.

Proposition 3.31. *Let $(X, \widetilde{g}_1, \widetilde{g}_2, E)$ be a SBGT−space, (F, E) and (G, E) be two soft set over X and $x_e \widetilde{\in} SS(X)_E$. Then,*

1. *If $(F, E) \in \widetilde{N}_{\widetilde{g}_{1,2}}(x_e)$, then $x_e \widetilde{\in} (F, E)$*
2. *If $(F, E) \in \widetilde{N}_{\widetilde{g}_{1,2}}(x_e)$ and $(F, E) \widetilde{\subseteq} (G, E)$, then $(G, E) \in \widetilde{N}_{\widetilde{g}_{1,2}}(x_e)$*
3. *(F, E) is a soft $\widetilde{g}_{1,2} - open$ set iff (F, E) is a soft $\widetilde{g}_{1,2} - nbd$ of each its soft points.*
4. *If $(F, E) \in \widetilde{N}_{\widetilde{g}_{1,2}}(x_e)$, then there exists a soft $\widetilde{g}_{1,2} - open$ set (U, E) such that $(U, E) \widetilde{\subseteq} (F, E)$ and $(U, E) \in \widetilde{N}_{\widetilde{g}_{1,2}}(y_{e'})$ for every $y_{e'} \widetilde{\in} (U, E)$.*

Proof. 1. Let (F, E) be a soft $\widetilde{g}_{1,2} - nbd$ of x_e, then $x_e \widetilde{\in} (U, E) \widetilde{\subseteq} (F, E)$ such that $(U, E) \in \widetilde{g}_{1,2}$. Therefore, $x_e \widetilde{\in} (F, E)$.

2. Assume that $(F, E) \in \widetilde{N}_{\widetilde{g}_{1,2}}(x_e)$ and $(F, E) \widetilde{\subseteq} (G, E)$. Then $x_e \widetilde{\in} (U, E) \widetilde{\subseteq} (F, E) \widetilde{\subseteq} (G, E)$ such that $(U, E) \in \widetilde{g}_{1,2}$. Therefore, $(G, E) \in \widetilde{N}_{\widetilde{g}_{1,2}}(x_e)$.

3. Suppose that (F, E) is a soft $\widetilde{g}_{1,2} - open$ set, then $x_e \widetilde{\in} (F, E) \widetilde{\subseteq} (F, E)$ for every $x_e \widetilde{\in} (F, E)$. Therefore, (F, E) is a soft $\widetilde{g}_{1,2} - nbd$ of each of its soft points.

Conversely, let (F,E) be a soft $\widetilde{g}_{1,2}-nbd$ of each its soft points and $x_e\widetilde{\in}(F,E)$. Then there exists a soft $\widetilde{g}_{1,2}-open$ set (U,E) such that $x_e\widetilde{\in}(U,E)\widetilde{\subseteq}(F,E)$. Since

$$(F,E)=\widetilde{\bigcup_{x_e\widetilde{\in}(F,E)}}\{x_e\}\widetilde{\subseteq}\widetilde{\bigcup_{x_e\widetilde{\in}(F,E)}}(U,E)\widetilde{\subseteq}\widetilde{\bigcup_{x_e\widetilde{\in}(F,E)}}(F,E),$$

it follows that (F,E) is a union of soft $\widetilde{g}_{1,2}-open$ sets. In this case, (F,E) is a soft $\widetilde{g}_{1,2}-open$ set.

4. Let $(F,E)\in\widetilde{N}_{\widetilde{g}_{1,2}}(x_e)$. Then $x_e\widetilde{\in}(U,E)\widetilde{\subseteq}(F,E)$ such that $(U,E)\in\widetilde{g}_{1,2}$. Hence, from the condition (3.), $(U,E)\in\widetilde{N}_{\widetilde{g}_{1,2}}(y_{e'})$ for every $y_{e'}\widetilde{\in}(U,E)$. □

Proposition 3.32. *Let $(X,\widetilde{g}_1,\widetilde{g}_2,E)$ be a SBGT−space. Then,*

$$\widetilde{N}_{\widetilde{g}_{1,2}}(x_e)=\widetilde{N}_{\widetilde{g}_1}(x_e)\widetilde{\cup}\widetilde{N}_{\widetilde{g}_2}(x_e)\ \textit{for each}\ x_e\widetilde{\in}SS(X)_E.$$

Proof. Let $x_e\widetilde{\in}SS(X)_E$ be a soft point and $(F,E)\in\widetilde{N}_{\widetilde{g}_{1,2}}(x_e)$ be any soft $\widetilde{g}_{1,2}-nbd$ of x_e. Then there exists a soft $\widetilde{g}_{1,2}-open$ set (G,E) such that $x_e\widetilde{\in}(G,E)\widetilde{\subseteq}(F,E)$. If $(G,E)\in\widetilde{g}_{1,2}$, there exist $(G_1,E)\in\widetilde{g}_1$ and $(G_2,E)\in\widetilde{g}_2$ such that $(G,E)=(G_1,E)\widetilde{\cup}(G_2,E)$. Since $x_e\widetilde{\in}(G,E)=(G_1,E)\widetilde{\cup}(G_2,E)$, then $x_e\widetilde{\in}(G_1,E)$ or $x_e\widetilde{\in}(G_2,E)$. So $x_e\widetilde{\in}(G_1,E)\widetilde{\subseteq}(G,E)\widetilde{\subseteq}(F,E)$ or $x_e\widetilde{\in}(G_2,E)\widetilde{\subseteq}(G,E)\widetilde{\subseteq}(F,E)$. In this case, $(F,E)\in\widetilde{N}_{\widetilde{g}_1}(x_e)$ or $(F,E)\in\widetilde{N}_{\widetilde{g}_2}(x_e)$, i. e., $(F,E)\in\widetilde{N}_{\widetilde{g}_1}(x_e)\widetilde{\cup}\widetilde{N}_{\widetilde{g}_2}(x_e)$.

Conversely, suppose that $(F,E)\in\widetilde{N}_{\widetilde{g}_1}(x_e)\widetilde{\cup}\widetilde{N}_{\widetilde{g}_2}(x_e)$. Then $(F,E)\in\widetilde{N}_{\widetilde{g}_1}(x_e)$ or $(F,E)\in\widetilde{N}_{\widetilde{g}_2}(x_e)$. Hence, there exists $x_e\widetilde{\in}(G_1,E)\in\widetilde{g}_1$ or $x_e\widetilde{\in}(G_2,E)\in\widetilde{g}_2$ such that $x_e\widetilde{\in}(G_1,E)\widetilde{\subseteq}(F,E)$ and $x_e\widetilde{\in}(G_2,E)\widetilde{\subseteq}(F,E)$. As a result, $x_e\widetilde{\in}(G_1,E)\widetilde{\cup}(G_2,E)=(G,E)\widetilde{\subseteq}(F,E)$ such that $(G,E)\in\widetilde{g}_{1,2}$ i. e., $(F,E)\in\widetilde{N}_{\widetilde{g}_{1,2}}(x_e)$. □

4. Conclusion

In this study, soft bigeneralized topological spaces have been introduced and the basic concepts of topology are adapted to this spaces. In other words, basic structure for topological studies on this spaces are established. We hope that results of this study will contribute to the topological studies on soft bigeneralized topological spaces.

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