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A New Aspect of Rectifying Curves and Ruled Surfaces in Galilean 3−Space

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Abstract. A curve is named as rectifying curve if its position vector always lies in its rectifying plane. There are lots of papers about rectifying curves in Euclidean and Minkowski spaces. In this paper, we give some relations between extended rectifying curves and their modified Darboux vector fields in Galilean 3−Space. The other aim of the paper is to introduce the ruled surfaces whose base curve is rectifying curve. Further, we prove that the parameter curve of the surface is a geodesic.

1. Introduction

B. Y. Chen introduced the notion of a rectifying curve in Euclidean 3−space as a space curve whose position vector always lies in its rectifying plane $Span(T, B)$ where T and B are the tangent and the binormal vector fields of the curve. Therefore, the position vector $\gamma(s)$ of the curve satisfies the equation

$$\gamma(s) = \lambda(s) T(s) + \mu(s) B(s)$$

for some differentiable functions λ and μ in arclength function s. To characterize rectifying curve, the harmonic curvature function of a *twisted curve* is congruent to the rectifying curve plays an important role. One of the most interesting characteristics of rectifying curves was given in [4] is that the harmonic curvature function is a non constant linear function of the arclength parameter s. Chen and Dillen [5] shown that there exists a simple relationship between the rectifying curves and the centrodes, which play some important roles in geometry, mechanics and kinematics. Some authors have extended the theory of the curves in three dimensional space forms [2, 10, 11, 14] and generalized rectifying curves in high dimensional spaces [3, 12].

Galilean space is the limit case of a pseudo-Euclidean space in which the isotropic cone degenerates to a plane. It is defined as a Klein geometry of the product space which plays an important part in physics. The main distinction of Galilean geometry is its relative simplicity, for it enables the student to study it in relative detail without losing a great deal of time and intellectual energy. Put differently, the simplicity of Galilean geometry makes its extensive development an easy matter, and extensive development of a new geometric system is a precondition for an effective comparison of it with Euclidean geometry. Also, extensive development is likely to give the student the psychological assurance of the consistency of

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the investigated structure. Another distinction of Galilean geometry is the fact that it exemplifies the fruitful geometric idea of duality. These reasons make me think that one should give serious thought to a mathematics program for teachers' colleges which would include a comparative study of three simple geometries, namely, Euclidean geometry, the geometry associated with the Galilean principle of relativity, and the geometry associated with Einstein's principle of relativity as well as an introduction to the special theory of relativity[20].

In recent years, Erjavec and Divjak have introduced some interesting properties of curves and surfaces in Galilean 3−space, such as curves, helices [6, 8] and some special curves on ruled surfaces [7]. Rectifying curve from various viewpoints has been studied in three dimensional Galilean space [17, 18]. Then Lone [13] defined rectifying curves in Galilean 4−space. In particular, he proved that there are no rectifying curves in Galilean 4−space with non-zero constant curvatures. After these studies, Uzunoğlu et.al [19] characterized some curves with the help of their harmonic curvature functions. Then, they investigated the relation between rectifying curves and Salkowski (or anti-Salkowski) curves in Galilean 3−space with the help of their harmonic curvature functions. Furthermore, the position vectors of them are obtained via the serial approach of the curves with polynomial coefficients.

The main object of this paper is to obtain some relationships between extended rectifying curves and their modified Darboux vector fields in in Galilean 3−Space. The other goal is to introduce the ruled surfaces whose base curve is rectifying curve. Finally, we prove that the parameter curve of the surface is a geodesic.

2. Basic concepts and notions

For 3-dimensional Galilean space $\mathbb{G}_3$, the Galilean scalar product between two vectors $\xi = (\xi_1, \xi_2, \xi_3)$ and $\zeta = (\zeta_1, \zeta_2, \zeta_3)$ is defined by

$$\langle \xi, \zeta \rangle_{G_3} = \begin{cases} \xi_1\zeta_1, & \text{if } \xi_1 \text{ or } \zeta_1 \text{ is not zero,} \\ \xi_2\zeta_2 + \xi_3\zeta_3, & \text{if } \xi_1 \text{ and } \zeta_1 \text{ are zero} \end{cases} \tag{1}$$

and the Galilean cross product is given as

$$(\xi \times \zeta)_{G_3} = \begin{cases} \begin{vmatrix} 0 & e_2 & e_3 \\ \xi_1 & \xi_2 & \xi_3 \\ \zeta_1 & \zeta_2 & \zeta_3 \end{vmatrix}, & \text{if } \xi_1 \text{ or } \zeta_1 \text{ is not zero,} \\ \\ \begin{vmatrix} e_1 & e_2 & e_3 \\ \xi_1 & \xi_2 & \xi_3 \\ \zeta_1 & \zeta_2 & \zeta_3 \end{vmatrix}, & \text{if } \xi_1 \text{ and } \zeta_1 \text{ are zero.} \end{cases} \tag{2}$$

where e_1, e_2, e_3 are Euclidean standard basis [1]. Let $\gamma : I \subset \mathbb{R} \to \mathbb{G}_3$, $\gamma(s) = (x(s), y(s), z(s))$ be an arbitrary curve with the Galilean invariant parameter s. If $x(s)$ is considered as the arc length parameter of the curve, we get the curve as $\gamma(s) = (s, y(s), z(s))$. Then the curvature $\kappa(s)$ and torsion $\tau(s)$ of the curve γ are defined by

$$\kappa(s) = \sqrt{(y'')^2(s) + (z'')^2(s)}$$

$$\tau(s) = \frac{\det(\gamma'(s), \gamma''(s), \gamma'''(s))}{(\kappa(s))^2}$$

and associated moving trihedron is given by

$$T(s) = \gamma'(s) = (1, y'(s), z'(s))$$

$$N(s) = \frac{1}{\kappa(s)}\gamma''(s) = \frac{1}{\kappa(s)}(0, y''(s), z''(s))$$

$$B(s) = \frac{1}{\kappa(s)}(0, -z''(s), y''(s)).$$

The vectors T, N, B are called the vectors of the tangent, principal normal and binormal line of γ, respectively. For their derivatives the following Frenet formulas are hold

$$\begin{bmatrix} T' \\ N' \\ B' \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ 0 & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix} \tag{3}$$

[1]

Definition 2.1. *Let $\gamma : I \subset \mathbb{R} \to \mathbb{G}^3$ be a unit speed curve with the Frenet frame apparatus $\{T, N, B, \kappa \neq 0, \tau\}$ in Galilean 3−Space. The harmonic curvature $\mathbf{H} : I \subset \mathbb{R} \to \mathbb{R}$ of the curve γ is defined by*

$$\mathbf{H} = \frac{\tau}{\kappa} \tag{4}$$

where κ and τ are curvature and torsion of the curve γ, respectively.

Definition 2.2. *Let $\gamma : I \subset \mathbb{R} \to \mathbb{G}^3$ be a unit speed curve with the Frenet frame apparatus $\{T, N, B, \kappa \neq 0, \tau\}$ in Galilean 3−Space. Then the curve γ is named as general helix if its harmonic curvature function is a constant function.[16]*

If parameter change is made as $\phi = \int \kappa(s)ds$, $d\phi = \kappa(s)ds$ and $\frac{d\phi}{ds} = \kappa(s)$, then the Frenet formulas are

$$\frac{d}{d\phi}\begin{bmatrix} T \\ N \\ B \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \mathbf{H} \\ 0 & -\mathbf{H} & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix} \tag{5}$$

where $\mathbf{H}$ is the harmonic curvature function of the curve. Izumiya and Takeuchi defined the modified Darboux vector field $\overline{D} = \mathbf{H}(s)T(s) + B(s)$, $\kappa \neq 0$ and another modified Darboux vector field is defined as $\widetilde{D} = T(s) + \frac{1}{\mathbf{H}(s)}B(s)$, $\tau(s) \neq 0$. [9]

3. Ruled Surfaces With The Base Rectifying Curves in Galilean 3-Space

In this part, we present the relation between rectifying curves and ruled surfaces according to the Frenet frame apparatus in three dimensional Galilean space. Furthermore, we obtain a characterization of the parameter curves of the ruled surfaces with the base rectifying curves are geodesics.

Theorem 3.1. *Let $\gamma : I \subset \mathbb{R} \to \mathbb{G}^3$ be a unit speed curve with the Frenet frame apparatus $\{T, N, B, \kappa, \tau, \mathbf{H}' \neq -\frac{1}{u}\}$ in Galilean 3−Space. Then γ is a rectifying curve if and only if the s−parameter curve $\overline{\beta}(s) = \gamma(s) + u\overline{D}(s)$ of the ruled surface $\overline{\phi}(s, u) = \gamma(s) + u\overline{D}(s)$ is a rectifying curve where $\overline{D}$ is the modified Darboux vector field of γ .*

Proof. Consider $\gamma(s)$ be a unit speed rectifying curve associated with the Frenet frame apparatus $\{T, N, B, \kappa, \tau\}$ in three dimensional Galilean space. Since the parameter u is constant the s−parameter curve of the surface is given as $\overline{\beta}(s) = \gamma(s) + u\overline{D}(s)$ with its harmonic curvature function $\mathbf{H}_{\overline{\beta}}$. If we take the derivative of the curve $\overline{\beta}$ by arc-length parameter s and use the Eq.(3), then we have

$$T_{\overline{\beta}}\frac{d\overline{s}}{ds} = (1 + u\mathbf{H}')\,T$$

An easy computation gives us

$$\bar{s} = s + u\mathbf{H} + c, c \in \mathbb{R}. \tag{6}$$

Thus, we have

$$T_{\overline{\beta}} = T. \tag{7}$$

Differentiation of the last equation gives us

$$\begin{aligned} \kappa_{\overline{\beta}} N_{\overline{\beta}} \frac{d\bar{s}}{ds} &= \kappa N, \\ \kappa_{\overline{\beta}} N_{\overline{\beta}} (1 + u\mathbf{H}') &= \kappa N. \end{aligned}$$

From this, we can easily obtain that

$$\kappa_{\overline{\beta}} = \frac{\kappa}{1 + u\mathbf{H}'} \tag{8}$$

and

$$N_{\overline{\beta}} = N. \tag{9}$$

Similarly, if we take the derivative of the Eq.(9) and use the Eq.(3), we get

$$\tau_{\overline{\beta}} = \frac{\tau}{1 + u\mathbf{H}'} \tag{10}$$

Since the curve γ is a rectifying curve the Eqs.(8) and (10) give us

$$\mathbf{H}_{\overline{\beta}} = \mathbf{H} = as + b; a \neq 0, b \in \mathbb{R}. \tag{11}$$

Then if we consider the Eq.(6) in the Eq.(11), firstly it is easy to obtain that

$$s = \frac{\bar{s} - (ub + c)}{1 + ua}$$

and then

$$\mathbf{H}_{\overline{\beta}} = m\bar{s} + n$$

where $m = \frac{a}{1+ua}$ and $n = \frac{-ac+b}{1+ua}$ are real constants. Consequently, the $s-$parameter curve $\overline{\beta}$ of the ruled surface $\overline{\phi}(s,u)$ is a rectifying curve.

Conversely, let $\overline{\beta}(s) = \int T(s)ds + u(\mathbf{H}T + B)$ be a rectifying curve. Then $\mathbf{H}_{\overline{\beta}}$ is a non-constant linear function ,that is, $\mathbf{H}_{\overline{\beta}} = m\bar{s} + n$; $m, n \in \mathbb{R}$. From the Eqs.(6) and (11), we obtain $\mathbf{H}$ is a linear function of the parameter s. Thus, γ is a rectifying curve. The proof is completed. □

Theorem 3.2. *Let $\gamma : I \subset \mathbb{R} \to \mathbb{G}^3$ be a unit speed curve with the Frenet frame apparatus $\{T, N, B, \kappa, \tau, \mathbf{H}' \neq -\frac{1}{u}\}$ in Galilean $3-$Space. If γ is a rectifying curve then the $s-$parameter curve $\overline{\beta}(s)$ is a geodesic curve of the ruled surface $\overline{\phi}(s,u)$ where $\overline{D}$ is the modified Darboux vector field of γ .*

Proof. Let $\gamma(s)$ be a unit speed rectifying curve in three dimensional Galilean space. Then $\mathbf{H} = \frac{\tau}{\kappa} = as + b$, where b and $a \neq -\frac{1}{u}$ are constants. If we take the partial derivatives of the ruled surface $\overline{\phi}(s,u) = \gamma(s) + u\overline{D}(s)$ with respect to s and u we have

$$\overline{\phi}_s = (1 + u\mathbf{H}')T \;\; , \;\; \overline{\phi}_u = \mathbf{H}T + B.$$

Then the unit normal vector $\mathcal{N}_{\overline{\phi}}$ of the surface and the $s-$parameter curve $\overline{\beta}(s)$ are obtained as follows

$$\mathcal{N}_{\overline{\phi}} = -N$$

and

$$\overline{\beta}(s) = \gamma(s) + u\left(\mathbf{H}T + B\right)(s),$$

respectively. If we take the derivative of both sides twice with respect to s and use the fact that $\mathbf{H}$ is a linear function, then we get

$$\frac{d^2\overline{\beta}}{d\overline{s}^2} = -\kappa(1+ua)\mathcal{N}_{\overline{\phi}},$$

Consequently, we prove that the $s-$parameter curve $\overline{\beta}(s)$ is a geodesic curve of the ruled surface $\overline{\phi}(s,u)$. □

Theorem 3.3. *Let $\gamma : I \subset \mathbb{R} \to \mathbb{G}^3$ be a unit speed curve with the Frenet frame apparatus $\{T, N, B, \kappa, \tau, \mathbf{H}' \neq 0\}$ and $\overline{D}(s)$ be the modified Darboux vector field of γ in Galilean $3-$Space. Then the curve $\overline{D}$ is a rectifying curve .*

Proof. Let $\gamma(s)$ be a unit speed curve and

$$\overline{D}(s) = \mathbf{H(s)}T(s) + B(s) \tag{12}$$

be the modified Darboux vector field of the curve γ in three dimensional Galilean space.
If we take the derivative of the Eq.(12) with respect to s, we have

$$T_{\overline{D}}\frac{d\overline{s}}{ds} = \mathbf{H}'T$$

Then we can easily see that

$$\frac{d\overline{s}}{ds} = \mathbf{H}' \tag{13}$$

or

$$\overline{s} = \mathbf{H} + k, k \in \mathbb{R}.$$

Then the Eq.(13) gives us

$$T_{\overline{D}} = T$$

Again, if we differentiate the last equation and use the Eq.(3), we get

$$\kappa_{\overline{D}} = \frac{\kappa}{\mathbf{H}'} \tag{14}$$

and

$$N_{\overline{D}} = N.$$

Then, we obtain the torsion of the curve $\overline{D}$ with the help of the differentiation of this equation and using the Eq.(3) as follows

$$\tau_{\overline{D}} = \frac{\tau}{\mathbf{H}'} \tag{15}$$

Since the Eq.(14) in the Eq.(15) gives us

$$\frac{\tau_{\overline{D}}}{\kappa_{\overline{D}}} = \mathbf{H} = \overline{s} - k, k \in \mathbb{R}$$

the curve $\overline{D}$ is a rectifying curve. □

Theorem 3.4. *Let $\gamma : I \subset \mathbb{R} \to \mathbb{G}^3$ be a unit speed curve with the Frenet frame apparatus $\{T, N, B, \kappa, \tau, (\frac{1}{\mathbf{H}})' \neq -\frac{1}{u}\}$ and $\zeta(s)$ be B direction curve of γ in Galilean 3−space. If γ is a rectifying curve then the s−parameter curve $\widetilde{\beta}(s) = \zeta(s) + u\widetilde{D}(s)$ of the ruled surface $\widetilde{\phi}(s,u) = \zeta(s) + u\widetilde{D}(s)$ is a circular helix where $\widetilde{D}$ is the modified Darboux vector field of γ.*

Proof. Let $\zeta(s) = \int B(s)ds$ be B direction curve of γ in Galilean 3−space. Since the parameter u is constant the curve is given by the form $\widetilde{\beta}(s) = \int B(s)ds + u\widetilde{D}(s)$. If we take the derivative of $\widetilde{\beta}$ with respect to parameter s then we have

$$T_{\widetilde{\beta}}\frac{d\widetilde{s}}{ds} = B + u(\tfrac{1}{\mathbf{H}})'B$$

If we take the norms of both sides

$$\widetilde{s} = s + u\tfrac{1}{\mathbf{H}} + c$$

where $c \in \mathbb{R}$. And thus we get

$$T_{\widetilde{\beta}} = B$$

Again if we differentiate the last equation according to parameter s, we obtain

$$\begin{aligned} \kappa_{\widetilde{\beta}}N_{\widetilde{\beta}}\frac{d\widetilde{s}}{ds} &= -\tau N, \\ \kappa_{\widetilde{\beta}}N_{\widetilde{\beta}}(1 + u(\tfrac{1}{\mathbf{H}})') &= -\tau N. \end{aligned} \tag{16}$$

The norm of both sides of the above equality gives us

$$\kappa_{\widetilde{\beta}} = \frac{\tau}{1 + u(\frac{1}{\mathbf{H}})'} \tag{17}$$

and considering the Eq.(17) in the Eq.(16), we get

$$N_{\widetilde{\beta}} = -N \tag{18}$$

Similarly if we take the derivative of the Eq.(18) and use the Eq.(3), we have

$$\tau_{\widetilde{\beta}}B_{\widetilde{\beta}}(1 + u(\tfrac{1}{\mathbf{H}})') = -\tau B$$

and the norms of both sides gives us

$$\tau_{\widetilde{\beta}} = \frac{\tau}{1 + u(\frac{\kappa}{\tau})'} \tag{19}$$

Finally, if we use the Eq.(17) in the Eq.(19) then we obtain

$$\mathbf{H}_{\widetilde{\beta}} = 1$$

Hence the s−parameter curve $\widetilde{\beta}(s)$ of the surface $\widetilde{\phi}(s,u)$ is a circular helix. Which completes the proof. □

Theorem 3.5. *Let $\gamma : I \subset \mathbb{R} \to \mathbb{G}^3$ be a unit speed curve with the Frenet frame apparatus $\{T, N, B, \kappa, \tau, (\frac{1}{\mathbf{H}})' \neq -\frac{1}{u}\}$ and $\zeta(s)$ be B direction curve of γ in Galilean 3−space. If γ is a rectifying curve then the s−parameter curve $\widetilde{\beta}(s)$ is a geodesic curve of the ruled surface $\widetilde{\phi}(s,u)$ where $\widetilde{D}$ is the modified Darboux vector field.*

Proof. The proof is similar to Theorem 3.2. □

Theorem 3.6. *Let $\gamma : I \subset \mathbb{R} \to \mathbb{G}^3$ be a unit speed curve with the Frenet frame apparatus $\{T, N, B, \kappa, \tau, \mathbf{H} \neq const.\}$ and $\widetilde{D}(s)$ be the modified Darboux vector field of γ. Then the curve $\widetilde{D}$ is a circular helix.*

Proof. Let $\gamma(s)$ be a unit speed curve and

$$\widetilde{D}(s) = T(s) + \frac{1}{\mathbf{H(s)}} B(s) \tag{20}$$

be the modified Darboux vector field of the curve γ in three dimensional Galilean space.
If we take the derivative of the Eq.(20) with respect to s, we have

$$T_{\widetilde{D}} \frac{d\widetilde{s}}{ds} = (\tfrac{1}{\mathbf{H}})' B$$

Then we can easily see that

$$\frac{d\widetilde{s}}{ds} = (\tfrac{1}{\mathbf{H}})' \tag{21}$$

or

$$\widetilde{s} = \frac{1}{\mathbf{H}} + k, k \in \mathbb{R}.$$

Then the Eq.(21) gives us

$$T_{\widetilde{D}} = B$$

Again, if we differentiate the last equation and use the Eq.(3), we get

$$\kappa_{\widetilde{D}} = \frac{\tau}{(\frac{1}{\mathbf{H}})'} \tag{22}$$

and

$$N_{\widetilde{D}} = -N.$$

Then, we obtain the torsion of the curve $\widetilde{D}$ with the help of the differentiation of this equation and use the Eq.(3) as follows

$$\tau_{\widetilde{D}} = \frac{\tau}{(\frac{1}{\mathbf{H}})'} \tag{23}$$

Since the Eq.(22) in the Eq.(23) gives us

$$\mathbf{H}_{\widetilde{D}} = \mathbf{1}$$

the curve $\widetilde{D}$ is circular helix. □

Example 3.7. *Let $\gamma(s)$ be an arbitrary curve with the curvature $\kappa(s) = \dfrac{1}{s\sqrt{1-s^2}}$ and the torsion $\tau(s) = \dfrac{1}{\sqrt{1-s^2}}$ in 3 dimensional Galillean space. Here the Frenet frame apparatus of γ is $\{T, N, B, \kappa, \tau\}$. The position vector of γ is*

$$\gamma(s) = (s, \int(\int \kappa(s)\cos(\int \tau(s)ds)ds)ds, \int(\int \kappa(s)\sin(\int \tau(s)ds)ds)ds)$$

Then $\gamma(s) = (s, -s + s\ln s, s\arcsin s + \sqrt{1-s^2})$ and the parameter curve of the ruled surface $\overline{\phi}(s, u) = \gamma(s) + u\bar{D}(s)$ is

$$\bar{\beta}(s) = (s, -s + s\ln s, s\arcsin s + \sqrt{1-s^2}) + u(s, -s + s\ln s, s\arcsin s + \sqrt{1-s^2})$$

with the Frenet frame apparatus $\{T_{\bar{\beta}}, N_{\bar{\beta}}, B_{\bar{\beta}}, \kappa_{\bar{\beta}}, \tau_{\bar{\beta}}\}$. *If we do the necessary computations we have*

$$\frac{\tau_{\bar{\beta}}}{\kappa_{\bar{\beta}}} = \frac{\tau}{\kappa} = s = m\bar{s} + n$$

where $m = \frac{1}{1+u}$ *and* $n = \frac{-c}{1+u}$ *are real constants. Thus the parameter curve of the ruled surface* $\overline{\phi}(s,u) = \gamma(s) + u\bar{D}(s)$ *is rectifying. The parameter curve* $\bar{\beta}(s)$ *can be seen on Figure 1.*

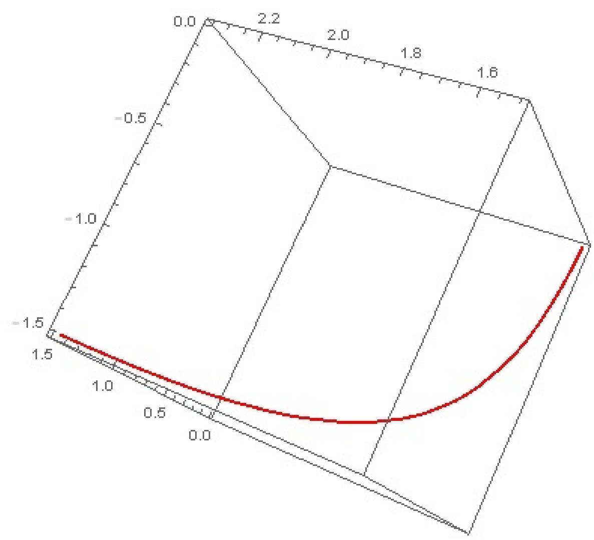

Figure 1

Along with that from Theorem 3.2, $\overline{\beta}(s)$ *must be a geodesic on the surface* $\overline{\phi}(s,u)$. *With an easy computation we have*

$$\frac{d^2\overline{\beta}}{d\bar{s}} = -\kappa(u+1)\mathcal{N}_{\overline{\phi}}$$

Here $\mathcal{N}_{\overline{\phi}}$ *is the unit normal vector field of the surface* $\overline{\phi}(s,u)$. *Thus* $\bar{\beta}(s)$ *is a geodesic curve of the ruled surface* $\overline{\phi}(s,u)$. *The surface* $\overline{\phi}(s,u) = \gamma(s) + u\bar{D}(s)$ *and* $\bar{\beta}(s)$ *can be seen on Figure 2.*

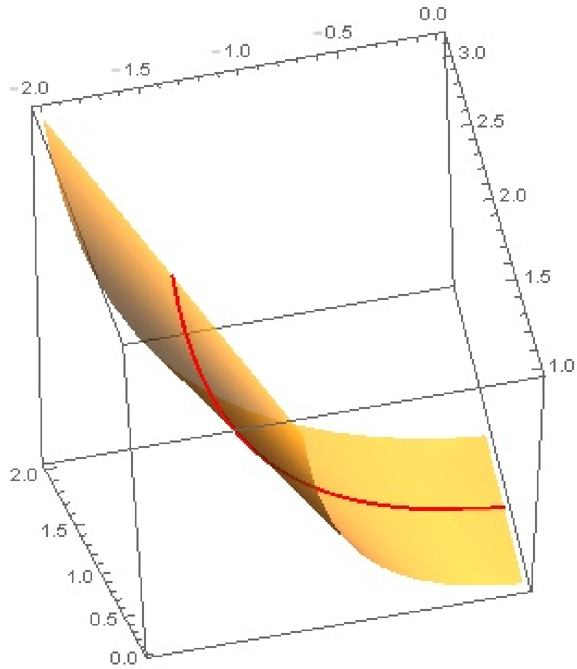

Figure 2

And finally from Theorem 3.3, since $\gamma(s)$ is an unit speed curve, the modified Darboux curve $\bar{D}(s)$ of γ is rectifying. That is

$$\bar{D}(s) = \frac{\tau}{\kappa}T + B = (s, -s + s\ln s, s\arcsin s + \sqrt{1 - s^2})$$

with the Frenet frame apparatus $\{T_{\bar{D}}, N_{\bar{D}}, B_{\bar{D}}, \kappa_{\bar{D}}, \tau_{\bar{D}}\}$. If we do the necessary computations we have

$$\frac{\tau_{\bar{D}}}{\kappa_{\bar{D}}} = \frac{\tau}{\kappa} = s = m\bar{s} + n$$

where $m = \frac{1}{1+u}$ and $n = \frac{-c}{1+u}$. Thus the modified Darboux curve $\bar{D}$ is rectifying. The ruled surface $\overline{\phi}(s, u) = \gamma(s) + u\bar{D}(s)$ and the modified Darboux $\bar{D}(s)$ can be seen on Figure 3.

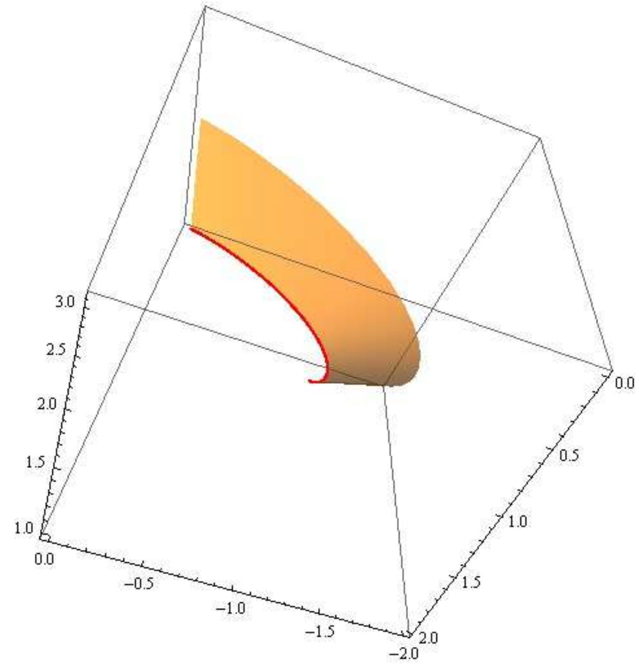

Figure 3

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