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On the Breather Waves, Rogue Waves and Solitary Waves to a Generalized (2+1)-dimensional Caudrey-Dodd-Gibbon-Kotera-Sawada Equation

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Abstract. In this paper, we consider a generalized (2+1)-dimensional Caudrey-Dodd-Gibbon-Kotera-Sawada (CDGKS) equation. By using the Bell polynomial, we derive its bilinear form. Based on the homoclinic breather limit method, we construct the homoclinic breather wave and the rational rogue wave solutions of the equation. Then by using its bilinear form, some solitary wave solutions of the CDGKS equation are provided by a very natural way. Moreover, some prominent characteristics for the dynamic behaviors of these solitons are analyzed by several graphics. Our results show that the breather wave can be transformed into rogue wave under the extreme behavior.

1. Introduction

In modern mathematics, the study of nonlinear evolution equations (NLEEs) plays a significant role in areas of physics and other sciences. It is well known that finding exact solutions of NLEEs is a popular topic, and more and more researchers are involved. After the development of these years, there are a number of methods to solve NLEEs, including Hirota bilinear method [24] and Lie group method [4], etc. Recently, rogue wave, noted for the nature of giant waves and extreme waves, has attracted a lot of attentions. Researchers notice it in the deep ocean for the first time. Since then, the rogue wave phenomenons can also be observed in some other fields, like nonlinear optic fibers, Bose-Einstein condensates, biophysics and sometimes finance [2, 26, 28, 29, 41, 64]. For the first time, Peregrine showed that the frst-order rational solution of nonlinear Schödinger equation can describe the rogue waves phenomenon [37]. Nowadays, according to the application of Darboux transformation and Hirota bilinear method, rogue wave solutions can be found in some other nonlinear equations [1, 3, 6, 11, 12, 16, 19, 20, 23, 38, 53–55, 57–61, 66, 71]. There are also recent systematical studies on lump solutions and interaction solutions to integrable equations by Ma and his collaborators [7, 32–35, 67, 68, 70, 72]. Especially, the lump solutions to nonlinear partial

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differential equations are systematically studied in [35] via Hirota bilinear forms. The existence of diverse lump and interaction solutions to linear partial differential equations in (3+1)-dimensions has been explored in [33].

In this paper, we consider a generalized (2+1)-dimensional Caudrey-Dodd-Gibbon-Kotera-Sawada (CDGKS) equation

$$36u_t = -\alpha\left(u_{xxxxx} + 15\left(uu_{xx}\right)_x + 45u^2u_x\right) + 5\beta\left(u_{xxy} + 3uu_y + 3u_x\int u_y dx\right) + \gamma\int u_{yy}dx, \tag{1}$$

which describes a large range of nonlinear dispersive physical phenomena as one of the most important integrable equations in soliton theory. Here $u = u(x, y, t)$, acts as the nonlinear dispersive wave, is a differentiable function with the scaled space variables x, y and time variable t, the real constant parameters α, β and γ are related to the dispersion, the subscripts represent partial derivatives, the operator $\int$ means integration to the corresponding variable. Eq.(1) was first presented by Konopelchenk and Dubovsky for the case $\alpha = \beta = 1$ and $\gamma = 5$ [8, 27]. On the other hand, when $\alpha = \beta = 1, \gamma = 5$ and $u_y = 0$, Eq.(1) reduces to the (1+1)-dimensional CDGKS equation, its N-soliton solutions has been researched by Sawada and Kotera [39]. Some systematic analysis were conducted via Gibbon, Dodd and Caudrey in [21, 22] and the quasi-periodic solutions were obtained in [5]. Recently, the interaction behaviours between solitons and cnoidal periodic waves for the CDGKS equation was studied in [9]. Eq.(1) is also called the BKP-type equation. Lump solutions for the BKP equation are perfectly presented by Ma and his collaborators in [35], and eight sets of mixed lump-soliton solutions which allow the separation of lumps and line solitons was derived for the (2+1)-dimensional BKP equation in [68].

To the best of our knowledge, There are a great deal of works performed about the generalized (2+1)-dimensional CDGKS equation. However, some phenomenons of rogue solitons for Eq.(1) have never been reported before. In this work, we provide the homoclinic breather waves based on the approach of the extended homoclinic test and the symbol calculation methods [10, 17, 18, 25, 40, 42–44, 50–52, 56, 62, 63, 65, 69]. Then taking extreme condition about the breather waves, we construct rogue solutions. Also, we study some soliton solutions by Hirota bilinear method in detail.

The outline of this paper is as follows. In section 2, we carry out the bilinear form analysis for Eq.(1). In section 3, the breather and rogue solutions are considered by bilinear form and the extended homoclinic test analysis. In section 4, we apply Hirota bilinear method to get some soliton solutions. In section 5, we discuss the solitons interactions. Finally, some conclusions of this paper are presented in the last section.

2. Bilinear Form

To begin with, we introduce a transformation in the form as follows

$$u = h(t)\, q_{2x}, \tag{2}$$

where $h = h(t)$ is a function with the variable of t which can be confirmed later. Putting (2) into Eq.(1), integrating the equation with respect to x twice, we have

$$E(q) = 5\beta\left(q_{3x,y} + 3q_{2x}q_{x,y}\right) + \gamma q_{2y} - \alpha\left(q_{6x} + 15q_{2x}q_{4x} + 15q_{2x}^3\right) - 36q_{x,t} = \delta, \tag{3}$$

under the condition of $h(t) = 1$. According to the following properties P-polynomial detailed cf. [13–15, 30, 31, 36, 45–49],

$$P_{x,t} = q_{x,t},\ \ P_{2y} = q_{2y},\ \ P_{3x,y} = q_{3x,y} + 3q_{2x}q_{x,y},$$
$$P_{6x} = q_{6x} + 15q_{2x}q_{4x} + 15q_{2x}^3,$$

we get

$$E(q) = 5\beta P_{3x,y} + \gamma P_{2y} - \alpha P_{6x} - 36P_{x,t} = \delta, \tag{4}$$

where δ is an integral constant. Considering $\delta = 0$ as a special circumstance, the above equation can be expressed by

$$E(q) = 5\beta P_{3x,y} + \gamma P_{2y} - \alpha P_{6x} - 36 P_{x,t} = 0. \tag{5}$$

Applying the transformation

$$q = 2\ln(F) \Leftrightarrow u = 2\ln(F)_{xx}, \tag{6}$$

where $F = F(x, y, t)$ is a real function with the variables of x, y, t, we have the bilinear form for Eq.(1)

$$\left(5\beta D_x^3 D_y + \gamma D_y^2 - \alpha D_x^6 - 36 D_x D_t\right) F \cdot F = 0, \tag{7}$$

with the D-operator being denoted by

$$D_x^m D_y^n (f \cdot g) = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^m \left(\frac{\partial}{\partial y} - \frac{\partial}{\partial y'}\right)^n f(x, y) \cdot g(x', y')\big|_{x=x', y=y'}. \tag{8}$$

3. Homoclinic Breather Waves and Rogue Waves

In this section, by employing the extended homoclinic test method (EHTM), we present the rogue wave solutions for Eq.(1). At first, making $\zeta = x + t$ and replacing it into Eq.(1), we have

$$36u_\zeta = -\alpha\left(u_{5\zeta} + 15\left(u u_{\zeta\zeta}\right)_\zeta + 45u^2 u_\zeta\right) + 5\beta\left(u_{\zeta\zeta y} + 3u u_y + 3u_\zeta \int u_y d_\zeta\right) + \gamma \int u_{yy} d_\zeta. \tag{9}$$

It is easy to see that Eq.(1) has a balanced solution u_0, which is an arbitrary constant. Performing the following assumption,

$$u = u_0 + 2(\ln F)_{\zeta\zeta}, \tag{10}$$

where $F = F(\zeta, y)$ is a real function , and substituting (10) into (9), we obtain the new bilinear form ,

$$\left[5\beta D_\zeta^3 D_y + \gamma D_y^2 - \left(36 + 45\alpha u_0^2\right) D_\zeta^2 - \alpha D_\zeta^6 - 15\alpha u_0 D_\zeta^4 + 15\beta u_0 D_\zeta D_y\right] F \cdot F = 0, \tag{11}$$

with

$$\begin{cases} D_\zeta^6 F \cdot F = 2\left(F_{6\zeta} F + 15 F_{4\zeta} F_{2\zeta} - 10 F_{3\zeta}^2 - 6 F_{5\zeta} F_\zeta\right), \\ D_\zeta^4 F \cdot F = 2\left(F_{4\zeta} F - 4 F_{3\zeta} F_\zeta + 3 F_{2\zeta}^2\right), \\ D_\zeta^3 D_y F \cdot F = 2\left(F_{3\zeta,y} F - F_{3\zeta} F_y - 3 F_{2\zeta,y} F_\zeta + 3 F_{2\zeta} F_{\zeta,y}\right), \\ D_\zeta D_y F \cdot F = 2\left(F_{\zeta,y} F - F_\zeta F_y\right), \\ D_\zeta^2 F \cdot F = 2\left(F_{2\zeta} F - F_\zeta^2\right), \\ D_y^2 F \cdot F = 2\left(F_{2y} F - F_y^2\right). \end{cases} \tag{12}$$

3.1. Homoclinic Breather Wave Solutions

According to the approach of the extended homoclinic test, the solution of Eq.(11) can be showed as follows

$$F = e^{-p_1(\zeta - \mu y)} + \kappa_1 \cos\left(p\left(\zeta + \nu y\right)\right) + \kappa_2 e^{p_1(\zeta - \mu y)}, \tag{13}$$

where $p_1, p, \mu, \nu, \kappa_1, \kappa_2$ are all real constants. Putting (13) into (11), and equating the each coefficient for $e^{mp_1(\zeta-\mu y)}$ $(m=-1,0,1)$ to zero, and let $p=p_1$, we obtain following results

$$
\begin{aligned}
& p^2 = \frac{15\beta u_0\mu + 15\beta u_0\nu - \gamma\mu^2 + \gamma\nu^2}{60\alpha u_0 + 10\beta\mu - 10\beta\nu}, \\
& -8\alpha p^4 + 10\beta\mu p^2 + 10\beta\nu p^2 + 90\alpha u_0^2 + 15\beta u_0\mu - 15\beta u_0\nu + 2\gamma\mu\nu + 72 = 0, \\
& 4\kappa_2\left(16\alpha p^4 + 60\alpha u_0 p^2 + 20\beta\mu p^2 + 45\alpha u_0^2 + 15\beta u_0\mu - \gamma\mu^2 + 36\right) \\
& = \kappa_1^2\left(16\alpha p^4 - 60\alpha u_0 p^2 + 20\beta\nu p^2 + 45\alpha u_0^2 - 15\beta u_0\nu - \gamma\nu^2 + 36\right),
\end{aligned}
\tag{14}
$$

where $\alpha, \beta, \gamma, \mu, \nu, \kappa_2$ are all constants to be selected later. Then, we provide another expression for Eq.(13) given by

$$
F = 2\sqrt{\kappa_2}\cosh\left(p\,(x+t-\mu y) + \frac{1}{2}\ln\kappa_2\right) + \kappa_1\cos\left(p\,(x+t+\nu y)\right). \tag{15}
$$

Through the substitution of Eq.(15) into Eq.(10), we have the following solution for Eq.(1)

$$
u\,(x,y,t) = u_0 + \frac{2p^2\left[4\kappa_2 - \kappa_1^2 + 4\kappa_1\sqrt{\kappa_2}\sinh(\Omega_1)\sin(\Omega_2)\right]}{\left[2\sqrt{\kappa_2}\cosh(\Omega_1) + \kappa_1\cos(\Omega_2)\right]^2}, \tag{16}
$$

where

$$
\Omega_1 = p\,(x+t-\mu y) + \frac{1}{2}\ln\kappa_2, \Omega_2 = p\,(x+t+\nu y),
$$

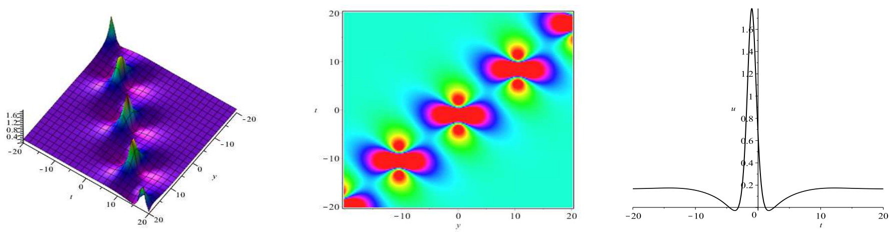
u_0 is a undetermined constant, p can be expressed by μ, ν based on the first formula of (14), and κ_1 is wrote by κ_2, μ, ν according to the last formula of (14).

The above solution u is homoclinic breather wave, which has been provided in (16), and when $t \to \infty$, the above wave will tend to a fixed point u_0. In fact, it also indicates that the breather wave propagates by the way of periodic oscillation. Considering one direction, the expression of the solution means that the homoclinic breather wave can be constituted via the mutual effect between the homoclinic waves and the breather waves.

Taking $\kappa_2 = 1$ leads to $\frac{1}{2}\ln\kappa_2 = 0$. Substituting it into Eq. (16), we have

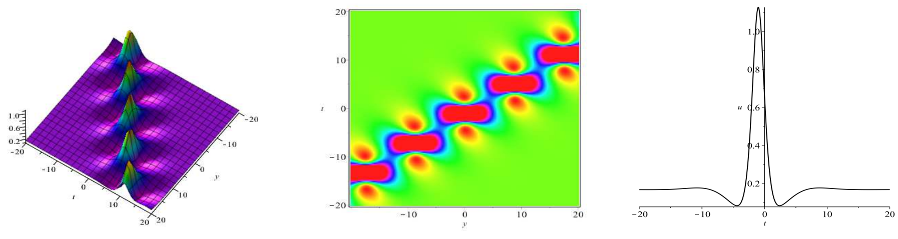
$$
u\,(x,y,t) = u_0 + \frac{2p^2\left[4 - \kappa_1^2 + 4\kappa_1\sinh\left(p\,(x+t-\mu y)\right)\sin\left(p\,(x+t+\nu y)\right)\right]}{\left[2\cosh\left(p\,(x+t-\mu y)\right) + \kappa_1\cos\left(p\,(x+t+\nu y)\right)\right]^2}. \tag{17}
$$

In the following, we present the propagation of the homoclinic breather waves by two graphics. Figs.1 and 2 reflect the homoclinic breather wave (17) based on the different parameters, respectively.



(a) (b) (c)

Figure 1. (Color online) Homoclinic breather waves (17) for Eq.(1) by selecting suitable parameters: $u_0 = \frac{1}{6}, \kappa_1 = -1.6, \kappa_2 = 1, p = 0.3, \mu = 0.9, \nu = 1.1, x = 1$. **(a)** Perspective view of the real part of the wave. **(b)** Density plot of the wave. **(c)** The wave propagation pattern of the wave along the t-axis.



(a) (b) (c)

Figure 2. (Color online) Homoclinic breather waves (17) for Eq.(1) by selecting suitable parameters: $u_0 = \frac{1}{6}, \kappa_1 = -1, \kappa_2 = 1, p = 0.4, \mu = 0.7, \nu = 1.1, x = 1$. **(a)** Perspective view of the real part of the wave. **(b)** Density plot of the wave. **(c)** The wave propagation pattern of the wave along the t-axis.

3.2. Rogue-Wave Solutions

Obviously, $\frac{2\pi}{p}$ denotes the period of Eq. (17). When the period $\frac{2\pi}{p} \to \infty$, the breather waves can transform into the rogue waves. Therefore, we take $p \to 0$. Meanwhile, according to the principle of Taylor expansion about the two-wave function $H(x, y, t)$ at $p = 0$, we have

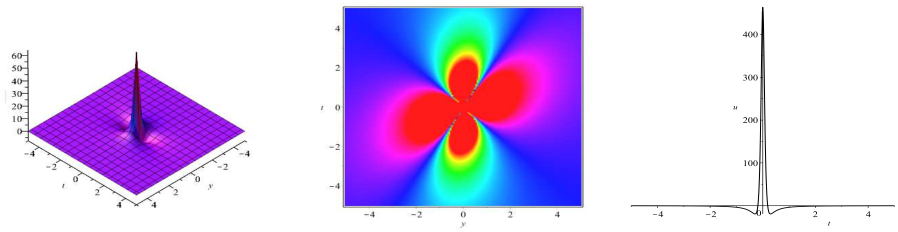
$$
\begin{aligned}
&e^{p(x+t-\mu y)} = 1 + p(x + t - \mu y) + \frac{p^2}{2}(x + t - \mu y)^2 + O\left(p^3\right), \\
&\cos\left(p\left(x + t + \nu y\right)\right) = 1 - \frac{p^2}{2}(x + t + \nu y)^2 + O\left(p^3\right).
\end{aligned} \tag{18}
$$

Ultimately, we derive the rogue-wave solutions as follows

$$
u = u_0 + \frac{16\left[M - (x + t - \mu y)(x + t + \nu y)\right]}{\left[(x + t - \mu y)^2 + (x + t + \nu y)^2 + M\right]^2}, \tag{19}
$$

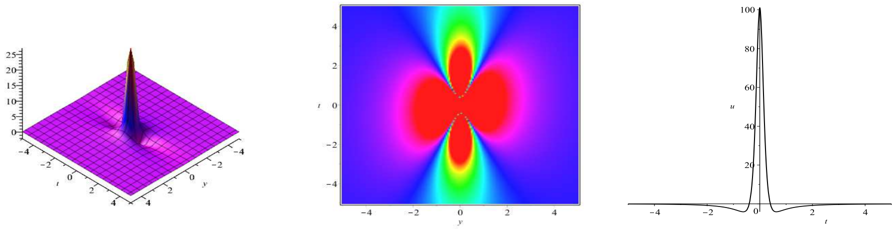
where $M = \frac{30\alpha u_0 + 5\beta\mu - 5\beta\nu}{45\alpha u_0^2 + 15\beta u_0 \mu - \gamma\mu + 36}$, and $\alpha, \beta, \gamma, u_0, \mu, \nu$ are constants.

In the following, we present the propagation of the rogue waves by two graphics. Figs.3 and 4 reflect the rogue waves (19) based on the different parameters, respectively.



(a) (b) (c)

Figure 3. (Color online) Rogue waves (19) for Eq.(1) by selecting suitable parameters: $u_0 = \frac{1}{10}, \alpha = 0.1, \beta = 0.25, \gamma = 0.5, \nu = 1$. **(a)** Perspective view of the real part of the wave $(x = 0)$. **(b)** Density plot of the wave. **(c)** The wave propagation pattern of the wave along the t-axis.



(a) (b) (c)

Figure 4. (Color online) Rogue waves (19) for Eq.(1) by selecting suitable parameters: $u_0 = \frac{1}{10}, \alpha = 1, \beta = 1, \gamma = 5, \nu = 2$. **(a)** Perspective view of the real part of the wave ($x = 0$). **(b)** Density plot of the wave. **(c)** The wave propagation pattern of the wave along the t-axis.

4. Solitary Waves

Theorem 4.1. *Based on the bilinear equation* (7), *Eq.*(1) *demits the following N-soliton solution with* $N = 1, 2$

$$u = 2\left[\ln(F)\right]_{xx}, \qquad F = \sum_{\sigma=0,1} \exp\left(\sum_{j=1}^{N} \sigma_i \xi_i + \sum_{1\le i<j\le N}^{N} \sigma_i \sigma_j \Delta_{ij}\right), \qquad N = 1, 2, \tag{20}$$

$$\xi_i = a_i x + b_i y + c_i t + \theta_i, \qquad c_i = \frac{5\beta a_i^3 b_i + \gamma b_i^2 - \alpha a_i^6}{36 a_i}, \tag{21}$$

and

$$e^{\Delta_{ij}} = -\frac{\left[5\beta\left(a_i - a_j\right)^3\left(b_i - b_j\right) + \gamma\left(b_i - b_j\right)^2 - \alpha\left(a_i - a_j\right)^6 - 36\left(a_i - a_j\right)\left(c_i - c_j\right)\right]}{\left[5\beta\left(a_i + a_j\right)^3\left(b_i + b_j\right) + \gamma\left(b_i + b_j\right)^2 - \alpha\left(a_i + a_j\right)^6 - 36\left(a_i + a_j\right)\left(c_i + c_j\right)\right]}, \tag{22}$$

here $\alpha, \beta, \gamma, a_i, b_i, \theta_i\, (i = 1, 2, \ldots, N)$ *are all arbitrary real constants, and* $\sum_{\sigma=0,1}$ *means the summation over all possible combinations of* $\sigma_i, \sigma_j = 0, 1\, (i, j = 1, 2, \ldots, N)$.

Taking $N = 1$, the one-soliton solution of Eq.(1) reads

$$u = 2\left[\ln\left(1 + e^{\xi_1}\right)\right]_{xx} = \frac{a_1^2}{2}\operatorname{sech}^2\frac{a_1 x + b_1 y + c_1 t + \theta_1}{2}, \tag{23}$$

with

$$c_1 = \frac{5\beta a_1^3 b_1 + \gamma b_1^2 - \alpha a_1^6}{36 a_1}, \tag{24}$$

where a_1, b_1, θ_1 are arbitrary constants, and c_1 is denoted by a_1, b_1.

In the same way, taking $N = 2$, the two-soliton solutions for eq.(1) have the following form

$$u = 2\left[\ln\left(1 + e^{\xi_1} + e^{\xi_2} + e^{\xi_1+\xi_2+\Delta_{12}}\right)\right]_{xx}, \tag{25}$$

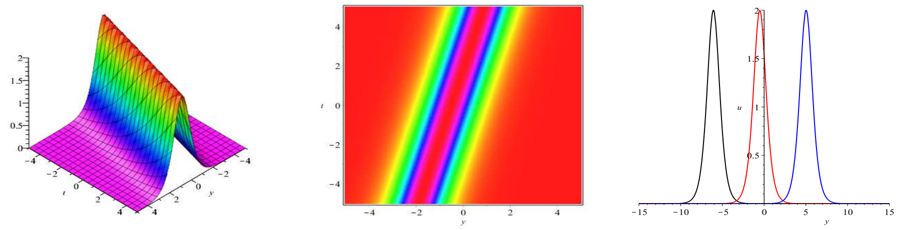
with

$$\xi_i = a_i x + b_i y + c_i t + \theta_i, \quad c_i = \frac{5\beta a_i^3 b_i + \gamma b_i^2 - \alpha a_i^6}{36 a_i}, \quad i = 1, 2, \tag{26}$$

$$e^{\Delta_{12}} = -\frac{\left[5\beta\left(a_1 - a_2\right)^3\left(b_1 - b_2\right) + \gamma\left(b_1 - b_2\right)^2 - \alpha\left(a_1 - a_2\right)^6 - 36\left(a_1 - a_2\right)\left(c_1 - c_2\right)\right]}{\left[5\beta\left(a_1 + a_2\right)^3\left(b_1 + b_2\right) + \gamma\left(b_1 + b_2\right)^2 - \alpha\left(a_1 + a_2\right)^6 - 36\left(a_1 + a_2\right)\left(c_1 + c_2\right)\right]}, \tag{27}$$

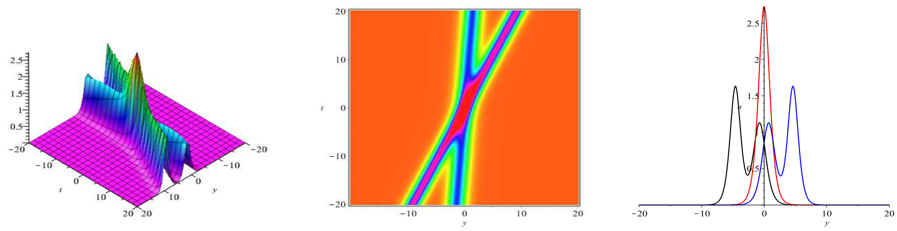
where a_i, b_i, θ_i, and c_i are are free constants denoted by a_i, b_i.

In what follows, we present the propagation of the one-soliton waves and two-soliton waves by Figs. 5 and Figs. 6 based on the suitable parameters, respectively.



(a) (b) (c)

Figure 5. (Color online) One-soliton waves (23) for Eq.(1) by selecting suitable parameters: $\alpha = 1, \beta = 0.2, \gamma = 2, a_1 = b_1 = 2, \theta_1 = \frac{\pi}{3}$. **(a)** Perspective view of the real part of the wave ($x = 0$). **(b)** Density plot of the wave. **(c)** The wave propagation pattern of the wave along the y-axis at $t = -20, t = 0, t = 20$.



(a) (b) (c)

Figure 6. (Color online) Two-soliton waves (25) for Eq.(1) by selecting suitable parameters: $\alpha = 1, \beta = -0.5, \gamma = 2, a_1 = b_1 = 1.8, a_2 = -b_2 = 1.5, \theta_1 = \theta_2 = 0$. **(a)** Perspective view of the real part of the wave ($x = 0$). **(b)** Density plot of the wave. **(c)** The wave propagation pattern of the wave along the y-axis at $t = -10, t = 0, t = 10$.

5. Analysis on Solitons Interaction

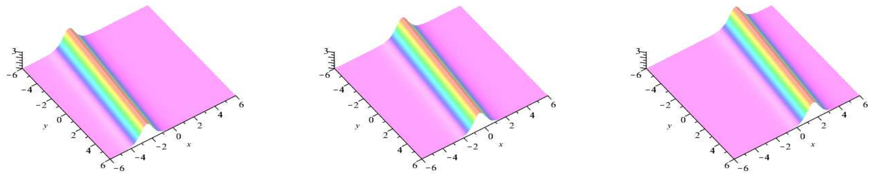
In this section, we discuss the solitons interactions of Eq. (1). In terms of one-soliton solution (23), it is not hard to obtain the amplitude A and velocity v along the x and y planes respectively,

$$A = \frac{a_1^2}{2}, \qquad v = \left(v_x, v_y\right)^T, \tag{28}$$

with

$$v_x = -\frac{5\beta a_1^3 b_1 + \gamma b_1^2 - \alpha a_1^6}{36 a_1^2}, \qquad v_y = -\frac{5\beta a_1^3 b_1 + \gamma b_1^2 - \alpha a_1^6}{36 a_1 b_1}. \tag{29}$$

Fig. 7 presents the shape and propagation of one-soliton solutions (23) on $x - y$ plane at $t = -20, t = 0$, and $t = 20$. As we can see from Fig.7, amplitude and shape are invariable with the change of time, which implies the energy of soliton (23) is steady.



(a) (b) (c)

Figure 7. (Color online) One-soliton waves (23) for Eq.(1) with parameters: $\alpha = 0.1, \beta = 0.1, \gamma = 0.5, a_1 = 2.5, b_1 = 0.2, \theta_1 = 0$. **(a)** t=-20. **(b)** t=0. **(c)** t=20.

According to two-soliton solution (25), we can analyze the asymptotic properties of two-solitons under the circumstance of long time. Firstly, for one case, fixing ξ_1, we derive ξ_2 in the following form:

$$\xi_2 = a_2\left[\frac{\xi_1}{a_1} + \left(\frac{b_2}{a_2} - \frac{b_1}{a_1}\right)y + \left(\frac{c_2}{a_2} - \frac{c_1}{a_1}\right)t + \frac{\theta_2}{a_2} - \frac{\theta_1}{a_1}\right], \tag{30}$$

where we suppose a_1, a_2 are positive number, and $\frac{b_2}{a_2} > \frac{b_1}{a_1}, \frac{c_2}{a_2} > \frac{c_1}{a_1}$.

Let $t \to -\infty, y \to -\infty$, e^{ξ_2} and $e^{\xi_1+\xi_2+\Delta_{12}}$ tend to zero, and $t \to +\infty, y \to +\infty$, e^{ξ_2} and $e^{\xi_1+\xi_2+\Delta_{12}}$ tend to infinity. In terms of the above case, we have

$$\begin{aligned} \lim_{t\to-\infty, y\to-\infty} u(x,y,t) &= \frac{a_1^2}{2}\operatorname{sech}^2\frac{a_1x + b_1y + c_1t + \theta_1}{2}, \\ \lim_{t\to+\infty, y\to+\infty} u(x,y,t) &= \frac{a_1^2}{2}\operatorname{sech}^2\frac{a_1x + b_1y + c_1t + \theta_1 + \Delta_{12}}{2}. \end{aligned} \tag{31}$$

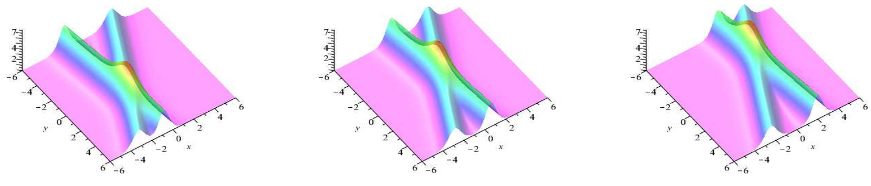
In the same way, for another case, ξ_2 is fixed, ξ_1 can be wrote as follows

$$\xi_1 = a_1\left[\frac{\xi_2}{a_2} + \left(\frac{b_1}{a_1} - \frac{b_2}{a_2}\right)y + \left(\frac{c_1}{a_1} - \frac{c_2}{a_2}\right)t + \frac{\theta_1}{a_1} - \frac{\theta_2}{a_2}\right]. \tag{32}$$

Let $t \to -\infty, y \to -\infty$, e^{ξ_1} and $e^{\xi_1+\xi_2+\Delta_{12}}$ tend to infinity, and $t \to +\infty, y \to +\infty$, e^{ξ_2} and $e^{\xi_1+\xi_2+\Delta_{12}}$ tend to zero. In terms of the above case, we have

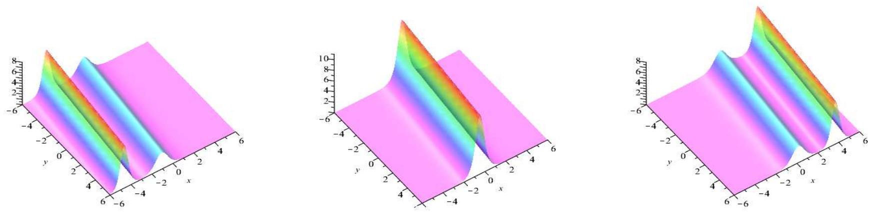
$$\begin{aligned} \lim_{t\to+\infty, y\to+\infty} u(x,y,t) &= \frac{a_2^2}{2}\operatorname{sech}^2\frac{a_2x + b_2y + c_2t + \theta_2 + \Delta_{12}}{2}, \\ \lim_{t\to-\infty, y\to-\infty} u(x,y,t) &= \frac{a_2^2}{2}\operatorname{sech}^2\frac{a_2x + b_2y + c_2t + \theta_2}{2}. \end{aligned} \tag{33}$$

By considering Eqs. (31) and (33), we realize that it is a an elastic interaction for solitons propagation. It doesn't make any difference between the previous period of interaction($t \to -\infty$) and the latter period of interaction($t \to +\infty$) in the amplitude, width and velocity of u, which can be corroborated in Figs. 8 and 9. As presented in Fig 8, by plotting the pictures of diagonal interaction (means that the propagation directions of two soliton are different), we know that there are not any change before and after each interaction besides the moving of phase. As for Fig. 9, by plotting the pictures of two parallel solitons (same propagation directions), it shows that tall-thin wave spreads faster than dwarf-fat wave and then catches up with it. When the interval between the two waves shrinks to zero ($t = 0$), their amplitudes are linearly superimposed and reach to a maximum point(see Fig 9 (b)). Then, as time goes on, they continue to move in their original state.



(**a**) (**b**) (**c**)

Figure 8. (Color online) Elastic interaction between the two soliton waves for (25) with parameters: $\alpha = 0.1, \beta = 0.1, \gamma = 0.5, a_1 = 2.5, b_1 = 1.2, a_2 = 3, b_2 = -0.28, \theta_1 = \theta_2 = 0$. **(a)** t=-8. **(b)** t=0. **(c)** t=8.



(**a**) (**b**) (**c**)

Figure 9. (Color online) Interaction between the two parallel soliton waves for (25) with parameters: $\alpha = 0.1, \beta = 0.1, \gamma = 0.5, a_1 = 2.5, b_1 = 0.2, a_2 = 4, b_2 = 0.32, \theta_1 = \theta_2 = 0$. **(a)** t=-8. **(b)** t=0. **(c)** t=8.

6. Conclusions and Discussions

In this work, we have studied the generalized (2+1)-dimensional CDGKS equation. By employing the homoclinic breather method, we have derived its breather waves. Then according to the the principle of Taylor expansion, we have got the rogue waves from the extremity of breather waves. Furthermore, based on Hirota bilinear method, the solitary waves of Eq. (1) have been provided. Additionally, in order to analysis the characteristic of the dynamical behavior for these solition solutions, we have drawn some figures, see Figs. 1-6. Finally, we have discussed the solitons interaction for Eq. (1) with Figs.7-9. The topic of the paper is of current interest and the results will enrich exact solutions to equations of mathematical physics. These effective methods that we show in this paper should play a important role in studying other issues in the filed of mathematical physics and engineering.

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