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Separation Axioms in Supra Soft Bitopological Spaces

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Abstract. In 1999, Russian researcher Molodtsov proposed the new concept of a soft set which can be considered as a new mathematical approach for vagueness. Topological structures of soft set have been studied by some authors in recent years. In this paper we define separation axioms in supra soft bitopological space using only soft point in [2] and investigate some of their important characterizations.

1. Introduction

The theory of generalized topological spaces (briefly *GT*), introduced by Á. Császár [4], is one of the most important developments of general topology in recent years. Á. Császár defined some basic operators on generalized topological spaces and studied some simplest separation axioms in [5]. G.Xun and G.Ying [20] gave some characterizations of separation axioms in generalized topological space. Later, W. K. Min and Y. K. Kim [13] introduced the notion of bigeneralized topological spaces and quasi generalized open sets and studied some basic properties for the sets. There are many definitions for separation axioms in bigeneralized topological spaces. P. Torton et al. [19] studied some separation axioms in bigeneralized topological spaces and defined the notions of regular, normal in these spaces.

In recent years the soft set theory, initiated D. Molodtsov [14], is one of the branches of mathematics, which aims to describe phenomena and concepts of an ambiguous, vague, undefined and imprecise meaning. Since the soft set theory has a rich potential, researches on soft set theory and its applications in various fields are progressing rapidly.

Topological structures of soft set have been studied by some authors. M. Shabir and M. Naz [17] have initiated the study of soft topological spaces which are defined over an initial universe with a fixed set of parameters and showed that a soft topological space gives a parameterized family of topological spaces. Theoretical studies of soft topological spaces have also been researched by some authors in [2, 6, 7, 12, 15, 18, 21] etc. S. Bayramov and C. Gunduz Aras [3] gave separation axioms and compactness in soft topological spaces. As a generalized of soft topological spaces, S.A.El-Sheikh and A.M.Abd-El-Latif [8] introduced the notion of supra soft topological spaces by neglecting only the soft intersection condition. After the concept of bitopological spaces was introduced by J.C. Kelly [10] as an extension of topological spaces in 1963, B.M. Ittanagi [9] defined the notion of soft bitopological space. A.F. Sayed [16] studied separation axioms in fuzzy soft bitopological spaces. A study of soft bitopological spaces is a generalization of the study of soft

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topological spaces as every soft bitopological space (X, τ_1, τ_2, E) can be regarded as a soft topological space (X, τ, E) if $\tau_1 = \tau_2 = \tau$.

In this paper we define separation axioms in supra soft bitopological space using only soft point in [2] and investigate some of their important characterizations.

We now state certain useful definitions and several existing results that we require in the next section.

2. Preliminaries

In this section we will introduce necessary definitions and theorems for soft sets. Throughout this paper X denotes initial universe, E denotes the set of all parameters, $P(X)$ denotes the power set of X.

Definition 2.1. ([14]) A pair (F, E) is called a soft set over X, where F is a mapping given by $F : E \to P(X)$.

In other words, the soft set is a parameterized family of subsets of the set X. For $e \in E$, $F(e)$ may be considered as the set of $e-$elements of the soft set (F, E), or as the set of $e-$approximate elements of the soft set.

After this, $SS(X)_E$ denotes the family of all soft sets over X with a fixed set of parameters E.

Definition 2.2. ([1]) For two soft sets (F, E) and (G, E) over X, (F, E) is called a soft subset of (G, E) if $\forall e \in E$, $F(e) \subseteq G(e)$. This relationship is denoted by $(F, E)\widetilde{\subseteq}(G, E)$.

Similarly, (F, E) is called a soft superset of (G, E) if (G, E) is a soft subset of (F, E). This relationship is denoted by $(F, E)\widetilde{\supseteq}(G, E)$. Two soft sets (F, E) and (G, E) over X are called soft equal if (F, E) is a soft subset of (G, E) and (G, E) is a soft subset of (F, E).

Definition 2.3. ([1]) The intersection of two soft sets (F, E) and (G, E) over X is the soft set (H, E), where $\forall e \in E$, $H(e) = F(e) \cap G(e)$. This is denoted by $(F, E)\widetilde{\cap}(G, E) = (H, E)$.

Definition 2.4. ([1]) The union of two soft sets (F, E) and (G, E) over X is the soft set (H, E), where $\forall e \in E$, $H(e) = F(e) \cup G(e)$. This is denoted by $(F, E)\widetilde{\cup}(G, E) = (H, E)$.

Definition 2.5. ([11]) A soft set (F, E) over X is said to be a null soft set denoted by Φ if for all $e \in E$, $F(e) = \emptyset$.

Definition 2.6. ([11]) A soft set (F, E) over X is said to be an absolute soft set denoted by $\widetilde{X}$ if for all $e \in E$, $F(e) = X$.

Definition 2.7. ([17]) The difference (H, E) of two soft sets (F, E) and (G, E) over X, denoted by $(F, E)\backslash(G, E)$, is defined as $H(e) = F(e)\backslash G(e)$ for all $e \in E$.

Definition 2.8. ([17]) The complement of a soft set (F, E), denoted by $(F, E)^c$, is defined $(F, E)^c = (F^c, E)$, where $F^c : E \to P(X)$ is a mapping given by $F^c(e) = X\backslash F(e)$, $\forall e \in E$ and F^c is called the soft complement function of F.

Definition 2.9. ([17]) Let τ be the collection of soft sets over X, then $\widetilde{\tau}$ is said to be a soft topology on X if

1) $\Phi, \widetilde{X}$ belongs to $\widetilde{\tau}$;
2) the union of any number of soft sets in $\widetilde{\tau}$ belongs to $\widetilde{\tau}$;
3) the intersection of any two soft sets in $\widetilde{\tau}$ belongs to $\widetilde{\tau}$.

The triplet $(X, \widetilde{\tau}, E)$ is called a soft topological space over X.

Definition 2.10. ([17]) Let $(X, \widetilde{\tau}, E)$ be a soft topological space over X, then members of $\widetilde{\tau}$ are said to be a soft open sets in X.

Definition 2.11. ([17]) Let $(X, \widetilde{\tau}, E)$ be a soft topological space over X. A soft set (F, E) over X is said to be a soft closed set in X, if its complement $(F, E)^c$ belongs to $\widetilde{\tau}$.

Proposition 2.12. ([17]) *Let $(X, \widetilde{\tau}, E)$ be a soft topological space over X. Then the collection $\widetilde{\tau}_e = \{F(e) : (F, E) \in \widetilde{\tau}\}$ for each $e \in E$, defines a topology on X.*

Definition 2.13. ([17]) Let $(X, \widetilde{\tau}, E)$ be a soft topological space over X and (F, E) be a soft set over X. Then the soft closure of (F, E), denoted by $cl_{\widetilde{\tau}}(F, E)$ is the intersection of all soft closed super sets of (F, E). Clearly $cl_{\widetilde{\tau}}(F, E)$ is the smallest soft closed set over X which contains (F, E).

Definition 2.14. ([2]) Let (F, E) be a soft set over X. The soft set (F, E) is called a soft point, denoted by (x_e, E), if for the element $e \in E$, $F(e) = \{x\}$ and $F(e') = \emptyset$ for all $e' \in E - \{e\}$ (briefly denoted by x_e).

It is obvious that each soft set can be expressed as a union of soft points. For this reason, to give the family of all soft sets on X it is sufficient to give only soft points on X.

Definition 2.15. ([2]) Two soft points x_e and $y_{e'}$ over a common universe X, we say that the soft points are different if $x \neq y$ or $e \neq e'$.

Definition 2.16. ([2]) The soft point x_e is said to be belonging to the soft set (F, E), denoted by $x_e \widetilde{\in} (F, E)$, if $x_e(e) \in F(e)$, i.e., $\{x\} \subseteq F(e)$.

Definition 2.17. ([2]) Let $(X, \widetilde{\tau}, E)$ be a soft topological space over X. A soft set $(F, E) \widetilde{\subseteq} (X, E)$ is called a soft neighborhood of the soft point $x_e \widetilde{\in} (F, E)$ if there exists a soft open set (G, E) such that $x_e \widetilde{\in} (G, E) \widetilde{\subset} (F, E)$.

3. Separation Axioms in Supra Soft Bitopological Spaces

In this section, we introduce different separation axioms on a supra soft bitopological space and establish their interrelations. Let X be a nonempty set E be a set of parameters and τ_1, τ_2 be two supra soft topologies on X. Then (X, τ_1, τ_2, E) is said to be a supra soft bitopological space (briefly *SSBTS*).

Let (X, τ_1, τ_2, E) be a supra soft bitopological space and (F, E) be a soft set. The closure of (F, E) with respect to τ_m are denoted by $cl_{\tau_m}(F, E)$, for $m = 1, 2$.

Definition 3.1. a) Let (X, τ_1, τ_2, E) be a supra soft bitopological space over X. (X, τ_1, τ_2, E) is said to be a soft $\tau_{(m,n)}$-T_0 space if for any soft points $x_e, y_{e'}$ with $x_e \neq y_{e'}$, there exist soft open sets $(F, E), (G, E), (F_1, E), (G_1, E)$ such that

$$x_e \in (F, E) \in \tau_1, y_{e'} \notin (F, E) \text{ and } x_e \in (G, E) \in \tau_2, y_{e'} \notin (G, E)$$

or

$$y_{e'} \in (F_1, E) \in \tau_1, x_e \notin (F_1, E) \text{ and } y_{e'} \in (G_1, E) \in \tau_2, x_e \notin (G_1, E), \text{ for } m, n = 1, 2.$$

b) (X, τ_1, τ_2, E) is said to be a soft $\tau_{(m,n)} - T_1$ space if for any soft points $x_e, y_{e'}$ with $x_e \neq y_{e'}$, there exist soft open sets $(F, E) \in \tau_m, (G, E) \in \tau_n$ such that

$$x_e \in (F, E) \in \tau_m, y_{e'} \notin (F, E) \text{ and } y_{e'} \in (G, E) \in \tau_n, x_e \notin (G, E), \text{for } m, n = 1, 2.$$

Theorem 3.2. *Let (X, τ_1, τ_2, E) be a SSBTS over X. Then (X, τ_1, τ_2, E) is a soft $\tau_{(m,n)} - T_1$ space if and only if the soft point $\{x_e\}$ is a τ_m- supra soft closed set and a τ_n- supra soft closed set, for all $x_e \in (X, \tau_1, \tau_2, E)$.*

Proof. Assume that (X, τ_1, τ_2, E) is a soft $\tau_{(m,n)} - T_1$ space and $x_e \in (X, \tau_1, \tau_2, E)$. For each $y_{e'} \in \{x_e\}^c$, $x_e \neq y_{e'}$. By assumption, there exists soft open sets $\left(F_{y_{e'}}, E\right), \left(G_{y_{e'}}, E\right)$ such that $y_{e'} \in \left(F_{y_{e'}}, E\right) \in \tau_m, x_e \notin \left(F_{y_{e'}}, E\right)$ and $x_e \notin \left(G_{y_{e'}}, E\right) \in \tau_n, y_{e'} \in \left(G_{y_{e'}}, E\right)$. Then the soft sets

$\{x_e\}^c = \underset{y_{e'} \neq x_e}{\cup} \left(F_{y_{e'}}, E\right)$, $\{x_e\}^c = \underset{y_{e'} \neq x_e}{\cup} \left(G_{y_{e'}}, E\right)$ are supra soft open sets, respectively τ_m and τ_n. Thus the soft point $\{x_e\}$ is a τ_m- supra soft closed set and a τ_n- supra soft closed set.

Conversely, assume that $\{x_e\}$ is a τ_m- supra soft closed set and τ_n- supra soft closed set, for all $x_e \in (X, \tau_1, \tau_2, E)$. Let $\{x_e\}, \{y_{e'}\} \in (X, \tau_1, \tau_2, E)$ with $x_e \neq y_{e'}$. By assumption, we obtain that $\{x_e\}$ is a τ_n- supra soft closed set, $\{y_{e'}\}$ is a τ_m- supra soft closed set. We set $(F, E) = \widetilde{X} \setminus \{y_{e'}\}$ and $(G, E) = \widetilde{X} \setminus \{x_e\}$. Then (F, E) is a τ_m- supra soft open set and (G, E) is a τ_n- supra soft open set. Therefore $x_e \in (F, E), y_{e'} \notin (F, E)$ and $y_{e'} \in (G, E), x_e \notin (G, E)$ are obtained. Then (X, τ_1, τ_2, E) is a soft $\tau_{(m,n)} - T_1$ space. □

Proposition 3.3. *Let (X, τ_1, τ_2, E) be a SSBTS over X.*

a) If (X, τ_1, τ_2, E) is a soft $\tau_{(m,n)} - T_0$ space, then $(X, \tau_{1_e}, \tau_{2_e})$ is a $\tau_{(m,n)} - T_0$ space, for each $e \in E$.
b) If (X, τ_1, τ_2, E) is a soft $\tau_{(m,n)} - T_1$ space, then $(X, \tau_{1_e}, \tau_{2_e})$ is a $\tau_{(m,n)} - T_1$ space, for each $e \in E$.

Definition 3.4. Let (X, τ_1, τ_2, E) be a *SSBTS* over X. (X, τ_1, τ_2, E) is said to be a soft $\tau_{(m,n)} - T_2$ space if for any soft points $x_e, y_{e'}$ with $x_e \neq y_{e'}$, there exist soft open sets $(F, E) \in \tau_m, (G, E) \in \tau_n$ such that

$$x_e \in (F, E), y_{e'} \in (G, E) \text{ and } (F, E) \cap (G, E) = \Phi.$$

Proposition 3.5. *Let (X, τ_1, τ_2, E) be a SSBTS over X. If (X, τ_1, τ_2, E) is a soft $\tau_{(m,n)} - T_2$ space, then $(X, \tau_{1_e}, \tau_{2_e})$ is a $\tau_{(m,n)} - T_2$ space, for each $e \in E$.*

Remark 3.6. a) Every soft $\tau_{(m,n)} - T_1$ space is a soft $\tau_{(m,n)} - T_0$ space.

b) Every soft $\tau_{(m,n)} - T_2$ space is a soft $\tau_{(m,n)} - T_1$ space.

Theorem 3.7. *Let (X, τ_1, τ_2, E) be a soft $\tau_{(m,n)} - T_1$ space and $x_e \in (F, E) \in \tau_m$, for every soft point x_e. If there exists a supra soft open set $(G, E) \in \tau_m$ such that $x_e \in (G, E) \subset cl_{\tau_n}(G, E) \subset (F, E)$, then (X, τ_1, τ_2, E) is a soft $\tau_{(m,n)} - T_2$ space.*

Proof. Suppose that $x_e \neq y_{e'}$. Since (X, τ_1, τ_2, E) is a soft $\tau_{(m,n)} - T_1$ space, $\{x_e\}, \{y_{e'}\}$ are τ_m-supra soft closed sets and τ_n-supra soft closed sets. Thus $x_e \in \{y_{e'}\}^c$ and $\{y_{e'}\}^c$ is a τ_m- soft open set and τ_n- soft open set. Then there exists a τ_m- soft open set (G, E) such that

$$x_e \in (G, E) \subset cl_{\tau_n}(G, E) \subset \{y_{e'}\}^c.$$

Hence we have $\{y_{e'}\} \in (cl_{\tau_n}(G, E))^c \in \tau_n$, $\{x_e\} \in (G, E) \in \tau_m$ and $(G, E) \cap (cl_{\tau_n}(G, E))^c = \Phi$, i.e., (X, τ_1, τ_2, E) is a soft $\tau_{(m,n)} - T_2$ space. □

Definition 3.8. Let (X, τ_1, τ_2, E) be a *SSBTS* over X. Then (X, τ_1, τ_2, E) is said to be a soft $\tau_{(m,n)}-$regular space if for any soft point x_e and for any τ_m- soft closed set (F, E) with $x_e \notin (F, E)$, there exist $(G, E) \in \tau_m$ and $(H, E) \in \tau_n$ such that $x_e \in (G, E), (F, E) \subset (H, E)$ and $(G, E) \cap (H, E) = \Phi$. Also (X, τ_1, τ_2, E) is said to be a soft $\tau_{(m,n)} - T_3$ space if (X, τ_1, τ_2, E) is a soft $\tau_{(m,n)} - T_1$ space and soft $\tau_{(m,n)}-$regular space.

Proposition 3.9. *Let (X, τ_1, τ_2, E) be a SSBTS over X. If (X, τ_1, τ_2, E) is a soft $\tau_{(m,n)} - T_3$ space, then $(X, \tau_{1_e}, \tau_{2_e})$ is a $\tau_{(m,n)} - T_3$ space, for each $e \in E$.*

Theorem 3.10. *Let (X, τ_1, τ_2, E) be a SSBTS over X. Then the following are equivalent:*

1. *(X, τ_1, τ_2, E) is a soft $\tau_{(m,n)}-$regular space,*
2. *For any soft point $x_e \in (X, \tau_1, \tau_2, E)$ and for any τ_m- soft closed set (F, E) with $x_e \notin (F, E)$, there exist $(G, E) \in \tau_m$ and $(H, E) \in \tau_n$ such that $x_e \in (G, E), (F, E) \subset (H, E)$ and $cl_{\tau_n}(G, E) \cap (H, E) = \Phi$,*
3. *If $x_e \in (X, \tau_1, \tau_2, E)$ and (F, E) is a τ_m- soft closed set with $x_e \notin (F, E)$, then there is a τ_m- soft open set (G, E) containing x_e such that $cl_{\tau_n}(G, E) \cap (F, E) = \Phi$,*
4. *If $x_e \in (X, \tau_1, \tau_2, E)$ and $(G, E) \in \tau_m$ with $x_e \in (G, E)$, then there is a τ_m- soft open set (H, E) containing x_e such that $x_e \in (H, E) \subset cl_{\tau_n}(H, E) \subset (G, E)$.*

Proof. 1) ⇒ 2) Let $x_e \in (X, \tau_1, \tau_2, E)$ and (F, E) be a τ_m- soft closed set such that $x_e \notin (F, E)$. Since (X, τ_1, τ_2, E) is a soft $\tau_{(m,n)}-$regular, there exist $(G, E) \in \tau_m$ and $(H, E) \in \tau_n$ such that $x_e \in (G, E), (F, E) \subset (H, E)$ and $(G, E) \cap (H, E) = \Phi$.

Suppose that $cl_{\tau_n}(G, E) \cap (H, E) \neq \Phi$. Then the soft point $y_e \in cl_{\tau_n}(G, E) \cap (H, E)$ is a soft tangency point of (G, E) in τ_n- supra soft topology. Since $y_e \in (H, E) \in \tau_n$, $(G, E) \cap (H, E) \neq \Phi$ is obtained. This is a contradiction.

2) ⇒ 3) It is clear.

3) ⇒ 4) Assume that $x_e \in (X, \tau_1, \tau_2, E)$ and $(G, E) \in \tau_m$ with $x_e \in (G, E)$. Then $(G, E)^c$ is a τ_m- soft closed set and $x_e \notin (G, E)^c$. By 3), there exists a τ_m- soft open set (H, E) containing x_e such that $cl_{\tau_n}(H, E) \cap (G, E)^c = \Phi$. Hence $x_e \in (H, E) \subset cl_{\tau_n}(H, E) \subset (G, E)$ is obtained.

4) ⇒ 1) Let $x_e \in (X, \tau_1, \tau_2, E)$ and (F, E) be a τ_m- soft closed set with $x_e \notin (F, E)$. Then $x_e \in (F, E)^c$ and $(F, E)^c \in \tau_m$. By 4) there exists $(H, E) \in \tau_n$ such that $x_e \in (H, E) \subset cl_{\tau_n}(H, E) \subset (F, E)^c$.

Moreover, $x_e \in (H, E), (F, E) \subset (cl_{\tau_n}(H, E))^c \in \tau_n$ and $(H, E) \cap (cl_{\tau_n}(H, E))^c = \Phi$ is satisfied. Thus (X, τ_1, τ_2, E) is a soft $\tau_{(m,n)}-$regular space. □

Definition 3.11. Let (X, τ_1, τ_2, E) be a *SSBTS* over X. Then (X, τ_1, τ_2, E) is said to be a soft $\tau_{(m,n)}-$normal space if for any τ_m- soft closed set (F_1, E) and for any τ_n- soft closed set (F_2, E) with $(F_1, E) \cap (F_2, E) = \Phi$, there exist $(G_1, E) \in \tau_m$ and $(G_2, E) \in \tau_n$ such that $(F_1, E) \subset (G_2, E), (F_2, E) \subset (G_1, E)$ and $(G_1, E) \cap (G_2, E) = \Phi$.

If (X, τ_1, τ_2, E) is a soft $\tau_{(m,n)}-$normal space and soft $\tau_{(m,n)} - T_1$ space, then (X, τ_1, τ_2, E) is said to be a soft $\tau_{(m,n)} - T_4$ space.

Proposition 3.12. *Let (X, τ_1, τ_2, E) be a SSBTS over X. If (X, τ_1, τ_2, E) is a soft $\tau_{(m,n)} - T_4$ space, then (X, τ_1, τ_2, E) is a soft $\tau_{(m,n)} - T_3$ space.*

Proof. Assume that (X, τ_1, τ_2, E) is a soft $\tau_{(m,n)} - T_4$ space. We show that (X, τ_1, τ_2, E) is a soft $\tau_{(m,n)} - T_3$ space. Let $x_e \in (X, \tau_1, \tau_2, E)$ be a soft point and (F, E) be a τ_m- soft closed set such that $x_e \notin (F, E)$. Since (X, τ_1, τ_2, E) is a soft $\tau_{(m,n)} - T_1$ space, $\{x_e\}$ is a τ_n- soft closed set. Since $\{x_e\} \cap (F, E) = \Phi$ and (X, τ_1, τ_2, E) is a soft $\tau_{(m,n)}-$ normal space, there exist a τ_m- soft open set (G, E) and τ_n- soft open set (H, E) such that $x_e \in (G, E), (F, E) \subset (H, E)$ and $(G, E) \cap (H, E) = \Phi$.

Hence (X, τ_1, τ_2, E) is a soft $\tau_{(m,n)} - T_3$ space. □

Theorem 3.13. *Let (X, τ_1, τ_2, E) be a SSBTS over X. Then the following conditions are equivalent:*

1) (X, τ_1, τ_2, E) is a soft $\tau_{(m,n)}-$ normal space,

2) If (F_1, E) is a τ_m- soft closed set and (F_2, E) is a τ_n- soft closed set such that $(F_1, E) \cap (F_2, E) = \Phi$, then there are a τ_m- soft open set (G_1, E) and τ_n- soft open set (G_2, E) such that $(F_1, E) \subset (G_2, E), (F_2, E) \subset (G_1, E)$ and $cl_{\tau_n}(G_1, E) \cap (G_2, E) = \Phi$ is satisfied.

3) If (F_1, E) is a τ_m-soft closed set and (F_2, E) is a τ_n-soft closed set such that $(F_1, E) \cap (F_2, E) = \Phi$, then there exists a τ_m- soft open set (G, E) such that $(F_2, E) \subset (G, E)$ and $cl_{\tau_n}(G, E) \cap (F_1, E) = \Phi$.

4) If (F, E) is a τ_m- soft closed set (G, E) is a τ_n- soft open set such that $(F, E) \subset (G, E)$, then there exists a τ_n- soft open set (H, E) such that $(F, E) \subset (H, E) \subset cl_{\tau_m}(H, E) \subset (G, E)$.

Proof. 1) ⇒ 2) Let (X, τ_1, τ_2, E) be a soft $\tau_{(m,n)}-$ normal space, (F_1, E) be a τ_m- soft closed set, (F_2, E) be a τ_n-soft closed set and $(F_1, E) \cap (F_2, E) = \Phi$. Since (X, τ_1, τ_2, E) be a soft $\tau_{(m,n)}-$ normal space, there exist $(G_1, E) \in \tau_m$ and $(G_2, E) \in \tau_n$ such that $(F_1, E) \subset (G_2, E), (F_2, E) \subset (G_1, E)$ and $(G_1, E) \cap (G_2, E) = \Phi$. Assume that $cl_{\tau_n}(G_1, E) \cap (G_2, E) \neq \Phi$, then soft point $y_e \in cl_{\tau_n}(G_1, E) \cap (G_2, E)$. The soft point y_e is a soft tangency point of (G_1, E) in supra soft topology τ_n. Then $y_e \in (G_2, E) \in \tau_n$, this implies that $(G_1, E) \cap (G_2, E) \neq \Phi$. This is a contradiction. Hence $cl_{\tau_n}(G_1, E) \cap (G_2, E) = \Phi$ is obtained.

2) ⇒ 3) It is clear.

3) ⇒ 4) Assume that (F, E) is a τ_m- soft closed set and (G, E) is a τ_n-soft open set and $(F, E) \subset (G, E)$. Then $(G, E)^c$ is a τ_n-soft closed set and $(F, E) \cap (G, E)^c = \Phi$. By 3), there exists a τ_m-soft open set (H, E) such that $(G, E)^c \subset (H, E)$ and $cl_{\tau_n}(H, E) \cap (F, E) = \Phi$. Hence $(F, E) \subset (cl_{\tau_n}(H, E))^c \subset (H, E)^c \subset (G, E)$. Let $(P, E) = (cl_{\tau_n}(H, E))^c$. Hence (P, E) is a τ_n- soft open set and $(F, E) \subset (P, E) \subset cl_{\tau_m}(P, E) \subset (G, E)$.

4) ⇒ 1) Let (F_1, E) be a τ_m- soft closed set and (F_2, E) be a τ_n-soft closed set such that $(F_1, E) \cap (F_2, E) = \Phi$. Then $(F_2, E)^c$ is a τ_n-soft open set and $(F_1, E) \subset (F_2, E)^c$. By 4), there exists a τ_n- soft open set (H, E) such that $(F_1, E) \subset (H, E) \subset cl_{\tau_m}(H, E) \subset (F_2, E)^c$. Then $(F_1, E) \subset (H, E) \in \tau_n$, $(F_2, E) \subset (cl_{\tau_m}(H, E))^c \in \tau_m$ and $(H, E) \cap (cl_{\tau_m}(H, E))^c = \Phi$. Hence (X, τ_1, τ_2, E) is a soft $\tau_{(m,n)}-$ normal space. □

Definition 3.14. Let (X, τ, E) be a supra soft topological space, $\tau = \{(F, E)\}$ and $Y \subset X$. Then the family $\tau_Y = \left\{\widetilde{Y} \cap (F, E)\right\}$ constitute a soft supra topology on Y. (Y, τ_Y, E) is said to be a supra soft subtopological space of (X, τ, E) .

Definition 3.15. Let (X, τ_1, τ_2, E) be a supra soft bitopological space and $Y \subset X$. Then $(Y, \tau_{1_Y}, \tau_{2_Y}, E)$ is called a supra soft bitopological subspace of (X, τ_1, τ_2, E).

Lemma 3.16. *Let (X, τ, E) be a supra soft topological space and (Y, τ_Y, E) be a supra soft subspace. Then $(H, E) \subset (Y, \tau_Y, E)$ is a supra soft closed set if and only if there exists a supra soft closed set $(F, E) \subset (X, \tau, E)$ such that $(H, E) = \widetilde{Y} \cap (F, E)$.*

Proof. Assume that let $(H, E) \subset (Y, \tau_Y, E)$ be a supra soft closed set. Then $(H, E)^c = \widetilde{Y} \backslash (H, E)$ is a supra soft open set. Thus there exists a supra soft open set (G, E) such that $(H, E)^c = \widetilde{Y} \cap (G, E)$. This implies that $(H, E) = \widetilde{Y} \cap (G, E)^c$ and $(G, E)^c$ is a supra soft closed set.

Conversely, let $(H, E) = \widetilde{Y} \cap (F, E)$ and (F, E) be a supra soft closed set. Then $(H, E)^c = \widetilde{Y} \cap (F, E)^c$ is a supra soft open set. Thus (H, E) is a supra soft closed set in (Y, τ_Y, E). □

Theorem 3.17. *Let (X, τ_1, τ_2, E) be a SSBTS over X. If (X, τ_1, τ_2, E) is a soft $\tau_{(m,n)} - T_i$ space, then $(Y, \tau_{1_Y}, \tau_{2_Y}, E)$ is a soft $\tau_{(m,n)} - T_i$ space, for $i = 0, 1, 2, 3$.*

Proof. Let $x_e, y_{e'} \in (Y, \tau_{1_Y}, \tau_{2_Y}, E)$ such that $x_e \neq y_{e'}$.Thus there exists $(F, E) \in \tau_m$, $(G, E) \in \tau_n$ which satisfying conditions of T_i space, for $i = 0, 1, 2$. Let $x_e \in (F, E)$, $y_{e'} \in (G, E)$. Then $x_e \in (F, E) \cap \widetilde{Y}$, $y_{e'} \in (G, E) \cap \widetilde{Y}$. Also the supra soft open sets $(F, E) \cap \widetilde{Y}$, $(G, E) \cap \widetilde{Y}$ in $(Y, \tau_{1_Y}, \tau_{2_Y}, E)$ satisfying conditions of soft $\tau_{(m,n)} - T_i$ space, for $i = 0, 1, 2$.

If $i = 3$, the proof is done similarly. □

Theorem 3.18. *Let (X, τ_1, τ_2, E) be a SSBTS over X. If (X, τ_1, τ_2, E) is a soft $\tau_{(m,n)} - T_4$ space and $\widetilde{Y}$ be a τ_m- soft closed set, τ_n- soft closed set, then $(Y, \tau_{1_Y}, \tau_{2_Y}, E)$ is a soft $\tau_{(m,n)} - T_4$ space.*

Proof. Let (X, τ_1, τ_2, E) be a soft $\tau_{(m,n)} - T_4$ space, $\widetilde{Y}$ be a τ_m- soft closed set and τ_n- soft closed set. Let (F_1, E) be a τ_m- soft closed set and (F_2, E) be a τ_n- soft closed set over $\widetilde{Y}$ such that $(F_1, E) \cap (F_2, E) = \Phi$. Since $\widetilde{Y}$ is a τ_m- soft closed set, (F_1, E) is a τ_m- soft closed set in $\widetilde{X}$. Similiarly, since $\widetilde{Y}$ is a τ_n- soft closed set, (F_2, E) is a τ_n- soft closed set in $\widetilde{X}$. Since (X, τ_1, τ_2, E) is a soft $\tau_{(m,n)} - T_4$ space, there exist soft open sets $(G_1, E) \in \tau_m$, $(G_2, E) \in \tau_n$ such that $(F_1, E) \subset (G_2, E)$, $(F_2, E) \subset (G_1, E)$ and $(G_1, E) \cap (G_2, E) = \Phi$. Then $(F_1, E) \subset (G_2, E) \cap \widetilde{Y} \in \tau_{n_Y}$, $(F_2, E) \subset (G_1, E) \cap \widetilde{Y} \in \tau_{m_Y}$ and $\left((G_2, E) \cap \widetilde{Y}\right) \cap \left((G_1, E) \cap \widetilde{Y}\right) = \Phi$ is satisfied. This completes the proof. □

Definition 3.19. Let (X, τ, E) be a supra soft topological space and $(F, E) \in SS(X)_E$. Then (F, E) is called $g\tau-$soft closed set if $cl_\tau(F, E) \subset (G, E)$ whenever $(F, E) \subset (G, E)$ and $(G, E) \in \tau$. (F, E) is called $g\tau-$soft open set if $(F, E)^c$ is $g\tau-$ soft closed set.

$cl^*_\tau(F, E)$ is the intersection of all $g\tau-$soft closed sets containing (F, E). $i^*_\tau(F, E)$ denotes the union of all $g\tau-$soft open sets contained in (F, E).

Lemma 3.20. *Let (X, τ, E) be a supra soft topological space and $(F, E) \in SS(X)_E$.Then*

1) $x_e \in cl^*_\tau(F, E)$ if and only if $(G, E) \cap (F, E) \neq \Phi$ for every $g\tau-$soft open set (G, E) containing x_e.
2) $x_e \in i^*_\tau(F, E)$ if and only if there exists a $g\tau-$soft open set (G, E) containing x_e such that $(G, E) \subset (F, E)$.

Proof. 1) Assume that there exists a $g\tau-$soft open set (G, E) containing x_e such that $(G, E) \cap (F, E) = \Phi$. Then $(G, E)^c$ is a $g\tau-$soft closed set contained (F, E) and $x_e \notin (G, E)^c$. Hence $x_e \notin cl^*_\tau(F, E)$.

Conversely, suppose that $x_e \notin cl^*_\tau(F, E)$. Then there exists a $g\tau-$soft open set (G, E) contained (F, E) such that $x_e \in (G, E)$. Set $(H, E) = (G, E)^c$ .Then (H, E) is a $g\tau-$soft open set containing x_e and $(H, E) \cap (F, E) = \Phi$.

2) It is clear. □

Definition 3.21. a) Let (X, τ_1, τ_2, E) be a $SSBTS$ over X. Then (X, τ_1, τ_2, E) is said to a soft $g\tau_{(m,n)}-$regular space if for any soft point x_e and for any τ_m- soft closed set (F, E) with $x_e \notin (F, E)$, there exists a $g\tau_m-$ soft open set (G_1, E) ,a $g\tau_n-$ soft open set (G_2, E) such that $x_e \in (G_1, E)$, $(F, E) \subset (G_2, E)$ and $(G_1, E) \cap (G_2, E) = \Phi$.

b) (X, τ_1, τ_2, E) is said to a soft $g\tau_{(m,n)}-$normal space if for any τ_m- soft closed set (F_1, E) and for any τ_n- soft closed set (F_2, E) with $(F_1, E) \cap (F_2, E) = \Phi$, there exists a $g\tau_m-$ soft open set (G_1, E) ,a $g\tau_n-$ soft open set (G_2, E) such that $(F_1, E) \subset (G_2, E)$, $(F_2, E) \subset (G_1, E)$and $(G_1, E) \cap (G_2, E) = \Phi$.

Theorem 3.22. *Let (X, τ_1, τ_2, E) be a SSBTS over X. Then the following are equivalent:*

1) (X, τ_1, τ_2, E) is a soft $g\tau_{(m,n)}-$regular space,

2) For any soft point $x_e \in (X, \tau_1, \tau_2, E)$ and for any τ_m- soft closed set (F, E) with $x_e \in (F, E)$,there is a $g\tau_m-$ soft open set (G_1, E) and a $g\tau_n-$ soft open set (G_2, E) such that $x_e \in (G_1, E)$, $(F, E) \subset (G_2, E)$ and $cl^*_{\tau_n}(G_1, E) \cap (G_2, E) = \Phi$.

Proof. 1) $\Rightarrow$ 2) Let $x_e \in (X, \tau_1, \tau_2, E)$ and (F, E) be a τ_m- soft closed set such that $x_e \notin (F, E)$. Since (X, τ_1, τ_2, E) is a soft $g\tau_{(m,n)}-$regular space, there exist a $g\tau_m-$ soft open set (G_1, E) and a $g\tau_n-$ soft open set (G_2, E) such that $x_e \in (G_1, E)$, $(F, E) \subset (G_2, E)$ and $(G_1, E) \cap (G_2, E) = \Phi$. Suppose that $cl^*_{\tau_n}(G_1, E) \cap (G_2, E) \neq \Phi$, then $y_e \in cl^*_{\tau_n}(G_1, E) \cap (G_2, E)$. Hence $y_e \in cl^*_{\tau_n}(G_1, E)$ and $y_e \in (G_2, E)$. Since $(G_2, E) \in \tau_n$, $(G_1, E) \cap (G_2, E) \neq \Phi$, which is contradiction. Hence $cl^*_{\tau_n}(G_1, E) \cap (G_2, E) = \Phi$.

2) $\Rightarrow$ 1) It is obvious. □

Theorem 3.23. *Let (X, τ_1, τ_2, E) be a SSBTS over X. (X, τ_1, τ_2, E) is a soft $g\tau_{(m,n)}-$normal space if and only if for any τ_m- soft closed set (F_1, E) and for any τ_n- soft closed set (F_2, E) with $(F_1, E) \cap (F_2, E) = \Phi$, then there are a $g\tau_m-$ soft open set (G_1, E) and a $g\tau_n-$ soft open set (G_2, E) such that $(F_1, E) \subset (G_2, E)$, $(F_2, E) \subset (G_1, E)$ and $cl^*_{\tau_n}(G_1, E) \cap (G_2, E) = \Phi$.*

Proof. Suppose that (X, τ_1, τ_2, E) is a soft $g\tau_{(m,n)}-$normal space and (F_1, E) is a τ_m- soft closed set, (F_2, E) is a τ_n- soft closed set with $(F_1, E) \cap (F_2, E) = \Phi$. Then there exist $(G_1, E) \in g\tau_m-$ soft open set, $(G_2, E) \in g\tau_n-$ soft open setsuch that $(F_1, E) \subset (G_2, E)$, $(F_2, E) \subset (G_1, E)$ and $(G_1, E) \cap (G_2, E) = \Phi$. If $y_e \in cl^*_{\tau_n}(G_1, E) \cap (G_2, E)$, then $y_e \in (G_2, E)$. Since $y_e \in (G_2, E)$ is a $g\tau_n-$ soft open set, $(G_1, E) \cap (G_2, E) \neq \Phi$, which is contradiction. Hence $cl^*_{\tau_n}(G_1, E) \cap (G_2, E) = \Phi$.

Conversely, it is clear. □

4. Conclusion

We have introduced separation axioms in supra soft bitopological spaces which are defined over an initial universe with a fixed set of parameters and also some important properties of these axioms are investigated. Moreover, we think that the results of this paper can be generalized to pseudo-BCI algebras given in [21].

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