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Certain Properties of Generalized Einstein Spaces

Vladislava M. Stanković[a]

[a] *Department of Mathematics, Faculty of Sciences and Mathematics, University of Niš, Serbia*

Abstract. In the present paper are introduced generalized Einstein spaces. Einstein type tensors are represented in the generalized Einstein spaces. Some relations of Einstein type tensors of the first and the second kind in the generalized Riemannian space are obtained. Also, geodesic mappings of T-connected generalized Einstein spaces onto Riemannian space are considered.

1. Introduction

In 1922 Cartan was put forward a modification of General Relativity Theory (GRT), by relaxing the assumption that the affine connection has vanishing the antisymmetric part (torsion tensor), and relating the torsion to the density of intrinsic angular momentum. Also, the torsion is implicit in the 1928 Einstein theory of gravitation with teleparallelism. Afterwards, several mathematicians dealt with non-symmetric affine connection, for example, Eisenhart [6], [7], Prvanović [24], Minčić [14] -[20], Zlatanović [9], [21], [32] -[39].

Geodesic and almost geodesic lines, play an important role in geometry and physics. Sinyukov [25] introduced the concept of geodesic mappings between affine connected spaces without torsion. Mikeš [1], [8] -[13], [26], [31] gave some significant contributions to the study of geodesic and almost geodesic mappings of affine connected, Riemannian and Einstein spaces. Contribution to the theory of geodesic and almost geodesic mappings of spaces with non-symmetric affine connection and generalized Riemannian spaces gave Stanković [19], [20], [27] -[30], [35].

2. Notation and preliminaries

A generalized Riemannian space GR_N in the sense of Eisenhart's definition [5] is a differentiable N-dimensional manifold, equipped with a nonsymmetric basic tensor g_{ij}. Connection coefficients of this space are generalized Cristoffel's symbols of the second kind.

Generalized Cristoffel's symbols of the first kind of the space GR_N are given by the formula

$$\Gamma_{i.jk} = \frac{1}{2}(g_{ji,k} - g_{jk,i} + g_{ik,j}), \tag{1}$$

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Email address: vanja.dunja91@gmail.com (Vladislava M. Stanković)

where, for example, $g_{ij,k} = \frac{\partial g_{ij}}{\partial x^k}$. Connection coefficients of the space GR_N are the generalized Cristoffel's symbols of the second kind

$$\Gamma^i_{jk} = g^{\underline{ip}}\Gamma_{p.jk}, \tag{2}$$

where $\|g^{\underline{ij}}\| = \|g_{\underline{ij}}\|^{-1}$, and $g_{\underline{ij}} = \frac{1}{2}(g_{ij} + g_{ji})$. We suppose that $\det\|g_{ij}\| \neq 0$, $\det\|g_{\underline{ij}}\| \neq 0$. Generally, we have $\Gamma^i_{jk} \neq \Gamma^i_{kj}$. The symmetric and anti-symmetric part of Γ^i_{jk} are given by the formulas

$$\Gamma^i_{\underline{jk}} = \frac{1}{2}(\Gamma^i_{jk} + \Gamma^i_{kj}) = S^i_{jk}, \quad \Gamma^i_{\underset{\vee}{jk}} = \frac{1}{2}(\Gamma^i_{jk} - \Gamma^i_{kj}) = T^i_{jk} \tag{3}$$

The magnitude T^i_{jk} is *the torsion tensor* of the space GR_N. Obviously,

$$\Gamma^i_{jk} = S^i_{jk} + T^i_{jk}. \tag{4}$$

The use of non-symmetric basic tensor and non-symmetric connection became especially actual after appearance the works of A. Einstein [2]-[5] related to create the Unified Field Theory (UFT). Remark that at UFT the symmetric part $g_{\underline{ij}}$ of the basic tensor g_{ij} is related to the gravitation, and anisymmetric one $g_{\underset{\vee}{ij}}$ to the electromagnetism.

In a generalized Riemannian space one can define four kinds of covariant derivatives [14], [17]. In this paper, we consider only the first two kinds of covariant differentiation. For example, for a tensor a^i_j in GR_N we have

$$a^i_{j\underset{1}{|}m} = a^i_{j,m} + \Gamma^i_{pm}a^p_j - \Gamma^p_{jm}a^i_p, \qquad a^i_{j\underset{2}{|}m} = a^i_{j,m} + \Gamma^i_{mp}a^p_j - \Gamma^p_{mj}a^i_p, \tag{5}$$

where $\underset{\theta}{|}$ $(\theta = 1, 2)$ denotes a covariant derivative of the kind θ and $a^i_{j,m} = \frac{\partial a^i_j}{\partial x^m}$.

In the case of the space GR_N we have five independent curvature tensors [14]. In this paper we will consider only the first two curvature tensors:

$$\begin{aligned} \underset{1}{R}^i_{jmn} &= \Gamma^i_{jm,n} - \Gamma^i_{jn,m} + \Gamma^p_{jm}\Gamma^i_{pn} - \Gamma^p_{jn}\Gamma^i_{pm}, \\ \underset{2}{R}^i_{jmn} &= \Gamma^i_{mj,n} - \Gamma^i_{nj,m} + \Gamma^p_{mj}\Gamma^i_{np} - \Gamma^p_{nj}\Gamma^i_{mp}. \end{aligned} \tag{6}$$

Designating by semicolon (;) covariant derivative with respect to S^i_{jm}, we have [5]:

$$\begin{aligned} \underset{1}{R}^i_{jmn} &= R^i_{jmn} + T^i_{jm;n} - T^i_{jn;m} + T^p_{jm}T^i_{pn} - T^p_{jn}T^i_{pm}, \\ \underset{2}{R}^i_{jmn} &= R^i_{jmn} + T^i_{jn;m} - T^i_{jm;n} + T^p_{jm}T^i_{pn} - T^p_{jn}T^i_{pm}, \end{aligned} \tag{7}$$

where R^i_{jmn} is the curvature tensor with respect to the symmetric connection S^i_{jm}.

Contracting by indices i and n in curvature tensor of the first kind $\underset{1}{R}^i_{jmn}$ in the space GR_N we have the first type Ricci tensor

$$\underset{1}{R}_{jm} = \underset{1}{R}^p_{jmp} = R_{jm} + T^p_{jm;p} + T^p_{jq}T^q_{mp}, \tag{8}$$

where $R_{jm} = R^p_{jmp}$ is Ricci tensor with respect to the symmetric connection S^i_{jm}.

Now, contracting by indices i and n in curvature tensor of the second kind $\underset{2}{R}^i_{jmn}$ we get the second type Ricci tensor

$$\underset{2}{R}_{jm} = \underset{2}{R}^p_{jmp} = R_{jm} - T^p_{jm;p} + T^p_{jq}T^q_{mp}. \tag{9}$$

3. Geodesic mappings of generalized Riemannian spaces

One says that reciprocal one valued mapping $f : GR_N \to G\overline{R}_N$ is *geodesic* [19], [20], if geodesics of the generalized Riemannian space GR_N pass to geodesics of the generalized Riemannian space $G\overline{R}_N$. We can consider these spaces in the common by this mapping system of local coordinates. In the corresponding points $M(x)$ and $\overline{M}(x)$ we can put

$$\overline{\Gamma}^i_{jk}(x) = \Gamma^i_{jk}(x) + P^i_{jk}(x), \quad (i, j, k = 1, ..., N), \tag{10}$$

where $P^i_{jk}(x)$ is *the deformation tensor* of the connection Γ of GR_N according to the mapping $f : GR_N \to G\overline{R}_N$.

A necessary and sufficient condition that the mapping $f : GR_N \to G\overline{R}_N$ be geodesic (see [19], [20]) is that the deformation tensor P^i_{jk} in (10) at the mapping f has the form

$$P^i_{jk}(x) = \delta^i_j\, \psi_k(x) + \delta^i_k\, \psi_j(x) + \xi^i_{jk}(x), \tag{11}$$

where

$$\psi_i(x) = \frac{1}{N+1}(\overline{\Gamma}^\alpha_{i\alpha}(x) - \Gamma^\alpha_{i\alpha}(x)),\ \ \xi^i_{jk}(x) = P^i_{\underset{\vee}{jk}} = \frac{1}{2}(P^i_{jk} - P^i_{kj}). \tag{12}$$

4. Some relations of Einstein type tensors

4.1. Einstein type tensors of the first kind

Starting from the Bianchi type identity (see [18])

$$\underset{mnv}{Cicl}\, \underset{1}{R}{}^i_{jmn\underset{1}{|}v} = 2\, \underset{mnv}{Cicl}\, T^p_{mn} \underset{1}{R}{}^i_{jpv}, \tag{13}$$

i.e.

$$\underset{1}{R}{}^i_{jmn\underset{1}{|}v} + \underset{1}{R}{}^i_{jnv\underset{1}{|}m} + \underset{1}{R}{}^i_{jvm\underset{1}{|}n} = 2(T^p_{mn}\underset{1}{R}{}^i_{jpv} + T^p_{nv}\underset{1}{R}{}^i_{jpm} + T^p_{vm}\underset{1}{R}{}^i_{jpn}), \tag{14}$$

composing by $g_{\underline{ih}}$ in (14) and using property of antisymmetry

$$\underset{1}{R}_{ijmn} = -\underset{1}{R}_{jimn}, \quad \underset{1}{R}_{ijmn} = -\underset{1}{R}_{ijnm},$$

we get

$$\underset{1}{R}_{hjmn\underset{1}{|}v} + \underset{1}{R}_{hjnv\underset{1}{|}m} + \underset{1}{R}_{hjvm\underset{1}{|}n} = 2(T^p_{mn}\underset{1}{R}_{hjpv} + T^p_{nv}\underset{1}{R}_{hjpm} + T^p_{vm}\underset{1}{R}_{hjpn}). \tag{15}$$

Now, composing by $g^{\underline{hn}}g^{\underline{jm}}$ from (15) we have

$$(\underset{1}{R}{}^m_v - \frac{1}{2}\delta^m_v \underset{1}{R})_{\underset{1}{|}m} = \underset{1}{\overline{S}}_v - \frac{1}{2}\underset{1}{S}_v, \tag{16}$$

where we denote

$$\underset{1}{S}_v = g^{\underline{mj}}\underset{1}{S}_{mjv}, \quad \underset{1}{\overline{S}}_v = g^{\underline{mj}}\underset{1}{S}_{vjm} \text{ and } \underset{1}{S}_{mjv} = 2T^p_{mq}\underset{1}{R}{}^q_{jpv}. \tag{17}$$

Definition 4.1. *A tensor* $\underset{1}{E}{}^m_v = \underset{1}{R}{}^m_v - \frac{1}{2}\delta^m_v \underset{1}{R}$ *is called the Einstein tensor of the first kind.*

In this way, the following theorem is proven

Theorem 4.2. *The Einstein tensor of the first kind* $\underset{1}{E}{}^m_v$ *satisfied the relation*

$$\underset{1}{E}{}^m_{v|\underset{1}{m}} = \underset{1}{\overline{S}}_v - \frac{1}{2}\underset{1}{S}_v, \tag{18}$$

where vectors $\underset{1}{S}_v$ *and* $\underset{1}{\overline{S}}_v$ *are given by relations* (17).

Analogously, starting from identity (see [18])

$$\underset{mnv}{Cicl}\, \underset{1}{R}{}^i{}_{jmn|\underset{2}{v}} = 2\underset{mnv}{Cicl}\,(T^p_{jm}\underset{1}{R}{}^i{}_{pnv} + T^p_{mn}\underset{1}{R}{}^i{}_{jvp} + T^i_{mp}\underset{1}{R}{}^p{}_{jnv}), \tag{19}$$

we obtain that the following theorem is valid:

Theorem 4.3. *The Einstein tensor of the first kind* $\underset{1}{E}{}^m_v$ *satisfied the relation*

$$\underset{1}{E}{}^m_{v|\underset{2}{m}} = -\frac{1}{2}(\underset{1}{P}_v + \underset{1}{\overline{P}}_v + \underset{1}{\overline{S}}_v - \underset{1}{Q}_v), \tag{20}$$

where vector $\underset{1}{\overline{S}}_v$ *is given by relation* (17) *and*

$$\underset{1}{P}_v = 2g^{\underline{jm}}T^p_{jn}\underset{1}{R}{}^n{}_{pvm}, \quad \underset{1}{\overline{P}}_v = 2g^{\underline{jm}}T^p_{jv}\underset{1}{R}{}^n{}_{pmn}, \quad \underset{1}{Q}_v = 2g^{\underline{jm}}T^p_{jm}\underset{1}{R}{}^n{}_{pvn}.$$

4.2. Einstein type tensor of the second kind

Starting from the Bianchi type identity (see [18])

$$\underset{mnv}{Cicl}\, \underset{2}{R}{}^i{}_{jmn|\underset{1}{v}} = 2\underset{mnv}{Cicl}\,(T^p_{mj}\underset{2}{R}{}^i{}_{pnv} + T^p_{mn}\underset{2}{R}{}^i{}_{jpv} + T^i_{mp}\underset{2}{R}{}^p{}_{jvn}), \tag{21}$$

we obtain

$$(\underset{2}{R}{}^m_v - \frac{1}{2}\delta^m_v \underset{2}{R})_{|\underset{1}{m}} = \frac{1}{2}(\underset{2}{P}_v + \underset{2}{\overline{P}}_v + \underset{2}{\overline{S}}_v - \underset{2}{Q}_v), \tag{22}$$

where

$$\begin{aligned} &\underset{2}{S}_v = g^{\underline{mj}}\underset{2}{S}_{mjv}, \quad \underset{2}{\overline{S}}_v = g^{\underline{mj}}\underset{2}{S}_{vjm}, \quad \underset{2}{S}_{mjv} = 2T^p_{mq}\underset{2}{R}{}^q{}_{jpv}, \\ &\underset{2}{P}_v = 2g^{\underline{jm}}T^p_{jn}\underset{2}{R}{}^n{}_{pvm}, \quad \underset{2}{\overline{P}}_v = 2g^{\underline{jm}}T^p_{jv}\underset{2}{R}{}^n{}_{pmn}, \quad \underset{2}{Q}_v = 2g^{\underline{jm}}T^p_{jm}\underset{2}{R}{}^n{}_{pvn}. \end{aligned} \tag{23}$$

Definition 4.4. *A tensor* $\underset{2}{E}{}^m_v = \underset{2}{R}{}^m_v - \frac{1}{2}\delta^m_v \underset{2}{R}$ *is called the Einstein tensor of the second kind.*

Therefore, the following theorem is valid

Theorem 4.5. *The Einstein tensor of the second kind $\underset{2}{E}{}^m_v$ satisfied the relation*

$$\underset{2}{E}{}^m_{v|\underset{1}{m}} = \frac{1}{2}(\underset{2}{P}_v + \underset{2}{\overline{P}}_v + \underset{2}{\overline{S}}_v - \underset{2}{Q}_v), \tag{24}$$

where $\underset{2}{\overline{S}}_v$, $\underset{2}{P}_v$, $\underset{2}{\overline{P}}_v$ and $\underset{2}{Q}_v$ are given by relations (23)

Starting from the Bianchi type identity (see [18])

$$\underset{mnv}{Cicl}\, \underset{2}{R}{}^i_{jmn|\underset{2}{v}} = 2\,\underset{mnv}{Cicl}\, T^p_{mn} \underset{2}{R}{}^i_{jvp}, \tag{25}$$

i.e.

$$\underset{2}{R}{}^i_{jmn|\underset{2}{v}} + \underset{2}{R}{}^i_{jnv|\underset{2}{m}} + \underset{2}{R}{}^i_{jvm|\underset{2}{n}} = 2(T^p_{mn}\underset{2}{R}{}^i_{jvp} + T^p_{nv}\underset{2}{R}{}^i_{jmp} + T^p_{vm}\underset{2}{R}{}^i_{jnp}). \tag{26}$$

Composing by $g_{\underline{ih}}$ in (26) and using property of antisymmetry

$$\underset{2}{R}_{ijmn} = -\underset{2}{R}_{jimn}, \quad \underset{2}{R}_{ijmn} = -\underset{2}{R}_{ijnm},$$

we get

$$\underset{2}{R}_{hjmn|\underset{2}{v}} + \underset{2}{R}_{hjnv|\underset{2}{m}} + \underset{2}{R}_{hjvm|\underset{2}{n}} = 2(T^p_{mn}\underset{2}{R}_{hjvp} + T^p_{nv}\underset{2}{R}_{hjmp} + T^p_{vm}\underset{2}{R}_{hjnp}). \tag{27}$$

Now, composing by $g^{\underline{hn}}g^{\underline{jm}}$ from (27) we have

$$(\underset{2}{R}{}^m_v - \frac{1}{2}\delta^m_v \underset{2}{R})_{|\underset{2}{m}} = \frac{1}{2}\underset{2}{S}_v - \underset{2}{\overline{S}}_v, \tag{28}$$

where we denote

$$\underset{2}{S}_v = g^{\underline{mj}}\underset{2}{S}_{mjv}, \quad \underset{2}{\overline{S}}_v = g^{\underline{mj}}\underset{2}{S}_{vjm} \text{ and } \underset{2}{S}_{mjv} = 2T^p_{mq}\underset{2}{R}{}^q_{jpv}. \tag{29}$$

In this way, the following theorem is proven

Theorem 4.6. *The Einstein tensor of the second kind $\underset{2}{E}{}^m_v$ satisfied the relation*

$$\underset{2}{E}{}^m_{v|\underset{2}{m}} = \frac{1}{2}\underset{2}{S}_v - \underset{2}{\overline{S}}_v, \tag{30}$$

where vectors $\underset{2}{S}_v$ and $\underset{2}{\overline{S}}_v$ are given by relations (29).

5. Geodesic mappings of T-connected generalized Einstein spaces

Einstein space V_N is Riemanian space, with symmetric basic metric tensor g_{jm}, where Ricci tensor R_{jm} satisfies the relation

$$R_{jm} = K \cdot g_{jm}, \quad K = const. \tag{31}$$

After contraction of (31) we obtain $K = \dfrac{R}{N}$.

Einstein spaces provide simple, highly symmetric cosmological models. In general relativity the Einstein equations relate the curvature of space-time to the energy and momentum of all the mater present in following way

$$R_{jm} - \frac{1}{2}Rg_{jm} + \Lambda g_{jm} = GT_{jm}. \tag{32}$$

The first two summands on the left side of the relation (32) represent so-called Einstein tensor, Λ is the cosmological constant, T_{jm} is energy-momentum tensor and G is the gravitational constant.

In the case of generalized Riemannian spaces we can define *generalized Einstein spaces of the kind* θ in the following way:

Definition 5.1. *Generalized Einstein space of the kind* θ $(\theta = 1, 2)$ *is generalized Riemannian space if Ricci tensor of the kind* θ *satisfies the condition*

$$\underset{\theta}{R}_{jm} = \underset{\theta}{K} \cdot g_{jm} \quad (\theta = 1, 2), \tag{33}$$

where $\underset{\theta}{K}$ *are constants. Generalized Einstein space of the kind* θ $(\theta = 1, 2)$ *denoted by* $G\underset{\theta}{V}_N$.

From (8) for symmetric and antisymmetric part of Ricci tensor $\underset{1}{R}_{jm}$ we obtain

$$\underset{1}{R}_{\underline{jm}} = R_{jm} + T^p_{jq}T^q_{mp} \quad \text{and} \quad \underset{1}{R}_{\underset{\vee}{jm}} = T^p_{jm;p}. \tag{34}$$

In this way the following theorem is satisfied:

Theorem 5.2. *In generalized Einstein space* $G\underset{1}{V}_N$ *the next conditions are valid:*

$$R_{jm} + T^p_{jq}T^q_{mp} = \underset{1}{K}g_{\underline{jm}} \quad \textit{and} \quad T^p_{jm;p} = \underset{1}{K}g_{\underset{\vee}{jm}}, \tag{35}$$

where R_{jm} *is Ricci tensor with respect to the symmetric part* $g_{\underline{jm}}$ *of* g_{jm}, T^i_{jk} *is the torsion tensor,* $g_{\underset{\vee}{jm}}$ *is an antisymmetric part of* g_{jm} *and* $\underset{1}{K}$ *is a constant.*

Analogously, for generalized Einstein space of the second kind the next theorem is valid:

Theorem 5.3. *In generalized Einstein space* $G\underset{2}{V}_N$ *the next conditions are valid:*

$$R_{jm} + T^p_{jq}T^q_{mp} = \underset{2}{K}g_{\underline{jm}} \quad \textit{and} \quad T^p_{jm;p} = -\underset{2}{K}g_{\underset{\vee}{jm}}, \tag{36}$$

where R_{jm} *is Ricci tensor with respect to the symmetric part* $g_{\underline{jm}}$ *of* g_{jm}, T^i_{jk} *is a torsion tensor,* $g_{\underset{\vee}{jm}}$ *is an antisymmetric part of* g_{jm} *and* $\underset{2}{K}$ *is a constant.*

In the case of Riemanian space, when the basic metric tensor g_{ij} is a symmetric, the torsion tensor T^i_{jk} is a zero and the conditions (35) and (36) reduce to the condition $R_{jm} = Kg_{jm}$.

Definition 5.4. *Generalized Riemannian space* GR_n *is T-connected if the torsion tensor* T^i_{jm} *satisfies the condition:*

$$T^p_{jq}T^q_{mp} = \mu g_{\underline{jm}}, \quad \mu = const. \tag{37}$$

It is not difficult to conclude that the following assertion is true

Lemma 5.5. *If the generalized Einstein space* $\underset{\theta}{GV}_N$, $(\theta = 1, 2)$ *is T-connected, then:*

$$R_{jm} = \underset{\theta}{\mu} g_{\underline{jm}}, \tag{38}$$

where R_{jm} *is Ricci tensor with respect to* $g_{\underline{jm}}$ *and* $\underset{\theta}{\mu} = \underset{\theta}{K} - \mu$ *are constants.*

Therefore the following assertion holds:

Theorem 5.6. *For generalized Einstein tensors* $\underset{\nu}{E}{}^m_v$ *of the kind* ν $(\nu = 1, 2)$ *in generalized Einstein spaces* $\underset{\theta}{GV}_N$ *of the type* θ $(\theta = 1, 2)$ *the next relations are valid:*

$$\underset{\nu}{E}{}^m_v = \left(\underset{\theta}{K} - \frac{1}{2}\right)\delta^m_v + \underset{\theta}{K} g^{\underline{pm}} \underset{\vee}{g_{pv}}, \quad (\theta = 1, 2;\ \nu = 1, 2), \tag{39}$$

where $\underset{\theta}{R}$ *are scalar curvatures.*

The Einstein spaces consists a closed class in relation to geodesic mappings, i.e. satisfied the following theorem (see [10], [13]):

Theorem 5.7. *If Einstein space* V_N *permits nontrivial geodesic mapping onto* $\overline{V}_N$, *then* $\overline{V}_N$ *is an Einstein space.*

In general case associate space, with the symmetric basic metric tensor $g_{\underline{jm}}$, at the generalized Einstein space of the kind θ $(\theta = 1, 2)$, is not an Einstein space. According to the Lemma 5.5 we conclude that the associate space, at the T-connected generalized Einstein space of the kind θ $(\theta = 1, 2)$, is an Einstein space.

In this way we have proved the following theorem:

Theorem 5.8. *If T-connected generalized Einstein space* $\underset{\theta}{GV}_N$ *of the kind* θ $(\theta = 1, 2)$ *permits nontrivial geodesic mapping onto Riemanian space* $\overline{V}_N$, *then* $\overline{V}_N$ *is an Einstein space.*

References

[1] V.E. Berezovski, J. Mikeš, A. Vanžurová, Fundamental PDE's of the Canonical Almost Geodesic Mappings of Type $\tilde{\pi}_1$, Bull. Malays. Math. Sci. Soc. (2), Vol. 37, No.3, (2014) 647–659.
[2] A. Einstein, Bianchi identities in the generalized theory of gravitation, Canad. J. Math., **2**, (1950) 120–128.
[3] A. Einstein, Die Grundlagen der allgemeinen Relativitäts-teorie, Annale der Physic, **49**, (1916) 769.
[4] A. Einstein, Relativistic theory of the non-symmetic field, Appendix II in the book: The meaning of relativity 5^{th} edit., Princeton, **49**, 1955.
[5] A. Einstein, Generalization of the relativistic theory of gravitation, Ann. of math., Princeton, **46**, (1945) 576–584.
[6] L.P. Eisenhart, Non-Riemannian geometry, New York, 1927.
[7] L.P. Eisenhart, Generalized Riemannian spaces I, Proceeding of the National Academy of Sciences of the USA, 37, (1951) 311–315.
[8] I. Hinterleitner, J. Mikeš, Geodesic mappings and Einstein Spaces, Geometric Methods in Physics, Trends in Mathematics (2013) 331–335.
[9] S. Ivanov, M. Lj. Zlatanović, Connections on non-symmetric (generalized) Riemannian manifold and gravity, Classical and Quantum Gravity, Vol. 33, No. 7, 075016 (2016).
[10] J. Mikeš, Geodesic mappings of Einstein spaces, Math. Notes 28, (1981), 922-923. Transl. from Mat. Zametki 28, (1980) 935–938.
[11] J. Mikeš, Geodesic mappings of affine-connected and Riemannian spaces, J. Math. Sci., New York 78, 3 (1996) 311–333.
[12] J. Mikeš, O. Pokorná, G. Starko, On almost geodesic mappings $\pi_2(e)$ onto Riemannian spaces, Rendiconti del circolo matematico di Palermo, Serie II, Suppl.72 (2004) 151–157.
[13] J. Mikeš, A. Vanžurová, I. Hinterleitner, Geodesic Mappings and Some Generalizations, Olomouc, Univ. Print 2009.
[14] S.M. Minčić, Independent curvature tensors and pseudotensors of spaces with non-symmetric affine connexion, Colloquia Mathematica Societatis János Bolayai, 31, Differential Geometry, Budapest (Hungary), (1979) 445–460.
[15] S.M. Minčić, New commutation formulas in the non-symmetric affine connexion space, Publ. Inst. Math. (Beograd) (N.S.), 22(36), (1977) 189–199.
[16] S.M. Minčić, On Ricci type identities in manifolds with non-symmetric affine connection, Publ. Inst. Math. (Beograd) (N.S.), 94(108), (2013) 205–217.
[17] S.M. Minčić, Ricci identities in the space of non-symmetric affine connexion, Mat. Vesnik, 10(25), Sv. 2, (1973) 161–172.

[18] S.M. Minčić, Bianchi type identities in the space of nonsymmetric affine connection, Collection of scientific papers of the Faculty of Science, Kragujevac, Vol. 16, (1994) 53–60.
[19] S.M. Minčić, M.S. Stanković, On geodesic mappings of general affine connection spaces and of generalized Riemannian spaces, Mat. Vesnik 49 (1997) 27–33.
[20] S.M. Minčić, M.S. Stanković, Equitorsion geodesic mappings of generalized Riemannian spaces, Publ. Inst. Math. (N. S) 61 (75) (1997) 97–104.
[21] M. S. Najdanović, M. Lj. Zlatanović, I. Hinterleitner, Conformal and geodesic mappings of generalized equidistant spaces, Publications de l'institut mathematique, 98(112) (2015) 71-84.
[22] A.Z. Petrov, Einstein spaces, M., GIFML, 1961.
[23] A.Z. Petrov, New methods in the general theory of relativity, M., Nauka, 1966.
[24] M. Prvanović, Four curvature tensors of non-symmetric affine connexion, Proc. Conf. "150 years of Lobachevski geometry", Kazan 1976, Moscow (1997) 199–205 (in Russian).
[25] N.S. Sinyukov, Geodesic mappings of Riemannian spaces, "Nauka" Moscow, 1979 (in Russian).
[26] V.S. Sobchuk, J. Mikeš, O. Pokorná, On almost geodesic mappings π_2 between semisymmetric Riemannian spaces, Novi Sad J. Math., Vol. 29, No. 3, (1999) 309–312.
[27] M.S. Stanković, On a canonic almost geodesic mappings of the second type of affine spaces, FILOMAT 13, (1999) 105–114.
[28] M.S. Stanković, Special equitorsion almost geodesic mappings of the third type of non-symmetric affine connection spaces, Applied Mathematics and Computation, 244 (2014) 695–701.
[29] M.S. Stanković, N.O. Vesić, Some relations in non-symmetric affine connection spaces with regard to a special almost geodesic mappings of the third type, Filomat, 29:9 (2015) 1941–1951.
[30] M.S. Stanković, M.Lj. Zlatanović, N.O. Vesić, Basic equations of G-almost geodesic mappings of the second type, which have the property of reciprocity, Czechoslovak Mathematical Journal 65:3, (2015) 787–799.
[31] H. Vavříková, J. Mikeš, O. Pokorná, G. Starko, On fundamental equations of almost geodesic mappings of type $\pi_2(e)$, Russ. Math., Vol. 51, No. 1, (2007), 8–12. Transl. from Iz. VUZ Matematika, No. 1, (2007) 10–15.
[32] M. Lj. Zlatanovic, New projective tensors for equitorsion geodesic mappings, Applied Mathematics Letters, Vol. 25, No. 5 (2012) 890–897.
[33] M. Lj. Zlatanović, I. Hinterleitner, M. S. Najdanović, Geodesic mapping onto Kahlerian space of the first kind, Czech. Math. Journal, Vol. 64, No. 4, (2014) 1113–1122.
[34] M. Lj. Zlatanović, I. Hinterleitner, M.S. Najdanović, On equitorsion concircular tensors of generalized Riemannian spaces, Filomat, Vol. 28, No. 3, (2014) 463–471.
[35] M.Lj. Zlatanović, Lj.S. Velimirović, M.S. Stanković, Necessary and sufficient conditions for equitorsion geodesic mapping, Journal of Mathematical Analysis and Applications, Vol. 435 (2016) 578–592.
[36] M.Lj. Zlatanović, S.M. Minčić, V.M. Stanković, New integrability conditions of derivation equations in a subspace of asymmetric affine connection space, Miskolc Mathematical Notes, Vol. 18, No. 1, (2017) 525–536.
[37] M. Lj. Zlatanović, S. M. Minčić, Lj. S. Velimirović, On integrability conditions of derivation equations in a subspace of asymmetric affine connection space, Filomat 29:10 (2015) 2421-2427.
[38] M. Lj. Zlatanović, V. M. Stanković, Geodesic mapping onto Kahlerian space of the third kind, Journal of Mathematical Analysis and Applications, Vol. 450, issue 1 (2017) 480–489.
[39] M. Lj. Zlatanović, V. M. Stanković, Some invariants of holomorphically projective mappings of generalized Khlerian spaces, Journal of Mathematical Analysis and Applications, Vol. 458, Issue 1 (2018) 601–610.