Filomat 32:8 (2018), 2793–2812
https://doi.org/10.2298/FIL1808793W

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Cloud Service Reliability Assessment Approach based on Multi-valued Neutrosophic Cross-entropy and Entropy Measures

Yi Wang[a], Xiao-kang Wang[a], Jian-qiang Wang[a]

[a] *School of Business, Central South University, Changsha 410083, China*

Abstract. Cloud service reliability assessment is a vital decision-making activity for companies and individuals. In this assessment, the evaluation information can be represented by multi-valued neutrosophic numbers (MVNNs). MVNNs are regarded as an integration of single-valued neutrosophic numbers (SVNNs) and hesitant fuzzy numbers (HFNs); therefore, considering the defects of entropy and cross-entropy measures for SVNNs and HFNs, we first define a framework of entropy measures and a family of cross-entropy measures for MVNNs in this paper. Second, a novel extended VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje) method based on entropy and cross-entropy measures is developed to address the decision-making problems when information about criteria weights is absolutely unknown. Finally, we apply the proposed method to evaluate cloud service reliability; also, a sensitivity analysis and a comparative analysis are made to interpret the practicality and effectiveness of it. The results of analyses verify that the proposed method based on cross-entropy is much better than the methods using general distance measures.

1. Introduction

Cloud service is an important new technology that has been developed rapidly in recent years. Individuals and companies struggle to make selections of a cloud service because of the rapid changes to the new technology [1]. Thus, cloud service reliability assessment is a significant problem for individuals and companies wanting to use this technology. Many different multi-criteria decision-making (MCDM) methods have been studied to cope with cloud service reliability assessment problems [2]. In these studies, some scholars have utilized fuzzy logic and fuzzy sets (FSs) to describe fuzzy, incomplete, and uncertain evaluation information in cloud service reliability assessments [3]. FSs were first proposed by Zadeh [4], and since then, many extensions of FSs have been proposed. For example, Atanassov [5] defined intuitionistic fuzzy sets (IFSs), which simultaneously consider the membership degree and non-membership degree. They are more flexible at expressing fuzziness, incompleteness, and uncertainty than FSs [5]. Besides, hesitancy may exist in the process of MCDM problems when decision makers (DMs) identify the membership degree of one element in specific situations. Therefore, Torra [6, 7] introduced hesitant fuzzy sets (HFSs) to

2010 *Mathematics Subject Classification*. Primary 90B50; Secondary 03E72
Keywords. Multi-criteria decision-making, Multi-valued neutrosophic sets, Cloud service reliability assessment, Entropy, Cross-entropy
Received: 19 June 2017; Accepted: 03 July 2018
Communicated by Dijana Mosić
Corresponding author: Jian-qiang Wang
Research supported by the National Natural Science Foundation of China (No. 71571193)
Email address: jqwang@csu.edu.cn (Jian-qiang Wang)

describe this hesitant information. However, FSs, IFSs, and HFSs cannot entirely depict inconsistent and indeterminate information under most real-life circumstances [8]. For instance, when a specialist is asked to give an attitude about a statement, the expert may say the probability that the statement is true is 0.5, the possibility that the statement is false is 0.6, and the extent that he or she is not sure is 0.2 [8]. This type of case cannot be handled by FSs, IFSs, or HFSs. Based on this, neutrosophic sets (NSs) were investigated by Smarandache [9, 10] to express true, false, and indeterminate information. Simultaneously, the example can be described by NSs as $x\,(0.5, 0.2, 0.6)$. However, NSs are hard to apply to practical circumstances without a certain description [8, 10]. Therefore, single-valued neutrosophic sets (SVNSs) have been developed as particular cases of NSs [8, 10]. The truth-membership degree, indeterminacy-membership degree, and falsity-membership degree are all single values ranging from 0 to 1 in SVNSs. Moreover, other instances of NSs have also been proposed. Examples include multi-valued neutrosophic sets (MVNSs) [11], bipolar neutrosophic sets [12], single-valued neutrosophic linguistic sets [13], single-valued trapezoidal neutrosophic number [14, 15], and interval neutrosophic linguistic sets [16]. Further, the NSs theories have been proved to be useful in fields such as decision-making problems [17], physician selection problems [18, 19], market segment selection problems [20], and e-commerce websites evaluation problems [21].

The unique properties of MVNSs allow them to perfectly represent the uncertain information found in the cloud service reliability assessment process. MVNSs (also referred to as neutrosophic hesitant fuzzy sets [22, 23]) were originally defined by Wang and Li [11]. As the particular instances of NSs, MVNSs integrate the edges of SVNSs and HFSs [24]. There are three sets in MVNSs, and each set consists of different values assigned in $[0, 1]$, to describe the truth-membership degree, indeterminacy-membership degree, and falsity-membership degree, respectively [11]. When we evaluate cloud service reliability, several experts are usually needed to provide their preferences on satisfaction, indeterminacy, or dissatisfaction with a real number between 0 and 1. For example, suppose two experts are invited to give their evaluation information. One expert may consider the degree of satisfaction to be 0.7 while another expert may think it is 0.6; meanwhile, both of two experts may feel dissatisfied and consider the degree is 0.3; in addition, one expert may think the degree of his/her indeterminacy to be 0.1 while another expert may consider it is 0.2. In this case, SVNSs can express the evaluation information of only one expert, such as $\{\langle x, 0.7, 0.1, 0.3\rangle\}$ or $\{\langle x, 0.6, 0.2, 0.3\rangle\}$. Thereafter, HFSs can describe the assessment information of merely one type of preference, such as $\{\langle x, \{0.6, 0.7\}\rangle\}$, $\{\langle x, \{0.1, 0.2\}\rangle\}$, or $\{\langle x, \{0.3\}\rangle\}$. Thus, evaluation information of all experts cannot be represented by SVNSs or HFSs completely. Since a cloud service assessment problem contains truth-membership degree, indeterminacy-membership degree, falsity-membership degree, and every one of these membership degrees is a group of different values ranging from 0 to 1, MVNSs are better at denoting the fuzziness and hesitancy in cloud service reliability assessment problems than HFSs and SVNSs. The aforementioned example employs MVNSs to depict as $\{\langle x, \{0.6, 0.7\}, \{0.1, 0.2\}, \{0.3\}\rangle\}$.

There have been a lot of researches involving multi-valued neutrosophic information. Biswas et al. [25, 26] defined some distance measures for MVNSs, meanwhile, Şahin and Liu [27] developed some distance and similarity measures for multi-valued neutrosophic information. Then, Biswas et al. [25, 26] and Şahin and Liu [27] established the decision-making methods using these measures, separately. Thereafter, Liu and Zhang [28] defined Hamming distance of MVNSs, and constructed a new decision-making method. In addition, traditional MCDM methods were extended to MVNSs, including the GRA (grey relational analysis) method [26], ELECTRE (Elimination and Choice Translating Reality) method [29], TODIM (An acronym in Portuguese of interactive and decision-making method named Tomada de decisao interativa e multicritvio) [11, 24], and QUALIFLEX (qualitative flexible multiple criteria) method [30].

These methods are efficient and effective for multi-valued neutrosophic MCDM problems. However, they have the following gaps:

(1) The criteria weights are artificially assigned [11, 24-30]. Nevertheless, under multi-valued neutrosophic environment, there are few studies about weight determination methods with completely unknown information related to criteria weights.

(2) The MCDM methods [11, 25-28] evaluate and select alternatives using MVNSs discrimination information measures, such as Hamming distance [11, 27, 28], Normalized Hamming distance [26], and generalized distance [25]. However, a large amount of information may be lost or distorted in the decision-making process and the outcomes derived by these methods may be inaccurate and useless.

(3) The MCDM methods [11, 25-28] are based on distance measures under multi-valued neutrosophic environments. The results obtained by these methods usually have only one alternative for a solution. However, in some real-life cases, there may be two, or more than two, alternatives to select for a compromise solution under conflicting criteria.

Considering the drawbacks of the current researches, this study utilizes cross-entropy and entropy to propose a novel MCDM method.

Entropy is a significant measure for the uncertainty of information and it can be utilized to identify the criteria weights when information about criteria weights is absolutely unknown [31]. Burillo and Bustince [32] first proposed the notion of entropy of IFSs in 1996. Since then, entropy has been greatly extended into other fuzzy environments. For instance, Wei et al. [33] proposed entropy measures with hesitant fuzzy information. Further, Majumdar and Samanta [34] developed the extensions of entropy under single-valued neutrosophic environments. Meanwhile, cross-entropy is an effective tool for processing MCDM problems. It describes the information differences between two numbers. One advantage of cross-entropy is that it can express the information difference more easily and exactly than traditional distance measures [35]. Because of this advantage, cross-entropy has been extended to various fuzzy environments, including intuitionistic fuzzy environments [35], hesitant fuzzy environments [31], and single-valued neutrosophic environments [36]. Since multi-valued neutrosophic numbers (MVNNs) can be viewed as the integration of single-valued neutrosophic numbers (SVNNs) and hesitant fuzzy numbers (HFNs) [24]. Therefore, the entropy and cross-entropy measures of MVNNs are derived from that of SVNNs and HFNs. Due to the extant entropy and cross-entropy measures of SVNNs and HFNs have some issues in some circumstances, we should address these issues at first.

VIKOR was proposed by Opricovic and Tzeng in 1998 [37] to maximize "group utility" for the "majority" and to minimize "individual regrets" for the "opponent" [38]. Since then, the VIKOR method has been extended to numerous fuzzy environments [39, 40]. For example, Zhao et al. [39] used the VIKOR method to solve the selection problem for a virtual enterprise under interval-based intuitionistic fuzzy environments. Bausys and Zavadskas [40] extended the VIKOR method under interval neutrosophic environments. The VIKOR method ranks several alternatives and obtains a compromise solution for issues with inconsistent criteria; the resulting compromise solution is an acceptable result which is nearest to the ideal result [41]. Typically, there are several conflicting criteria in the process of cloud service reliability assessment. Therefore, this study extends the VIKOR method with multi-valued neutrosophic information.

This study offers a number of contributions. First, MVNNs are employed to depict fuzzy and hesitant information in cloud service reliability assessments. Then, in view of the defects of single-valued neutrosophic entropies and hesitant fuzzy entropies, a general framework of entropy measures for MVNNs is defined. The entropy method is also extended to multi-valued neutrosophic environments. Next, taking into account the shortcomings of cross-entropies within single-valued neutrosophic environment and hesitant fuzzy environment, a family of cross-entropy measures for MVNNs is developed. Finally, for situations where information about criteria weights is absolutely unknown, we propose a novel VIKOR method based on entropy and cross-entropy measures for MCDM problems.

The rest of this paper is arranged as follows. In Section 2, the basic definitions related to MVNSs are presented, SVNSs, HFSs, and their entropy and cross-entropy measures are reviewed. In Section 3, we define an entropy measure for MVNNs and a general framework of entropies is developed. Meanwhile, a family of cross-entropy measures for MVNNs is investigated. Section 4 proposes a novel extended VIKOR method based on entropy and cross-entropy. In Section 5, an example of cloud service reliability assessment is given and the results are analyzed and discussed. A conclusion is offered in Section 6.

2. Preliminaries

This section presents primary ideas and definitions related to SVNSs, HFSs, and MVNSs. The cross-entropy and entropy measures for SVNNs and HFNs are also introduced. These concepts will be utilized in subsequent analyses.

2.1. SVNSs, HFSs, and MVNSs

Definition 1 [8]. Let X be a non-empty fixed set, with a generic element in X denoted by x. An SVNS α in X is characterized by the truth-membership degree $T_\alpha(x)$, indeterminacy-membership degree $I_\alpha(x)$, and falsity-membership degree $F_\alpha(x)$, where $T_\alpha(x),\ I_\alpha(x),\ F_\alpha(x) \in [0,1]$, for any $x \in X$, as follows:

$$A_\alpha = \{\langle x, T_\alpha(x), I_\alpha(x), F_\alpha(x)\rangle | x \in X\}. \tag{1}$$

α is called a single-valued neutrosophic number (SVNN) if X has only one element, which can be defined by $\alpha = \langle T_\alpha, I_\alpha, F_\alpha\rangle$. For convenience, let us denote the collection of SVNNs as SN.

Definition 2 [6, 7]. Let X be a non-empty fixed set, the hesitant set on X is with respect to a function that will return a subset of values in $[0,1]$. For convenience, an HFS can be represented as a mathematical symbol:

$$\bar{\alpha} = \left\{\left\langle x, \tilde{h}_{\bar{\alpha}}(x)\right\rangle | x \in X\right\}, \tag{2}$$

where $\tilde{h}_{\bar{\alpha}}(x)$ is a group of real numbers within $[0,1]$, denoting a group of membership degrees for all $x \in X$. $\bar{\alpha}$ is called as a hesitant fuzzy number (HFN) if X has only one element, which can be denoted by $\bar{\alpha} = \left\langle \tilde{h}_{\bar{\alpha}}\right\rangle$ for notational convenience. Further, H is denoted as the set of all HFNs.

Definition 3 [11, 24]. Let X be a non-empty fixed set with a generic element in X denoted by x. An MVNS A in X is characterized by the truth-membership degree $\tilde{T}_A(x)$, indeterminacy-membership degree $\tilde{I}_A(x)$, and falsity-membership degree $\tilde{F}_A(x)$. $\tilde{T}_A(x)$, $\tilde{I}_A(x)$, and $\tilde{F}_A(x)$ are three sets of precise values in $[0,1]$, satisfying $0 \le \gamma, \eta, \xi \le 1$ and $0 \le \gamma^+ + \eta^+ + \xi^+ \le 3$ where $\gamma \in \tilde{T}_A(x)$, $\eta \in \tilde{I}_A(x)$, $\xi \in \tilde{F}_A(x)$, $\gamma^+ = \sup \tilde{T}_A(x)$, $\eta^+ = \sup \tilde{I}_A(x)$, $\xi^+ = \sup \tilde{F}_A(x)$. An MVNS can be defined as follows:

$$A = \left\{\left\langle x, \cup_{\gamma\in\tilde{T}_A}\{\gamma\}, \cup_{\eta\in\tilde{I}_A}\{\eta\}, \cup_{\xi\in\tilde{F}_A}\{\xi\}\right\rangle | x \in X\right\}. \tag{3}$$

A is called a multi-valued neutrosophic number (MVNN) if X has only one element, denoted by $\left\langle \cup_{\gamma\in\tilde{T}_A}\{\gamma\}, \cup_{\eta\in\tilde{I}_A}\{\eta\}, \cup_{\xi\in\tilde{F}_A}\{\xi\}\right\rangle$. Further, let us denote the sets of all MVNNs as MN.

MVNSs are degenerated to SVNSs if each of $\tilde{T}_A(x)$, $\tilde{I}_A(x)$, and $\tilde{F}_A(x)$ for any x has only one number, and MVNSs are degenerated to HFSs if $\tilde{I}_A(x) = \emptyset$ and $\tilde{F}_A(x) = \emptyset$ for any x.

Definition 4 [24]. Let $A \in MN$, the complement of A is denoted by A^c and defined as:

$$A^c = \left\langle \cup_{\xi\in\tilde{F}_A}\{\xi\}, \cup_{\eta\in\tilde{I}_A}\{1-\eta\}, \cup_{\gamma\in\tilde{T}_A}\{\gamma\}\right\rangle. \tag{4}$$

Definition 5 [42]. Let $A, B \in MN$. A is greater to or equal to B, denoted by $A \ge B$, if and only if $\gamma_A \ge \gamma_B$, $\eta_A \le \eta_B$, and $\xi_A \le \xi_B$ for every $\gamma_A \in \tilde{T}_A$, $\gamma_B \in \tilde{T}_B$, $\eta_A \in \tilde{I}_A$, $\eta_B \in \tilde{I}_B$, $\xi_A \in \tilde{F}_A$, and $\xi_B \in \tilde{F}_B$.

2.2. The entropy and cross-entropy measures for SVNNs

The entropy and cross-entropy measures for SVNNs are introduced in this subsection.

Definition 6 [34]. Let $\alpha \in SN$ and $E : SN \to [0,1]$, then $E(\alpha)$ is called an entropy of α if these four axioms can be satisfied:

(1) $E(\alpha) = 0$ if α is a crisp number;

(2) $E(\alpha) = 1$ if $\langle T_\alpha, I_\alpha, F_\alpha\rangle = \langle 0.5, 0.5, 0.5\rangle$;

(3) $E(\alpha) = E(\alpha^c)$, $\forall\alpha \in SN$, where $\alpha^c = \langle F_\alpha, 1 - I_\alpha, T_\alpha\rangle$;

(4) $E(\alpha) \ge E(\beta)$ if α is more uncertain than β, i.e., $T_\alpha + F_\alpha \le T_\beta + F_\beta$ and $|I_\alpha - I_{\alpha^c}| \le \left|I_\beta - I_{\beta^c}\right|$. And then the entropy measure of SVNSs in Ref. [34] can be defined as follows:

$$E(\alpha) = 1 - (T_\alpha + F_\alpha) \cdot |I_\alpha - I_{\alpha^c}|. \tag{5}$$

Nevertheless, there is a limitation in the fourth axiom of Definition 6 and the entropy measure for SVNNs, which is depicted in the following example.

Example 1. Let $\alpha = \langle 0.6, 0.3, 0.4\rangle$ and $\beta = \langle 0.8, 0.3, 0.1\rangle$ be two SVNNs. The data shows that α hovers between truth and falseness while β is more inclined to truth. In other words, α is more uncertain than β. i.e.,$E(\alpha) > E(\beta)$. Meanwhile, it is easy to get an inequality $T_\alpha + F_\alpha > T_\beta + F_\beta$. However, the inequality contradicts the fourth axiom of entropy for SVNNs which is $T_\alpha + F_\alpha \le T_\beta + F_\beta$. Further, $E(\alpha) = 0.6$ and $E(\beta) = 0.64$ are derived by Eq. (5). That is, $E(\alpha) < E(\beta)$. The calculated results are counterintuitive. Thus, the entropy measures defined in Ref. [34] cannot express the real uncertainty in certain extreme cases.

Similarly, there are some issues in the cross-entropy measures of SVNNs. Under single-valued neutrosophic environment, three cross-entropy measures are defined by Ye [43] and Wu et al. [36]. Ye [43] developed a cross-entropy for SVNNs. However, Meng and Chen [44] pointed out that there is an issue in the cross-entropy measure investigated by Ye [43]. That is, the cross-entropy [43] is an indefinite number when α and β are crisp numbers. Therefore, the cross-entropy defined by Ye [43] is unreasonable. Meanwhile, the cross-entropy measures provided by Wu et al. [36] are also difficult to accept in some cases.
Definition 7 [36]. Let $\alpha, \beta \in SN$, then two cross-entropies between α and β are summarized below::

$$\begin{aligned} CE_k(\alpha,\beta) = & g_k(T_\alpha)\times g_k\left(T_\alpha - T_\beta\right) \\ & + g_k(I_\alpha)\times g_k\left(I_\alpha - I_\beta\right) \\ & + g_k(F_\alpha)\times g_k\left(F_\alpha - F_\beta\right) \end{aligned}, \tag{6}$$

where $k = 1, 2$, $g_1(z) = \sin(z)$ and $g_2(z) = \tan(z)$ are two functions.

In order to make Eq. (6) symmetric, Wu et al. [36] modified the cross-entropy measures through the following equation:

$$DE_k(\alpha,\beta) = CE_k(\alpha,\beta) + CE_k(\beta,\alpha), \text{ where } k = 1, 2. \tag{7}$$

However, these modified cross-entropy measures have one shortcoming as seen in Example 2.
Example 2. Let $\alpha_1 = \langle 0.1, 0.1, 0.1\rangle$, $\alpha_2 = \langle 0.9, 0.9, 0.9\rangle$, and $\alpha_3 = \langle 0.5, 0.5, 0.5\rangle$ be three SVNNs. Theoretically, the modified cross-entropies between α_1 and α_3 should be equal to that between α_2 and α_3. By Eqs. (6) and (7), we can obtain that we can obtain that $DE_1(\alpha_1,\alpha_3) = 0.4435$, $DE_1(\alpha_2,\alpha_3) = 0.3550$, $DE_2(\alpha_1,\alpha_3) = 0.5657$, and $DE_2(\alpha_2,\alpha_3) = 0.9054$. That is, $DE_1(\alpha_1,\alpha_3) \neq DE_1(\alpha_2,\alpha_3)$ and $DE_2(\alpha_1,\alpha_3) \neq DE_2(\alpha_2,\alpha_3)$. Since discrepancies exist between reality and theory, the cross-entropy measures defined by Wu et al. [36] cannot be accepted.

2.3. The entropy and cross-entropy measures for HFNs

The details of entropy and cross-entropy measures for HFNs are also presented in the rest of this subsection.
Definition 8 [33]. Let $\bar{\alpha} \in H$, the entropy measure for HFNs is developed as follows:

$$E(\bar{\alpha}) = \frac{1 - |\cos(\theta(\bar{\alpha})\cdot\pi)| + \tau(\bar{\alpha})}{1 + \tau(\bar{\alpha})}, \tag{8}$$

where $\theta(\bar{\alpha}) = \frac{1}{l_{\bar{\alpha}}}\sum_{i=1}^{l_{\bar{\alpha}}}\bar{\alpha}^i$ is interpreted as the score value of the HFN $\bar{\alpha}$ and $l_{\bar{\alpha}}$ is the length of the HFN $\bar{\alpha}$, also, $\tau(\bar{\alpha}) = \frac{2}{l_{\bar{\alpha}}(l_{\bar{\alpha}}-1)}\sum_{i=1}^{l_{\bar{\alpha}}-1}\sum_{j=i+1}^{l_{\bar{\alpha}}}\left(\bar{\alpha}^j - \bar{\alpha}^i\right)$ is the deviation value of the HFN $\bar{\alpha}$ and $\bar{\alpha}^i$ is the i-th smallest value for every $\bar{\alpha} \in H$.

Wei et al. [33] only used two values, score value and deviation value, to describe an HFN as a set of different values. Obviously, there is an issue in the entropy measures based on these two values as shown in Example 3.
Example 3. Let $\bar{\alpha} = \langle 0.3, 0.7\rangle$ and $\bar{\beta} = \langle 0.2, 0.5, 0.8\rangle$ be two HFNs. It is clear that the uncertainty of $\bar{\alpha}$ is not equal to that of $\bar{\beta}$. That is, $E(\bar{\alpha}) \neq E(\bar{\beta})$. Nevertheless, the score value $\theta(\bar{\alpha}) = \theta(\bar{\beta}) = 0.5$ and deviation value $\tau(\bar{\alpha}) = \tau(\bar{\beta}) = 0.4$ can be obtained by their definitions. By Eq. (8), it can be calculated

that $E(\bar{\alpha}) = E(\bar{\beta}) = 0.2860$, which is counterintuitive. Therefore, the score value and deviation value may lose a mass of hesitant fuzzy information in a HFN, and the entropy measures using these two values may incompletely and inaccurately express the uncertainty of hesitant fuzzy information.

Definition 9 [31]. Let $\bar{\alpha}, \bar{\beta} \in H$ and $l_{\bar{\alpha}} = l_{\bar{\beta}}$ where $l_{\bar{\alpha}}$ and $l_{\bar{\beta}}$ are denoted as the number of elements in $\bar{\alpha}$ and $\bar{\beta}$, respectively. Further, assume $\bar{\alpha}_{\sigma(i)}$ is the i-th smallest values for every $\bar{\alpha} \in H$, and the elements in $\bar{\beta}$ are also arranged in ascending order. Then the cross-entropy measures between $\bar{\alpha}$ and $\bar{\beta}$ are defined in the following formulas:

$$\begin{aligned} CE_1(\bar{\alpha}, \bar{\beta}) = \\ \frac{1}{lT}\sum_{i=1}^{l}\Bigg(& \frac{\left(1+q\bar{\alpha}_{\sigma(i)}\right)\ln\left(1+q\bar{\alpha}_{\sigma(i)}\right)+\left(1+q\bar{\beta}_{\sigma(i)}\right)\ln\left(1+q\bar{\beta}_{\sigma(i)}\right)}{2} \\ & - \frac{2+q\bar{\alpha}_{\sigma(i)}+q\bar{\beta}_{\sigma(i)}}{2}\ln\frac{2+q\bar{\alpha}_{\sigma(i)}+q\bar{\beta}_{\sigma(i)}}{2} \\ & + \frac{\left(1+q\left(1-\bar{\alpha}_{\sigma(l-i+1)}\right)\right)\ln\left(1+q\left(1-\bar{\alpha}_{\sigma(l-i+1)}\right)\right)+\left(1+q\left(1-\bar{\beta}_{\sigma(l-i+1)}\right)\right)\ln\left(1+q\left(1-\bar{\beta}_{\sigma(l-i+1)}\right)\right)}{2} \\ & - \frac{2+q\left(1-\bar{\alpha}_{\sigma(l-i+1)}+1-\bar{\beta}_{\sigma(l-i+1)}\right)}{2}\ln\frac{2+q\left(1-\bar{\alpha}_{\sigma(l-i+1)}+1-\bar{\beta}_{\sigma(l-i+1)}\right)}{2}\Bigg), \end{aligned} \tag{9}$$

where $q \geq 0$ and $T = (1+q)\ln(1+q) - (2+q)(\ln(2+q) - \ln 2)$.

$$\begin{aligned} CE_2(\bar{\alpha}, \bar{\beta}) = \frac{1}{\left(1-2^{1-p}\right)l}\sum_{i=1}^{l}\Bigg(& \frac{\bar{\alpha}^p_{\sigma(i)}+\bar{\beta}^p_{\sigma(i)}}{2} + \frac{\left(1-\bar{\alpha}_{\sigma(l-i+1)}\right)^p+\left(1-\bar{\beta}_{\sigma(l-i+1)}\right)^p}{2} \\ & - \left(\frac{\bar{\alpha}_{\sigma(i)}+\bar{\beta}_{\sigma(i)}}{2}\right)^p - \left(\frac{1-\bar{\alpha}_{\sigma(l-i+1)}+1-\bar{\beta}_{\sigma(l-i+1)}}{2}\right)^p\Bigg), \end{aligned} \tag{10}$$

where $p > 1$.

Both Eqs. (9) and (10) are studied under the same hypothesis that $l_{\bar{\alpha}} = l_{\bar{\beta}}$. If the corresponding HFNs do not have the same length, then the HFN, which has the fewest elements, needs to be changed by repeating the smallest or maximum number in this HFN.

Example 4. Let $p, q = 2$, $\bar{\alpha} = \langle 0.1, 0.6\rangle$ and $\bar{\beta} = \langle 0.2, 0.5, 0.9\rangle$ be two HFNs. We should extend $\bar{\alpha}$ until it has an identical length to $\bar{\beta}$. We can obtain $\bar{\alpha} = \langle 0.1, 0.1, 0.6\rangle$ by repeating the smallest value in $\bar{\alpha}$. Based on Eqs. (9) and (10), $CE_1(\bar{\alpha}, \bar{\beta}) = 0.0879$ and $CE_2(\bar{\alpha}, \bar{\beta}) = 0.0867$. Similarly, we can obtain $\bar{\alpha} = \langle 0.1, 0.6, 0.6\rangle$ by repeating the maximum value. And $CE_1(\bar{\alpha}, \bar{\beta}) = 0.0376$ and $CE_2(\bar{\alpha}, \bar{\beta}) = 0.0367$ by Eqs. (9) and (10). That is, when different values are added to $\bar{\alpha}$, there may be different cross-entropies between $\bar{\alpha}$ and $\bar{\beta}$.

According to the above illustrations, there are some issues in the entropy and cross-entropy measures for SVNNs and HFNs. Since MVNNs are the integration of SVNNs and HFNs [24], the entropy and cross-entropy measures for MVNNs should handle the aforementioned problems.

3. Entropy and cross-entropy measures for multi-valued neutrosophic information

By analogy, we define the axioms of entropy and entropy measures for MVNNs in this section based on their use in SVNNs and HFNs. Meanwhile, when the axioms of entropy and entropy measures for MVNNs reduce to SVNNs or HFNs, the defects of entropies for SVNNs or HFNs shown in Section 2 are well covered. Then, according to the entropy measures for MVNNs, a general framework of entropy measures for MVNNs is developed. Finally, similar to the proposal of entropy measures, we develop a family of cross-entropy measures for MVNNs based on the cross-entropy measures of SVNNs and HFNs.

3.1. Entropy measures for MVNNs

Definition 10. Let $A \in MN$, the two variables Δ^i_A and σ^j_A with respect to A are defined as follows:

$$\Delta^i_A = \cup_{\gamma_A \in \tilde{T}_A, \xi_A \in \tilde{F}_A}\left|\gamma_A - \xi_A\right|, \ \ i = 1, 2, ..., l_{\tilde{T}} \cdot l_{\tilde{F}}, \tag{11}$$

$$\sigma^j_A = \cup_{\eta \in \tilde{I}_A}\left|\eta_A - \eta_{A^c}\right|, \ \ j = 1, 2, ..., l_{\tilde{I}}, \tag{12}$$

where $0 \leq \Delta_A^i, \sigma_A^j \leq 1$, and $l_{\tilde{T}}, l_{\tilde{I}}, l_{\tilde{F}}$ denote the length of $\tilde{T}_A$, $\tilde{I}_A$, and $\tilde{F}_A$, respectively.

The two variables Δ_A^i and σ_A^j concerning A are related to the uncertainties of multi-valued neutrosophic information. The first variable Δ_A^i means the difference between truth-membership degree and falsity-membership degree. Moreover, the more significant the difference is, the smaller the uncertainties are involved in MVNNs. For example, assume $B = \langle\{0.1\}, \{0.3, 0.4\}, \{0.7\}\rangle$ and $C = \langle\{0.3\}, \{0.3, 0.4\}, \{0.5\}\rangle$ be two MVNNs. Obviously, B is more inclined to falsity while C is wandering between truth and falsity. We can conclude that B is less uncertain than C. Simultaneously, it can be derived that $\Delta_B = \{0.6\}$ and $\Delta_C = \{0.2\}$, which are consistent with our conclusions. Similarly, the second variable σ_A^j means internal difference in indeterminacy-membership degree, and there is also a negative correlation between the degree of difference and uncertainties of MVNNs.

Definition 11. An entropy of MVNN A is a real-valued function $E : MN \to [0, 1]$, which satisfies the following axiomatic conditions and is related to Δ_A^i and σ_A^j:

(1) $E(A) = 0$ if and only if $\Delta_A^i = 1$ for $i = 1, 2, ..., l_{\tilde{T}} \cdot l_{\tilde{F}}$ and $\sigma_A^j = 1$ for $j = 1, 2, ..., l_{\tilde{I}}$;

(2) $E(A) = 1$ if and only if $\Delta_A^i = 0$ for $i = 1, 2, ..., l_{\tilde{T}} \cdot l_{\tilde{F}}$ and $\sigma_A^j = 0$ for $j = 1, 2, ..., l_{\tilde{I}}$;

(3) $E(A) = E(A^c)$;

(4) E decreases monotonically regarding Δ_A^i for all $i = 1, 2, ..., l_{\tilde{T}} \cdot l_{\tilde{F}}$, and also monotonically decreases regarding σ_A^j for all $j = 1, 2, ..., l_{\tilde{I}}$.

Definition 12. Let $A \in MN$, an entropy measure for A is denoted by the following equation:

$$E(A) = 1 - \frac{1}{l_{\tilde{T}} \cdot l_{\tilde{I}} \cdot l_{\tilde{F}}} \sum_{i=1}^{l_{\tilde{T}} \cdot l_{\tilde{F}}} \sum_{j=1}^{l_{\tilde{I}}} \frac{\Delta_A^i + \sigma_A^j}{2}. \tag{13}$$

Proposition 1. The measure defined by Eq. (13) is a multi-valued neutrosophic entropy, and satisfies conditions (1)-(4) given in Definition 11.

The proof of Proposition 1 is described concretely in the "Appendix A".

The defects of single-valued neutrosophic entropy and hesitant fuzzy entropy mentioned in Section 2 can be covered by entropy of MVNNs. The single-valued neutrosophic entropy and its fourth axiom, which are reduced from those of MVNNs, can be represented below:

$$E(\alpha) = 1 - \frac{|T_\alpha - F_\alpha| + |I_\alpha - I_{\alpha^c}|}{2}. \tag{14}$$

(4) The inequality $E(\alpha) \geq E(\beta)$ holds if α is more uncertain than β, i.e., $|T_\alpha - F_\alpha| \leq |T_\beta - F_\beta|$ and $|I_\alpha - I_{\alpha^c}| \leq |I_\beta - I_{\beta^c}|$.

Regarding the single-valued neutrosophic entropy developed by Majumdar and Samanta [34], the new entropy measure and its axiom conquer the issues in Ref. [34]. Our intuitions consider $E(\alpha) \geq E(\beta)$ when the inequality $|T_\alpha - F_\alpha| \leq |T_\beta - F_\beta|$ holds, and the reduced axiomatic condition (4) are in accordance with our intuitions. Meanwhile, using the data in Example 1, we can obtain that $E(\alpha) = 1 - \frac{|0.6-0.4|+|0.3-0.7|}{2} = 0.7$ and $E(\beta) = 1 - \frac{|0.8-0.1|+|0.3-0.7|}{2} = 0.45$, i.e., $E(\alpha) > E(\beta)$, which also confirms to our experience. As a result, the reduced entropy and its axiom can address the problems of entropy measures in Ref. [34].

With regard to the hesitant fuzzy entropy investigated by Wei et al. [33], we substitute the original multi-valued neutrosophic information into the entropy measure formulas while Wei et al. [33] used score value and deviation value, which are processed hesitant fuzzy information. Therefore, the entropy of MVNNs can express the fuzzy and hesitant information completely and accurately.

3.2. A general framework of entropy measures for MVNNs

A general framework of entropy measures for MVNNs is defined in this subsection. Subsequently, four different entropy measures for MVNNs are derived based on the general framework of entropy measures. Wei et al. [33] investigated a general form of entropy measures for HFNs and derived a family of concrete entropy formulas. Yao et al. [45] developed a general framework of type-2 fuzzy entropy measures. A general framework of entropies can allow us to view uncertainties with a fresh perspective. Simultaneously, we can obtain concrete entropy measures with simple function for MVNNs from the general form.

Theorem 1. For each $A \in MN$, let

$$E_f(A) = \frac{1}{l_{\tilde{T}} \cdot l_{\tilde{I}} \cdot l_{\tilde{F}}} \sum_{i=1}^{l_{\tilde{T}} \cdot l_{\tilde{F}}} \sum_{j=1}^{l_{\tilde{I}}} f\left(\Delta_A^i, \sigma_A^j\right). \tag{15}$$

Then $E_f(A)$ is an entropy measure for an MVNN A, if and only if the function $f : [0,1] \times [0,1] \to [0,1]$ has the following properties:

(1) $f(x,y) = 0$ if and only if $x = 1$ and $y = 1$;

(2) $f(x,y) = 1$ if and only if $x = 0$ and $y = 0$;

(3) f decreases monotonically with respect to the first variable and the second variable, respectively.

The evidentiary procedure of Theorem 1 is offered in the "Appendix B"

If we change the function f in $E_f(A)$ defined by Eq. (15), and this function possesses the properties listed in Theorem 1, we can obtain a series of entropy measures for MVNNs. For example, let $f(x,y) = \frac{(1-x)^\rho + (1-y)^\rho}{2}$, $\rho > 0$, $f(x,y) = 1 - \frac{\sin\left(\frac{\pi}{2} \cdot x\right) + \sin\left(\frac{\pi}{2} \cdot y\right)}{2}$, $f(x,y) = 1 - \frac{\tan\left(\frac{\pi}{4} \cdot x\right) + \tan\left(\frac{\pi}{4} \cdot y\right)}{2}$, and $f(x,y) = 1 - \frac{\log_2(x+1) + \log_2(y+1)}{2}$, respectively.

Then, four different entropy measure formulas are obtained:

$$E_{f1}(A) = \frac{1}{l_{\tilde{T}} \cdot l_{\tilde{I}} \cdot l_{\tilde{F}}} \sum_{i=1}^{l_{\tilde{T}} \cdot l_{\tilde{F}}} \sum_{j=1}^{l_{\tilde{I}}} \frac{\left(1 - \Delta_A^i\right)^\rho + \left(1 - \sigma_A^j\right)^\rho}{2}, \quad \rho > 0, \tag{16}$$

$$E_{f2}(A) = \frac{1}{l_{\tilde{T}} \cdot l_{\tilde{I}} \cdot l_{\tilde{F}}} \sum_{i=1}^{l_{\tilde{T}} \cdot l_{\tilde{F}}} \sum_{j=1}^{l_{\tilde{I}}} 1 - \frac{\sin\left(\frac{\pi}{2} \cdot \Delta_A^i\right) + \sin\left(\frac{\pi}{2} \cdot \sigma_A^j\right)}{2}, \tag{17}$$

$$E_{f3}(A) = \frac{1}{l_{\tilde{T}} \cdot l_{\tilde{I}} \cdot l_{\tilde{F}}} \sum_{i=1}^{l_{\tilde{T}} \cdot l_{\tilde{F}}} \sum_{j=1}^{l_{\tilde{I}}} 1 - \frac{\tan\left(\frac{\pi}{4} \cdot \Delta_A^i\right) + \tan\left(\frac{\pi}{4} \cdot \sigma_A^j\right)}{2}, \tag{18}$$

$$E_{f4}(A) = \frac{1}{l_{\tilde{T}} \cdot l_{\tilde{I}} \cdot l_{\tilde{F}}} \sum_{i=1}^{l_{\tilde{T}} \cdot l_{\tilde{F}}} \sum_{j=1}^{l_{\tilde{I}}} 1 - \frac{\log_2\left(\Delta_A^i + 1\right) + \log_2\left(\sigma_A^j + 1\right)}{2}, \tag{19}$$

$E(A)$ defined by Eq. (13) is equal to $E_{f1}(A)$ when $\rho = 1$.

Example 5. Let $A = \langle\{0.7\}, \{0.2, 0.3\}, \{0.1, 0.2\}\rangle$, $B = \langle\{0.4, 0.5, 0.6\}, \{0.2\}, \{0.3, 0.4\}\rangle$, and $C = \langle\{0.3, 0.4\}, \{0.2, 0.3\}, \{0.4, 0.5\}\rangle$ be three MVNNs. Suppose $\rho = 2$, we can obtain the following results by Eqs. (16)-(19):

$E_{f1}(A) = 0.2325$, $E_{f1}(B) = 0.4458$, $E_{f1}(C) = 0.5375$;
$E_{f2}(A) = 0.2718$, $E_{f2}(B) = 0.4801$, $E_{f2}(C) = 0.5731$;
$E_{f3}(A) = 0.5605$, $E_{f3}(B) = 0.6857$, $E_{f3}(C) = 0.7519$;
$E_{f4}(A) = 0.3934$, $E_{f4}(B) = 0.5627$, $E_{f4}(C) = 0.6419$.

3.3. A family of cross-entropy measures for MVNNs

Definition 13. Let $A, B, C \in MN$, $CE : MN \times MN \to R^+$, then the cross-entropy $CE(A,B)$ between A and B should satisfy the following axiomatic conditions:

(1) $CE(A,B) \geq 0$, and $CE(A,B) = 0$ if and only if $A = B$;

(2) $CE(A^c, B^c) = CE(A,B)$ where A^c and B^c are the complement of A and B, respectively , as defined in Definition 4.

(3) $CE(A,B) = CE(B,A)$;

(4) If $A \leq B \leq C$, then $CE(A,B) \leq CE(A,C)$ and $CE(B,C) \leq CE(A,C)$.

Definition 14. Let $A, B \in MN$, then cross-entropies between A and B are defined below:

$$\begin{aligned} CE_k(A,B) = & \max_{\gamma_A \in \tilde{T}_A} \min_{\gamma_B \in \tilde{T}_B} \left[g_k(\gamma_A - \gamma_B) \ln \tfrac{2+\gamma_A-\gamma_B}{2-\gamma_A+\gamma_B} \right] + \max_{\gamma_B \in \tilde{T}_B} \min_{\gamma_A \in \tilde{T}_A} \left[g_k(\gamma_B - \gamma_A) \ln \tfrac{2+\gamma_B-\gamma_A}{2-\gamma_B+\gamma_A} \right] \\ & + \max_{\eta_A \in \tilde{I}_A} \min_{\eta_B \in \tilde{I}_B} \left[g_k(\eta_A - \eta_B) \ln \tfrac{2+\eta_A-\eta_B}{2-\eta_A+\eta_B} \right] + \max_{\eta_B \in \tilde{I}_B} \min_{\eta_A \in \tilde{I}_A} \left[g_k(\eta_B - \eta_A) \ln \tfrac{2+\eta_B-\eta_A}{2-\eta_B+\eta_A} \right] \\ & + \max_{\xi_A \in \tilde{F}_A} \min_{\xi_B \in \tilde{F}_B} \left[g_k(\xi_A - \xi_B) \ln \tfrac{2+\xi_A-\xi_B}{2-\xi_A+\xi_B} \right] + \max_{\xi_B \in \tilde{F}_B} \min_{\xi_A \in \tilde{F}_A} \left[g_k(\xi_B - \xi_A) \ln \tfrac{2+\xi_B-\xi_A}{2-\xi_B+\xi_A} \right], \end{aligned} \tag{20}$$

where $k = 1, 2, 3$, $g_1(z) = z$, $g_2(z) = \sin(z)$, and $g_3(z) = \tan(z)$ are three functions.

Proposition 2. The measures defined by Eq. (20) are all multi-valued neutrosophic cross-entropies, and meet conditions (1)-(4) in Definition 13.

We give the detailed verification for Proposition 2 in the "Appendix C".

The cross-entropy measures for MVNNs cover the deficiencies of single-valued neutrosophic cross-entropies and hesitant fuzzy cross-entropies shown in Section 2. The cross-entropy measures for SVNNs, which are reduced from those for MVNNs defined above, are denoted below.

$$CE_k(A,B) = g_k(T_A - T_B) \ln \frac{2 + T_A - T_B}{2 - T_A + T_B} + g_k(I_A - I_B) \ln \frac{2 + I_A - I_B}{2 - I_A + I_B} + g_k(F_A - F_B) \ln \frac{2 + F_A - F_B}{2 - F_A + F_B}. \tag{21}$$

where $k = 1, 2, 3$, $g_1(z) = z$, $g_2(z) = \sin(z)$, and $g_3(z) = \tan(z)$ are three functions.

With regard to the cross-entropies of SVNNs developed by Ye [43], the reduced cross-entropy measures are not indefinite numbers when A and B are crisp numbers. Thus, the novel cross-entropy measures for SVNNs overcome the defects in Ref. [43]. The reduced cross-entropy measures also conquer the shortcomings in the modified cross-entropy measures defined by Wu et al. [36]. Using the data in Example 2, we can obtain that $CE_1(\alpha_1, \alpha_3) = CE_1(\alpha_2, \alpha_3) = 0.4866$, $CE_2(\alpha_1, \alpha_3) = CE_2(\alpha_2, \alpha_3) = 0.4737$, and $CE_3(\alpha_1, \alpha_3) = CE_3(\alpha_2, \alpha_3) = 0.5143$, which conforms to the theory. Therefore, the reduced cross-entropy measures are superior to the modified cross-entropies developed by Wu et al. [36].

Regarding the cross-entropies for HFNs developed by Xu and Xia [31], the proposed cross-entropy measures for MVNNs do not need to add a certain number into MVNNs to equalize the lengths of different MVNNs, which can avoid different results when different numbers are added. Therefore, the cross-entropy measures for MVNNs in this paper do not have the defects in the cross-entropy measures for HFNs.

Example 6. Let $A = \langle \{0.7\}, \{0.2, 0.3\}, \{0.1, 0.2\} \rangle$, $B = \langle \{0.4, 0.5, 0.6\}, \{0.2\}, \{0.3, 0.4\} \rangle$, and $C = \langle \{0.3, 0.4\}, \{0.2, 0.3\}, \{0.4, 0.5\} \rangle$, then based on the cross-entropy measures denoted by Eq. (20), we can obtain the following results:

$CE_1(A, D) = 0.3130$, $CE_1(B, D) = 0.2010$, $CE_1(C, D) = 0.3845$;
$CE_2(A, D) = 0.3071$, $CE_2(B, D) = 0.1990$, $CE_2(C, D) = 0.3756$;
$CE_3(A, D) = 0.3257$, $CE_3(B, D) = 0.2050$, $CE_3(C, D) = 0.4036$.

4. An extended VIKOR method for MCDM problems based on entropies and cross-entropies of MVNNs

In this section, an entropy weight method is constructed to identify the criteria weight vector. Then, we propose an extended VIKOR approach, according to the cross-entropies and weight vector, to solve MCDM problems when the information about criteria weights is completely unknown.

4.1. *The determination of criteria weights based on entropies of MVNNs*

Information about criteria weights is often completely unknown during the decision-making process. There are many reasons that this information is unknown, including time pressure, limited resources, and experts' limited professional knowledge [31]. The entropy weight method is an important way to obtain the criteria weight vector when their information is completely unknown [33]. However, there is no research related to the entropy method within multi-valued neutrosophic environments. Therefore, in this subsection, we investigate an extension of the entropy method under multi-valued neutrosophic environments to identify the criteria weight vector when the weights information is completely unknown.

For convenience, assume that there are m alternatives A_i $(i = 1, 2, ..., m)$ with regard to n criteria C_j $(j = 1, 2, ..., n)$. Subsequently, the weight vector of decision criteria is assumed to be $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$, where $0 \leq \omega_j \leq 1$ and $\sum_{j=1}^{n} \omega_j = 1$. Then the criteria weights can be obtained by the following equation:

$$\omega_j = \frac{1 - \frac{1}{m}\sum_{i=1}^{m} E_{ij}}{n - \frac{1}{m}\sum_{j=1}^{n}\sum_{i=1}^{m} E_{ij}}, \ i = 1, 2, ..., m, \ j = 1, 2, ..., n, \tag{22}$$

where E_{ij} is the entropy for the alternative A_i under the criterion C_j and it can be calculated by any one of the entropy measure formulas denoted by Eqs. (16)-(19).

4.2. The extended VIKOR method for MCDM problems

In this subsection, we establish an extended VIKOR approach on the basis of cross-entropies and entropies with multi-valued neutrosophic information.

For an MCDM problem with multi-valued neutrosophic information, suppose that there is a group of t DMs D_k $(k = 1, 2, ..., t)$ who have to evaluate m alternatives A_i $(i = 1, 2, ..., m)$ with respect to n criteria C_j $(j = 1, 2, ..., n)$. Assume that the weight vector of the decision criteria is $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$, where $0 \leq \omega_j \leq 1$ and $\sum_{j=1}^{n} \omega_j = 1$. Let $B = \left[b_{ij}\right]_{m \times n}$ be the decision matrix, and $b_{ij} = \left\langle \tilde{T}_{ij}, \tilde{I}_{ij}, \tilde{F}_{ij} \right\rangle = \left\langle \left\{\gamma_{ij}^1, \gamma_{ij}^2, ..., \gamma_{ij}^t\right\}, \left\{\eta_{ij}^1, \eta_{ij}^2, ..., \eta_{ij}^t\right\}, \left\{\xi_{ij}^1, \xi_{ij}^2, ..., \xi_{ij}^t\right\} \right\rangle$ is evaluation information for the alternative A_i regarding the criterion C_j by all DMs in the form of MVNNs.

The main procedures of the improved VIKOR method based on entropies and cross-entropies are studied in the remainder of this subsection.

Step 1. Normalize the decision matrix.

Since there are usually both cost and benefit criteria in an MCDM problem, the decision matrix should be normalized. b_{ij} does not need to be normalized when C_j is a benefit criterion while b_{ij} should be normalized to its complement when C_j is a cost criterion. The formula of the normalized transformation is defined as:

$$u_{ij} = \begin{cases} b_{ij}, & \text{if } C_j \text{ is a benefit criterion} \\ b_{ij}^c, & \text{if } C_j \text{ is a cost criterion} \end{cases}. \tag{23}$$

Step 2. Determine the weight vector.

Using Eq. (22) to derive the weight vector of the decision criteria $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$.

Step 3. Determine the best alternative and the worst alternative.

In this paper, the absolute positive ideal point and absolute negative ideal point are regarded as the best and the worst alternatives.

Step 4. Compute the values S_i and R_i.

According to the cross-entropies of MVNNs in Definition 14, S_i and R_i can be calculated using the following equations:

$$S_i = \sum_{j=1}^{n} \omega_j \frac{CE\left(f_j^*, u_{ij}\right)}{CE\left(f_j^*, f_j^-\right)} \text{ for } i = 1, 2, ..., m, \tag{24}$$

$$R_i = \max_j \omega_j \frac{CE\left(f_j^*, u_{ij}\right)}{CE\left(f_j^*, f_j^-\right)} \text{ for } i = 1, 2, ..., m, \tag{25}$$

where ω_j is the weight of criterion C_j for $j = 1, 2, ..., n$.

Table 1: Decision criteria and their descriptions

Criterion	Description
Flexibility	Flexibility is the resource allocation and de-allocation ability, including data exchange rate and system responsiveness.
Stability	Stability is the service control and monitor ability, including systematic error count and service state measure.
Security	Security is the data projection and management ability, including data permission control and data confidentiality.

Step 5. Calculate the values Q_i.

We can obtain Q_i for $i = 1, 2, ..., m$ by

$$Q_i = v\frac{S_i - S^*}{S^- - S^*} + (1 - v)\frac{R_i - R^*}{R^- - R^*} \text{ for } i = 1, 2, ..., m, \tag{26}$$

where $S^* = \min_i S_i$, $S^- = \max_i S_i$, $R^* = \min_i R_i$, and $R^- = \max_i R_i$. Particularly, v is the group utility weight, while $(1 - v)$ is the individual regret weight. Commonly, $v = 0.5$ [39].

Step 6. Sort the alternatives.

In this step, we rank the alternatives by the values S, R, and Q in decreasing order. It should be noted that the results are three ranking lists.

Step 7. Determine the compromise solution. The alternative A', which is ranked the optimal (minimum) by the measure Q, is the compromise solution if it satisfies the following two conditions:

Condition 1: Acceptable advantage: $Q(A'') - Q(A') \geq 1/(m-1)$, where A'' is the alternative which is ranked the second by Q and m is the amount of the alternatives.

Condition 2: Acceptable stability in decision-making: If A' is also ranked the optimal in S and R, it can be regarded that A' is stable in the process of decision-making, which could be voted on through majority-rule voting (when $v > 0.5$ is needed), or by consensus ($v \approx 0.5$), or with veto ($v < 0.5$). It should be noted that when either of the conditions cannot be met, a group of alternatives are obtained as a compromise solution based on the following rules:

Rule 1: If only Condition 1 is not met, all the alternatives $A', A'', ..., A^{(a)}$ are regarded as a compromise solution and $A^{(a)}$ is determined by $Q\left(A^{(a)}\right) - Q(A') < 1/(m-1)$.

Rule 2: If only Condition 2 is not met, both of the alternatives A' and A'' are considered as the a compromise solution.

Based on the above analyses, we can give a conceptual model of the new VIKOR method on the basis of entropy and cross-entropy measures in Figure 1.

5. Cloud service reliability assessment using the novel VIKOR method

In this section, the extended VIKOR method is applied to assess cloud service reliability and the applicability of the proposed method is demonstrated.

With the rapid development of information technology, more and more firms and individuals can access cloud computing and storage services through the internet without owning the actual technology infrastructures. However, until now, the cloud services provided by various providers has been varied reliabilities, which often affects the degree of satisfaction experienced by companies and individuals. Therefore, assessing cloud service reliability is an important issue for consumers. During the process of assessment, alternatives are evaluated by several experts on certain criteria. MVNNs are effective when they are used to denote the fuzziness and hesitancy in the evaluation process.

5.1. information collection

Three criteria in Table 1 are employed in cloud service reliability assessment.

Generally, the criteria in Table 1 are evaluated by a group of experts in cloud service reliability assessments. Suppose there are four cloud service providers (A_1, A_2, A_3, A_4) needing to be assessed regarding

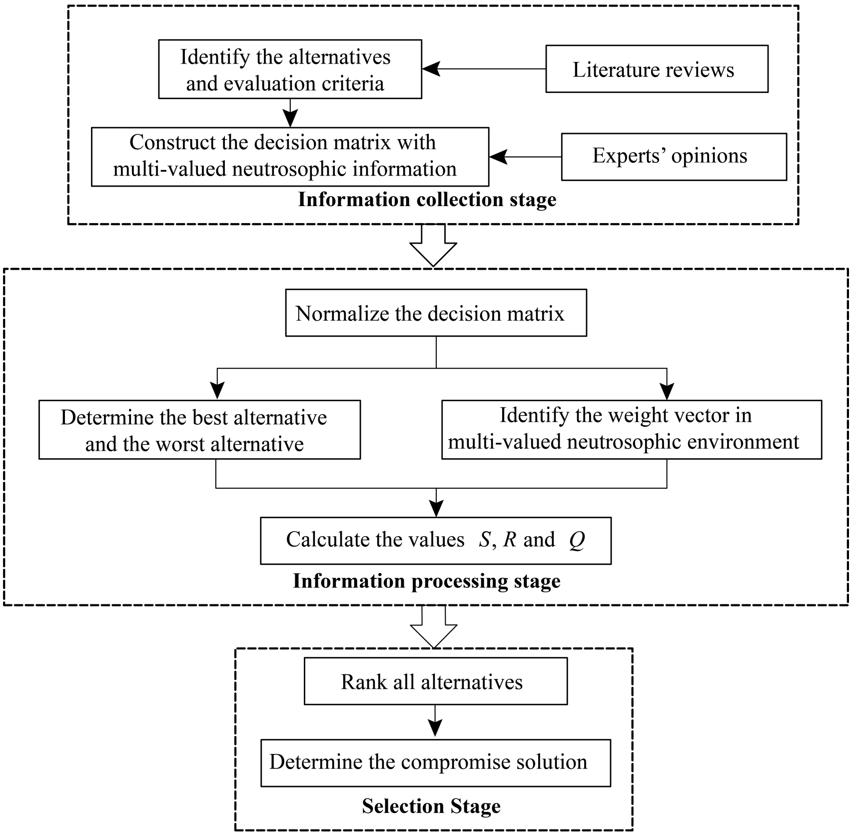

Figure 1: A flow chart of the new VIKOR method

Table 2: Questionnaire for expert e_k

Number	Criterion	Attitude	Evaluation values			
			A_1	A_2	A_3	A_4
C_1	Flexibility	Satisfaction	γ_{11}^k	γ_{21}^k	γ_{31}^k	γ_{41}^k
		Neutrality	η_{11}^k	η_{21}^k	η_{31}^k	η_{41}^k
		Dissatisfaction	ξ_{11}^k	ξ_{21}^k	ξ_{31}^k	ξ_{41}^k
C_2	Stability	Satisfaction	γ_{12}^k	γ_{22}^k	γ_{32}^k	γ_{42}^k
		Neutrality	η_{12}^k	η_{22}^k	η_{32}^k	η_{42}^k
		Dissatisfaction	ξ_{12}^k	ξ_{22}^k	ξ_{32}^k	ξ_{42}^k
C_3	Security	Satisfaction	γ_{13}^k	γ_{23}^k	γ_{33}^k	γ_{43}^k
		Neutrality	η_{13}^k	η_{23}^k	η_{33}^k	η_{43}^k
		Dissatisfaction	ξ_{13}^k	ξ_{23}^k	ξ_{33}^k	ξ_{43}^k

three criteria (C_1, C_2, C_3) by a group of three experts (e_1, e_2, e_3). To obtain the evaluation of the experts, Table 2 is designed and given to the expert e_k where $k = 1, 2, 3$. Experts need to evaluate the cloud service providers with respect to each criterion with a number in the set $\{0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5,\ 0.6,\ 0.7,\ 0.8,\ 0.9,\ 1.0\}$. The greater the evaluation value is, the higher the degree of satisfied, neutral, or dissatisfied attitude is. Subsequently, we transform the evaluation information of experts into a decision matrix with multi-valued neutrosophic information. For example, the experts evaluate the cloud service provider A_1 regarding the criterion C_1, and the assessment information is presented in Table 3. In this situation, the comprehensive evaluation information can be expressed by an MVNN $\langle \{0.6, 0.7, 0.8\}, \{0.1\}, \{0.1, 0.2\} \rangle$. Likewise, all the evaluation information of the alternatives with regard to the criteria from the three experts can be expressed in terms of multi-valued neutrosophic information, which is shown in Table 4.

Table 3: Evaluation information of the cloud service provider A_1 regarding the criterion C_1

Experts	e_1	e_2	e_3
Satisfaction	0.6	0.8	0.7
Neutrality	0.1	0.1	0.1
Dissatisfaction	0.2	0.1	0.2

Table 4: Decision-making matrix with multi-valued neutrosophic information

	C_1	C_2	C_3
A_1	$\langle\{0.6,0.7,0.8\},\{0.1\},\{0.1,0.2\}\rangle$	$\langle\{0.6,0.7\},\{0.2,0.3\},\{0.2,0.3\}\rangle$	$\langle\{0.3,0.4\},\{0.2,0.3\},\{0.4,0.5\}\rangle$
A_2	$\langle\{0.8\},\{0.2,0.3\},\{0.1,0.2\}\rangle$	$\langle\{0.6,0.7\},\{0.1\},\{0.3\}\rangle$	$\langle\{0.7,0.8\},\{0.1,0.2\},\{0.2,0.3\}\rangle$
A_3	$\langle\{0.9\},\{0.1,0.2\},\{0.2,0.3\}\rangle$	$\langle\{0.7\},\{0.3\},\{0.4\}\rangle$	$\langle\{0.6,0.7\},\{0.2\},\{0.3\}\rangle$
A_4	$\langle\{0.7\},\{0.1\},\{0.1,0.3\}\rangle$	$\langle\{0.7,0.8\},\{0.3\},\{0.3\}\rangle$	$\langle\{0.4,0.6\},\{0.3\},\{0.1,0.2,0.3\}\rangle$

5.2. Decision-making procedures based on MVNNs

In this subsection, the main procedures of an extended VIKOR method are introduced and the compromise solution is obtained under multi-valued neutrosophic environments.

Step 1. Normalize the decision matrix.

Since every criterion is a benefit one, we do not need to normalize the decision-making matrix.

Step 2. Determine the weight vector.

Using Eqs. (19) and (22), the weight vector of criteria can be calculated as $\omega = (0.41, 0.30, 0.29)$.

Step 3. Determine the best alternative and the worst alternative.

According to the absolute positive ideal point and absolute negative ideal point, the best and the worst alternatives can be defined as:

$$f_j^* = \{\langle 1,0,0\rangle, \langle 1,0,0\rangle, \langle 1,0,0\rangle\}; \tag{27}$$

$$f_j^- = \{\langle 0,1,1\rangle, \langle 0,1,1\rangle, \langle 0,1,1\rangle\}. \tag{28}$$

Step 4. Compute S_i and R_i.

Based on the first cross-entropy measure for MVNNs denoted by $CE_1(A,B)$, S_i and R_i can be obtained:

$S_1 = 0.1032$, $S_2 = 0.0507$, $S_3 = 0.0666$, $S_4 = 0.0771$;

$R_1 = 0.0627$, $R_2 = 0.0209$, $R_3 = 0.0316$, $R_4 = 0.0357$.

Step 5. Calculate Q_i.

According to S_i and R_i calculated above, Q_i can be obtained as follows:

$Q_1 = 1.0000$, $Q_2 = 0$, $Q_3 = 0.2790$, $Q_4 = 0.4289$.

Step 6. Sort the alternatives.

We can get three ranking lists of alternatives by listing the values S, R, and Q in decreasing order; the ranking results are shown in Table 5.

Step 7. Determine the compromise solution.

Since Condition 2 is satisfied by the results and Condition 1 is not met, a group of alternatives A_2 and A_3 is the compromise solution in this cloud service assessment problem.

5.3. Analysis and discussion

To further demonstrate the effectiveness of the new VIKOR method, we first analyze how the results change according to different entropy and cross-entropy measures for MVNNs. Thereafter, a comparative

Table 5: Ranking results and the compromise solution for all alternatives

	A_1	A_2	A_3	A_4	Ranking orders	Compromise solution
S	0.1032	0.0507	0.0666	0.0771	$A_2 > A_3 > A_4 > A_1$	
R	0.0627	0.0209	0.0316	0.0357	$A_2 > A_3 > A_4 > A_1$	A_2, A_3
$Q\,(v = 0.5)$	1.0000	0	0.2790	0.4289	$A_2 > A_3 > A_4 > A_1$	

Table 6: The results based on cross-entropy CE_1

	Ranking list-S	Ranking list-R	Q values				Ranking list-Q	Results
			A_1	A_2	A_3	A_4		
E_{f1}	$A_2 \succ A_3 \succ A_4 \succ A_1$	$A_2 \succ A_3 \succ A_4 \succ A_1$	1	0	0.2822	0.4271	$A_2 \succ A_3 \succ A_4 \succ A_1$	A_2, A_3
E_{f2}	$A_2 \succ A_3 \succ A_4 \succ A_1$	$A_2 \succ A_3 \succ A_4 \succ A_1$	1	0	0.2795	0.4286	$A_2 \succ A_3 \succ A_4 \succ A_1$	A_2, A_3
E_{f3}	$A_2 \succ A_3 \succ A_4 \succ A_1$	$A_2 \succ A_3 \succ A_4 \succ A_1$	1	0	0.2731	0.4321	$A_2 \succ A_3 \succ A_4 \succ A_1$	A_2, A_3
E_{f4}	$A_2 \succ A_3 \succ A_4 \succ A_1$	$A_2 \succ A_3 \succ A_4 \succ A_1$	1	0	0.2790	0.4289	$A_2 \succ A_3 \succ A_4 \succ A_1$	A_2, A_3

Table 7: The results based on cross-entropy CE_2

	Ranking list-S	Ranking list-R	Q values				Ranking list-Q	Results
			A_1	A_2	A_3	A_4		
E_{f1}	$A_2 \succ A_3 \succ A_4 \succ A_1$	$A_2 \succ A_3 \succ A_4 \succ A_1$	1	0	0.2953	0.4345	$A_2 \succ A_3 \succ A_4 \succ A_1$	A_2, A_3
E_{f2}	$A_2 \succ A_3 \succ A_4 \succ A_1$	$A_2 \succ A_3 \succ A_4 \succ A_1$	1	0	0.2924	0.4361	$A_2 \succ A_3 \succ A_4 \succ A_1$	A_2, A_3
E_{f3}	$A_2 \succ A_3 \succ A_4 \succ A_1$	$A_2 \succ A_3 \succ A_4 \succ A_1$	1	0	0.2858	0.4397	$A_2 \succ A_3 \succ A_4 \succ A_1$	A_2, A_3
E_{f4}	$A_2 \succ A_3 \succ A_4 \succ A_1$	$A_2 \succ A_3 \succ A_4 \succ A_1$	1	0	0.2919	0.4364	$A_2 \succ A_3 \succ A_4 \succ A_1$	A_2, A_3

study is conducted with other MCDM methods in multi-valued neutrosophic environment.

5.3.1. Sensitive analysis

Suppose $\rho = 2$ and $v = 0.5$. The results, which are calculated by the proposed method based on different entropies and cross-entropies, are provided in Table 6-Table 8. It can be seen that A_2 is first and A_1 is last in the ranking list of the values S, R, and Q, no matter what the entropies and cross-entropies are. Simultaneously, A_2 and A_3 are the compromise solution for the alternatives regardless of the entropy and cross-entropy measures utilized. With the comprehensive analyses above, the results calculated by different cross-entropy and entropy measures have great stability. Therefore, the entropies and cross-entropies of MVNNs are valid and valuable.

5.3.2. Comparative analysis

In order to verify the effectiveness and the advantages of the proposed method, a comparative analysis is made with other actual multi-valued neutrosophic MCDM methods in the rest of this subsection.

A brief introduction of existing methods with MVNNs is presented in Table 9. Since criteria weights are all artificially specified in these MCDM methods, we utilize the extended entropy weight method in this study to determine criteria weights in the above methods for comparative convenience. The results obtained by the different methods are given in Table 10. Moreover, Figure 2 is provided to visualize the comparative results.

As we can see in Table 10 and Figure 2, alternative A_3 ranks the first in Biswas et al.'s method [25] and A_2 in the other existing methods [11, 26-28]. Moreover, the proposed method derives a compromise solution which includes both A_2 and A_3. Thereafter, the alternative A_1 is the worst one in all the multi-valued neutrosophic methods. In addition, the ranking orders by the proposed method are the same as the results by methods in Refs. [11, 26, 27], while they are inconsistent with the ranking results calculated by methods in Refs. [25, 28].

The decision-making results obtained by extant multi-valued neutrosophic MCDM methods [11, 25-28] are always one alternative merely, which is A_2 or A_3. However, the proposed method can derive a compromise solution including both A_2 and A_3. In some practical circumstances, the proposed method

Table 8: The results based on cross-entropy CE_3

	Ranking list-S	Ranking list-R	Q values				Ranking list-Q	Results
			A_1	A_2	A_3	A_4		
E_{f1}	$A_2 \succ A_3 \succ A_4 \succ A_1$	$A_2 \succ A_3 \succ A_4 \succ A_1$	1	0	0.2549	0.4108	$A_2 \succ A_3 \succ A_4 \succ A_1$	A_2, A_3
E_{f2}	$A_2 \succ A_3 \succ A_4 \succ A_1$	$A_2 \succ A_3 \succ A_4 \succ A_1$	1	0	0.2524	0.4121	$A_2 \succ A_3 \succ A_4 \succ A_1$	A_2, A_3
E_{f3}	$A_2 \succ A_3 \succ A_4 \succ A_1$	$A_2 \succ A_3 \succ A_4 \succ A_1$	1	0	0.2469	0.4152	$A_2 \succ A_3 \succ A_4 \succ A_1$	A_2, A_3
E_{f4}	$A_2 \succ A_3 \succ A_4 \succ A_1$	$A_2 \succ A_3 \succ A_4 \succ A_1$	1	0	0.2520	0.4124	$A_2 \succ A_3 \succ A_4 \succ A_1$	A_2, A_3

Table 9: The comparative analysis with existing method based on distances

Literature	Method
Biswas et al. [25]	Weighted generalized distance-based method
Şahin and Liu [27]	Weighted Hamming distance-based method
Biswas et al. [26]	Normalized Hamming distance-based GRA method
Wang and Li [11]	Hamming distance-based TODIM method
Liu and Zhang [28]	Hamming distance-based VIKOR method

Table 10: The ranking results of the six methods

Methods	Ranking results	Compromise solution
Biswas et al.'s method ($\lambda = 1$) [25]	$A_3 \succ A_2 \succ A_4 \succ A_1$	None
Şahin and Liu's method [27]	$A_2 \succ A_3 \succ A_4 \succ A_1$	None
Biswas et al.'s method ($\rho = 1$) [26]	$A_2 \succ A_3 \succ A_4 \succ A_1$	None
Wang and Li's method ($t = 1$) [11]	$A_2 \succ A_3 \succ A_4 \succ A_1$	None
Liu and Zhang's method ($v = 0.5$) [28]	$A_2 \succ A_4 \succ A_3 \succ A_1$	None
The proposed method ($v = 0.5$)	$A_2 \succ A_3 \succ A_4 \succ A_1$	A_2, A_3

takes maximum "group utility" and minimum "individual regret" into accounts at the same time. Therefore, the compromise solution can be more easily accepted by decision makers. To some extent, the comparative results prove the effectiveness and practicability of the proposed methods in this paper.

Simultaneously, though the ranking orders of proposed method are consistent with that calculated by the multi-valued neutrosophic methods in Refs. [11, 26, 27], as well as have only slight difference from that with other extant methods in Refs. [25, 28], the computational processes of these methods have several dissimilarities:

(1) The measurements employed in the proposed method are different from that utilized in existing MCDM methods. The distances investigated in Refs. [11, 25-28] may cause loss and distortion of the preference information during the process of decision-making. For instance, let $A = \langle \{0.6\}, \{0.1, 0.3\}, \{0.5\} \rangle$, $B = \langle \{0.8\}, \{0.1, 0.3\}, \{0.5\} \rangle$, and $C = \langle \{0.5\}, \{0.1, 0.3\}, \{0.4\} \rangle$ be three MVNNs, then the comparative results are obtained in Table 11. As is seen to us, whichever distances in Refs. [11, 25-28] are used, the same results $d(A, B) = d(A, C)$ are computed. Subsequently, if we use the first proposed cross-entropy in this paper, we derive $CE_1(A, B) \neq CE_1(A, C)$. Thus, the distances developed in Refs. [11, 25-28] cannot distinguish two

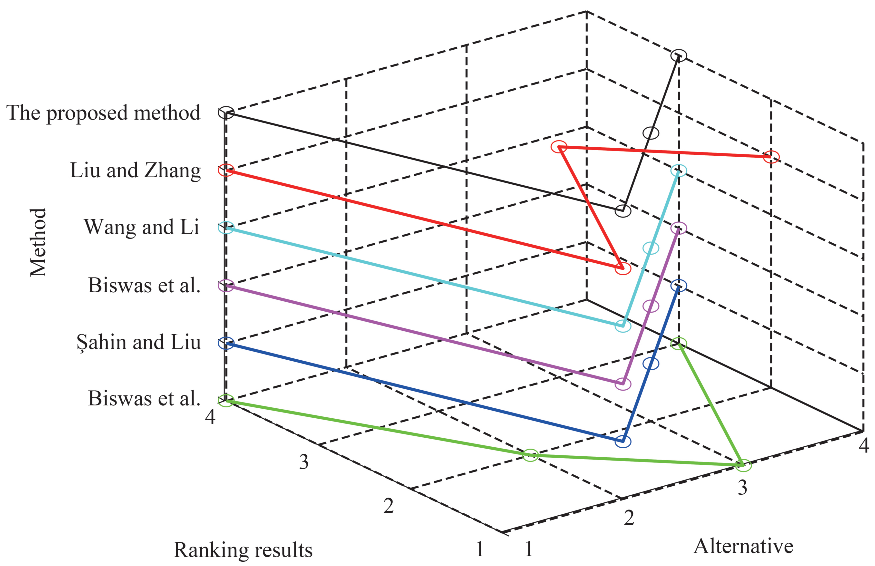

Figure 2: The comparative analyses with extant method

Table 11: The comparative analysis between distances and cross-entropy

Literatures	Measurements	$A \to B$	$A \to C$	Results
Biswas et al. ($\lambda = 1$) [25]	Weighted generalized distance	0.1000	0.1000	Equality
Şahin and Liu [27]	Weighted Hamming distance	0.1000	0.1000	Equality
Biswas et al. [26]	Normalized Hamming distance	0.0667	0.0667	Equality
Wang and Li [11]	Hamming distance	0.2000	0.2000	Equality
Liu and Zhang [28]	Hamming distance	0.1000	0.1000	Equality
This study	Cross-entropy	0.0401	0.0200	Inequality

MVNNs completely and accurately. By contrast, the cross-entropies defined in this study can differentiate one MVNN from another easily. In view of this, the methods based on cross-entropies can keep the DMs' preference information as completely as possible in decision-making processes. Further, the results acquired by the proposed method are more reasonable than extant methods on the basis of distances under multi-valued neutrosophic environment.

(2) The proposed multi-valued neutrosophic VIKOR method can rank alternatives better than existing methods, and get a compromise solution with one or more alternatives. The MCDM methods in Refs. [11, 25-27] only consider the similarities or grey relational coefficients to ideal solution. In contrast with these methods, this study takes maximum "group utility" and minimum "individual regret" into considerations under conflicting criteria, which are often involved during the evaluation process in some real-life cases. Thus, the proposed method is more likely to be applied in practical circumstances. Subsequently, Liu and Zhang [28] also extended VIKOR method into multi-valued neutrosophic environment. However, only the alternative A_2 is selected as the optimal alternative while a compromise solution is provided with both A_2 and A_3 in this study. This reveals the cross-entropies are more superior than distance defined in Liu and Zhang [28] from another perspective.

(3) In addition, the determination approaches of criteria weights are distinct between the proposed method and the existing MCDM methods. The criteria weights are all designated in the Refs. [11, 25-28]. However, in most actual situations, experts are hard to give an exact weight vector in view of the complexities of decision-making environment. The proposed method based on entropy measures can calculate criteria weights when the weight information is completely unknown. Accordingly, the proposed method can have useful and efficient applications for real life.

In summary, the proposed method employs cross-entropy measures so that the initial information can be kept and used well. The results obtained based on cross-entropies may be more accurate than the results derived by methods that use distances. Meanwhile, the proposed VIKOR method on the basis of entropy weight method can get an appropriate result validly and objectively in real-life decision-making activities.

6. Conclusions

This paper presents a multi-valued neutrosophic MCDM method for cloud service reliability assessment problems. A general framework of entropy measures is proposed by considering the weakness of entropies of SVNNs and HFNs. Meanwhile, a family of cross-entropy measures is developed in the analogous way. Further, an extended VIKOR method is proposed on the basis of entropy and cross-entropy measures. Different from the classical VIKOR method, the proposed VIKOR method can consider the evaluation information adequately under conflicting criteria with absolutely unknown weight information. Moreover, the influences of different entropy and cross-entropy formulas are discussed, and a comparative analysis between the proposed method and existing methods with MVNNs is also conducted.

The proposed approach can depict the evaluation information effectively in cloud service reliability assessment with MVNNs. What's more, compared with the researches using general distance measures, this paper can keep the multi-valued neutrosophic information unbroken and the results may be more accurate than in previous methods. Also, this study enables DMs to select the compromise solution when the information about criteria weights is completely unknown.

In future studies, at the beginning, multi-valued neutrosophic entropy and cross-entropy measures can be extended to other neutrosophic environments, such as n-valued refined neutrosophic environments, interval-valued neutrosophic environments, and n-valued interval neutrosophic environments. Then, many other criteria should be comprehensively considered in cloud service reliability assessments to make the outcomes more reasonable and applicable. Finally, the novel extended VIKOR method based on entropy and cross-entropy measures can be employed in other fields, such as medical diagnosis, pattern recognition, decision-making problems, and third-party logistic evaluation.

Acknowledgements

The authors would like to thank the editors and anonymous reviewers for their helpful comments that improved the paper. This work was supported by the National Natural Science Foundation of China (No. 71571193).

Appendix A

Proof of Proposition 1

It can be easily verified that the entropy of MVNNs satisfies principles (1) and (2) in Definition 11. Next, the proof of principles (3) and (4) is given below.

(3) Assume $A \in MN$, then $A^c = \left\langle \cup_{\xi \in \tilde{F}_A} \{\xi\}, \cup_{\eta \in \tilde{I}_A} \{1-\eta\}, \cup_{\gamma \in \tilde{T}_A} \{\gamma\} \right\rangle$ can be acquired based on Definition 4. The two variables $\Delta^i_{A^c}$ and $\sigma^j_{A^c}$ with regard to A^c are calculated below:

$\Delta^i_{A^c} = \cup_{\xi_A \in \tilde{T}_A, \gamma_A \in \tilde{F}_A} \left|\xi_A - \gamma_A\right| = \cup_{\gamma_A \in \tilde{T}_A, \xi_A \in \tilde{F}_A} \left|\gamma_A - \xi_A\right| = \Delta^i_A$ for all $i = 1, 2, ..., l_{\tilde{T}} \cdot l_{\tilde{F}}$,

$\sigma^j_{A^c} = \cup_{\eta \in \tilde{I}_A} \left|\eta_{A^c} - \eta_A\right| = \cup_{\eta \in \tilde{I}_A} \left|\eta_A - \eta_{A^c}\right| = \sigma^j_A$ for all $j = 1, 2, ..., l_{\tilde{I}}$.

The variables $\Delta^i_{A^c}$ and $\sigma^j_{A^c}$ with respect to A^c are equal to Δ^i_A and σ^j_A regarding A. Thus, the entropy $E(A)$, which is based on variables Δ^i_A and σ^j_A, is identical to $E(A^c)$.

(4) Obviously, Eq. (13) satisfies principle (4) in Definition 11 if we can demonstrate that the function $1 - \frac{\Delta^i_A + \sigma^j_A}{2}$, where $0 \le \Delta^i_A, \sigma^j_A \le 1$, decreases monotonically with regard to Δ^i_A and σ^j_A, separately.

Assume $g(x, y) = 1 - \frac{x+y}{2}$, where $0 \le x, y \le 1$. Then we take the partial derivative with x and y, respectively. $\frac{\partial g(x,y)}{\partial x} = \frac{\partial g(x,y)}{\partial y} = -\frac{1}{2} < 0$ can be derived. That is, $g(x, y)$ decrease monotonically regarding x and y, respectively. Therefore, Eq. (13) satisfies principle (4) in Definition 11.

Appendix B

Proof of Theorem 1

Assume that $E_f(A)$ meets axiomatic conditions (1)-(4) defined in Definition 11. We can demonstrate that the function f has the properties (1)-(3) defined in Theorem 1 below.

(1) Suppose that $f(x, y) = 0$ where $0 \le x, y \le 1$, then for the MVNN A with $\Delta^i_A = x$ for $i = 1, 2, ..., l_{\tilde{T}} \cdot l_{\tilde{F}}$ and $\sigma^j_A = y$ for $j = 1, 2, ..., l_{\tilde{I}}$, we have $E_f(A) = \frac{1}{l_{\tilde{T}} \cdot l_{\tilde{I}} \cdot l_{\tilde{F}}} \sum_{i=1}^{l_{\tilde{T}} \cdot l_{\tilde{F}}} \sum_{j=1}^{l_{\tilde{I}}} f\left(\Delta^i_A, \sigma^j_A\right) = 0$. By condition (1) defined in Definition 11, because $E_f(A) = 0$, we obtain $\Delta^i_A = 1$ for $i = 1, 2, ..., l_{\tilde{T}} \cdot l_{\tilde{F}}$ and $\sigma^j_A = 1$ for $j = 1, 2, ..., l_{\tilde{I}}$, i.e. $x = 1$ and $y = 1$.

(2) Suppose that $f(x, y) = 1$ where $0 \le x, y \le 1$, then for the MVNN A with $\Delta^i_A = x$ for $i = 1, 2, ..., l_{\tilde{T}} \cdot l_{\tilde{F}}$ and $\sigma^j_A = y$ for $j = 1, 2, ..., l_{\tilde{I}}$, we have $E_f(A) = \frac{1}{l_{\tilde{T}} \cdot l_{\tilde{I}} \cdot l_{\tilde{F}}} \sum_{i=1}^{l_{\tilde{T}} \cdot l_{\tilde{F}}} \sum_{j=1}^{l_{\tilde{I}}} f\left(\Delta^i_A, \sigma^j_A\right) = 1$. By condition (2) defined in Definition 11, since $E_f(A) = 1$, we obtain $\Delta^i_A = 0$ for $i = 1, 2, ..., l_{\tilde{T}} \cdot l_{\tilde{F}}$ and $\sigma^j_A = 0$ for $j = 1, 2, ..., l_{\tilde{I}}$, i.e. $x = 0$ and $y = 0$.

(3) Suppose there is $x_1 \le x_2$ and $f(x_1, y) \ge f(x_2, y)$, where $0 \le x_1, x_2, y \le 1$. Considering the MVNN A_1 with $\Delta^i_{A_1} = x_1$ for $i = 1, 2, ..., l_{\tilde{T}} \cdot l_{\tilde{F}}$ and $\sigma^j_{A_1} = y$ for $j = 1, 2, ..., l_{\tilde{I}}$, and the MVNN A_1 with $\Delta^i_{A_2} = x_2$ for $i = 1, 2, ..., l_{\tilde{T}} \cdot l_{\tilde{F}}$ and $\sigma^j_{A_2} = y$ for $j = 1, 2, ..., l_{\tilde{I}}$. We obtain: $E_f(A_1) = \frac{1}{l_{\tilde{T}} \cdot l_{\tilde{I}} \cdot l_{\tilde{F}}} \sum_{i=1}^{l_{\tilde{T}} \cdot l_{\tilde{F}}} \sum_{j=1}^{l_{\tilde{I}}} f\left(\Delta^i_{A_1}, \sigma^j_{A_1}\right) = f(x_1, y) \ge f(x_2, y) = \frac{1}{l_{\tilde{T}} \cdot l_{\tilde{I}} \cdot l_{\tilde{F}}} \sum_{i=1}^{l_{\tilde{T}} \cdot l_{\tilde{F}}} \sum_{j=1}^{l_{\tilde{I}}} f\left(\Delta^i_{A_2}, \sigma^j_{A_2}\right) = E_f(A_2)$, which corresponds to condition (4) defined in Definition 11. Therefore, f decreases monotonically with respect to x. Similarly, we can also prove that f decreases

monotonically with respect to y.

Appendix C

Proof of Proposition 2

It can be easily demonstrated that the axiomatic conditions (2) and (3) hold. The evidentiary process for conditions (1) and (4) is shown below.

(1) Let us consider the function $f(x,y) = (x-y)\ln\frac{2+x-y}{2-x+y}$ where $x, y \in [0,1]$.

If $x \geq y$, then $x - y \geq 0$. We obtain $2 \leq 2 + x - y \leq 3$ and $1 \leq 2 - x + y \leq 2$. Therefore, $1 \leq \frac{2+x-y}{2-x+y} \leq 3$. That is, $0 \leq \ln\frac{2+x-y}{2-x+y} \leq \ln 3$, since $x - y \geq 0$, $f(x,y) = (x-y)\ln\frac{2+x-y}{2-x+y} \geq 0$. We can similarly prove that $f(x,y) \geq 0$ when $x \leq y$. As a result, the function $f(x,y) \geq 0$ always holds. Moreover, the equality $f(x,y) = 0$ holds if and only if $x = y$.

For Eq. (20), since $\forall \gamma_A, \gamma_B, \eta_A, \eta_B, \xi_A, \xi_B \in [0,1]$ where $\gamma_A \in \tilde{T}_A$, $\gamma_B \in \tilde{T}_B$, $\eta_A \in \tilde{I}_A$, $\eta_B \in \tilde{I}_B$, $\xi_A \in \tilde{F}_A$, and $\xi_B \in \tilde{F}_B$, $CE_1(A,B) \geq 0$ holds. $CE_1(A,B) = 0$ holds if and only if $\tilde{T}_A = \tilde{T}_B$, $\tilde{I}_A = \tilde{I}_B$, and $\tilde{F}_A = \tilde{F}_B$, namely $A = B$.

The proof above can demonstrate that cross-entropy measures $CE_2(A,B)$ and $CE_3(A,B)$ also meet condition (1).

(4) When $A \leq B \leq C$, by Definition 5, the inequalities $\gamma_A \leq \gamma_B \leq \gamma_C$, $\eta_A \geq \eta_B \geq \eta_C$, and $\xi_A \geq \xi_B \geq \xi_C$ hold for every $\gamma_A \in \tilde{T}_A$, $\gamma_B \in \tilde{T}_B$, $\gamma_C \in \tilde{T}_C$, $\eta_A \in \tilde{I}_A$, $\eta_B \in \tilde{I}_B$, $\eta_C \in \tilde{I}_C$, $\xi_A \in \tilde{F}_A$, $\xi_B \in \tilde{F}_B$, and $\xi_C \in \tilde{F}_C$. Then the following inequalities can be obtained:

$$\begin{aligned}
&\gamma_A \leq \gamma_B \leq \gamma_C\\
&\Rightarrow 0 \leq \gamma_B - \gamma_A \leq \gamma_C - \gamma_A\\
&\Rightarrow 2 \leq 2 + \gamma_B - \gamma_A \leq 2 + \gamma_C - \gamma_A\\
&\Rightarrow \ln 2 \leq \ln(2 + \gamma_B - \gamma_A) \leq \ln(2 + \gamma_C - \gamma_A)\\
&\Rightarrow (\gamma_B - \gamma_A)\ln(2 + \gamma_B - \gamma_A) \leq (\gamma_C - \gamma_A)\ln(2 + \gamma_C - \gamma_A)
\end{aligned}$$

and

$$\begin{aligned}
&\gamma_A \leq \gamma_B \leq \gamma_C\\
&\Rightarrow -1 \leq \gamma_A - \gamma_C \leq \gamma_A - \gamma_B \leq 0\\
&\Rightarrow 1 \leq 2 + \gamma_A - \gamma_C \leq 2 + \gamma_A - \gamma_B \leq 2\\
&\Rightarrow 0 \leq \ln(2 + \gamma_A - \gamma_C) \leq \ln(2 + \gamma_A - \gamma_B) \leq \ln 2\\
&\Rightarrow (\gamma_A - \gamma_B)\ln(2 + \gamma_A - \gamma_B) \geq (\gamma_A - \gamma_C)\ln(2 + \gamma_A - \gamma_C)\\
&\Rightarrow (\gamma_B - \gamma_A)\ln(2 + \gamma_A - \gamma_B) \leq (\gamma_C - \gamma_A)\ln(2 + \gamma_A - \gamma_C).
\end{aligned}$$

Then, $(\gamma_B - \gamma_A)\ln(2 + \gamma_B - \gamma_A) - (\gamma_B - \gamma_A)\ln(2 + \gamma_A - \gamma_B) \leq (\gamma_C - \gamma_A)\ln(2 + \gamma_C - \gamma_A) - (\gamma_C - \gamma_A)$ $\ln(2 + \gamma_A - \gamma_C)$,

namely: $(\gamma_A - \gamma_B)\ln\frac{2+\gamma_A-\gamma_B}{2+\gamma_B-\gamma_A} \leq (\gamma_A - \gamma_C)\ln\frac{2+\gamma_A-\gamma_C}{2+\gamma_C-\gamma_A}$.

Consequently, $\max\limits_{\gamma_A\in\tilde{T}_A}\min\limits_{\gamma_B\in\tilde{T}_B}(\gamma_A - \gamma_B)\ln\frac{2+\gamma_A-\gamma_B}{2+\gamma_B-\gamma_A} \leq \max\limits_{\gamma_A\in\tilde{T}_A}\min\limits_{\gamma_C\in\tilde{T}_C}(\gamma_A - \gamma_C)\ln\frac{2+\gamma_A-\gamma_C}{2+\gamma_C-\gamma_A}$.

In the same way, it can be also obtained that $\max\limits_{\eta_A\in\tilde{I}_A}\min\limits_{\eta_B\in\tilde{I}_B}(\eta_A - \eta_B)\ln\frac{2+\eta_A-\eta_B}{2+\eta_B-\eta_A} \leq \max\limits_{\eta_A\in\tilde{I}_A}\min\limits_{\eta_C\in\tilde{I}_C}(\eta_A - \eta_C)\ln\frac{2+\eta_A-\eta_C}{2+\eta_C-\eta_A}$ and $\max\limits_{\xi_A\in\tilde{F}_A}\min\limits_{\xi_B\in\tilde{F}_B}(\xi_A - \xi_B)\ln\frac{2+\xi_A-\xi_B}{2+\xi_B-\xi_A} \leq \max\limits_{\xi_A\in\tilde{F}_A}\min\limits_{\xi_C\in\tilde{F}_C}(\xi_A - \xi_C)\ln\frac{2+\xi_A-\xi_C}{2+\xi_C-\xi_A}$.

Therefore, $CE_1(A,B) \leq CE_1(A,C)$ holds. In the previously shown analogical method, it can be proven that the inequality $CE_1(B,C) \leq CE_1(A,C)$ holds too.

Similarly, we can prove that the cross-entropies $CE_2(A,B)$ and $CE_3(A,B)$ also meet condition (4).

References

[1] S. Ding, Z. Wang, D. Wu, D.L. Olson, Utilizing customer satisfaction in ranking prediction for personalized cloud service selection, Decision Support Systems, 93 (2017) 1-10.

[2] C.H. Su, G.-H. Tzeng, S.-K. Hu, Cloud e-learning service strategies for improving e-learning innovation performance in a fuzzy environment by using a new hybrid fuzzy multiple attribute decision-making model, Interactive Learning Environments, 24 (2016) 1812-1835.

[3] C. Esposito, M. Ficco, F. Palmieri, A. Castiglione, Smart cloud storage service selection based on fuzzy logic, theory of evidence and game theory, IEEE Transactions on Computers, 65 (2015) 2348-2362.
[4] L. Zadeh, Fuzzy sets, Information and Control, 8 (1965) 338-353.
[5] K.T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20 (1986) 87-96.
[6] V. Torra, Hesitant fuzzy sets, International Journal of Intelligent Systems, 25 (2010) 529-539.
[7] V. Torra, Y. Narukawa, On hesitant fuzzy sets and decision, IEEE International Conference on Fuzzy Systems, (2009) 1378-1382.
[8] H.-b. Wang, F. Smarandache, Y.-q. Zhang, R. Sunderraman, Single valued neutrosophic sets, Muiltispace and Multistrunctures, 4 (2010) 410-413.
[9] F. Smarandache, A unifying field in logics: neutrosophic logic. Neutrosophy, neutrosophic set, probability, American Research Press, Rehoboth, 1999.
[10] F. Smarandache, Neutrosophy: neutrosophic probability, set, and logic, American Research Press, Rehoboth, 1998.
[11] J.-q. Wang, X.-e. Li, TODIM method with multi-valued neutrosophic sets, Control & Decision, 30 (2015) 1139-1142.
[12] L. Wang, H.Y. Zhang, J.Q. Wang, Frank Choquet Bonferroni Mean Operators of Bipolar Neutrosophic Sets and Their Application to Multi-criteria Decision-Making Problems, International Journal of Fuzzy Systems, 20 (2018) 13–28.
[13] J.-q. Wang, Y. Yang, L. Li, Multi-criteria decision-making method based on single-valued neutrosophic linguistic Maclaurin symmetric mean operators, Neural Computing and Applications, 30 (2018) 1529–1547.
[14] P. Ji, H.-y. Zhang, J.-q. Wang, Fuzzy decision-making framework for treatment selection based on the combined QUALIFLEXTODIM method, International Journal of Systems Science, 48 (2017) 3072-3086.
[15] R.-x. Liang, J.-q. Wang, L. Li, Multi-criteria group decision-making method based on interdependent inputs of single-valued trapezoidal neutrosophic information, Neural Computing and Applications, 30 (2018) 241–260.
[16] J.-q. Wang, X. Zhang, H.-y. Zhang, Hotel recommendation approach based on the online consumer reviews using interval neutrosophic linguistic numbers, Journal of Intelligent & Fuzzy Systems, 34 (1) (2018) 381–394.
[17] R.-x. Liang, J.-q. Wang, H.-y. Zhang, A multi-criteria decision-making method based on single-valued trapezoidal neutrosophic preference relations with complete weight information, Neural Computing and Applications, (2017). doi:10.1007/s00521-017-2925-8
[18] R. Sun, J. Hu, X. Chen, Novel single-valued neutrosophic decision-making approaches based on prospect theory and their applications in physician selection, Soft Computing, (2017). doi:10.1007/s00500-017-2949-0
[19] J. Hu, L. Pan, X. Chen, An Interval neutrosophic projection-based VIKOR method for selecting doctors, Cognitive Computation, 9 (2017) 801-816.
[20] Z.-p. Tian, J.-q. Wang, H.-y. Zhang, Hybrid single-valued neutrosophic MCGDM with QFD for market segment evaluation and selection, Journal of Intelligent & Fuzzy Systems, 34 (1) (2018) 177–187.
[21] R.-x. Liang, J.-q. Wang, H.-y. Zhang, Evaluation of e-commerce websites: An integrated approach under a single-valued trapezoidal neutrosophic environment, Knowledge-Based Systems, 135 (2017) 44-59.
[22] J. Ye, Multiple-attribute decision-making method under a single-valued neutrosophic hesitant fuzzy environment, Journal of Intelligent Systems, 24 (2014) 23-36.
[23] R. Şahin, P. Liu, Correlation coefficient of single-valued neutrosophic hesitant fuzzy sets and its applications in decision making, Neural Computing and Applications, 28 (2017) 1387-1395.
[24] P. Ji, H.-y. Zhang, J.-q. Wang, A projection-based TODIM method under multi-valued neutrosophic environments and its application in personnel selection, Neural Computing and Applications, 29 (2018) 221–234.
[25] P. Biswas, S. Pramanik, B.C. Giri, Some distance measures of single valued neutrosophic hesitant fuzzy sets and their applications to multiple attribute decision making, in: F. Smarandache, S. Pramanik (Eds.) New trends in neutrosophic theory and applications Brussels: Pons Editions, 2016, pp. 27-34.
[26] P. Biswas, S. Pramanik, B.C. Giri, GRA method of multiple attribute decision making with single valued neutrosophic hesitant fuzzy set information, in: F. Smarandache, S. Pramanik (Eds.) New trends in

neutrosophic theory and applications Brussels:Pons Editions, 2016, pp. 55-63.
[27] R. Şahin, P. Liu, Distance and similarity measure for multiple attribute with singlevalued neutrosophic hesitant fuzzy information, in: F. Smarandache, S. Pramanik (Eds.) New trends in neutrosophic theory and applications Brussels: Pons Editions, 2016, pp. 35-54.
[28] P. Liu, L. Zhang, An extended multiple criteria decision making method based on neutrosophic hesitant fuzzy information, Journal of Intelligent & Fuzzy Systems, 32 (2017) 4403-4413.
[29] J.-j. Peng, J.-q. Wang, X.-h. Wu, An extension of the ELECTRE approach with multi-valued neutrosophic information, Neural Computing and Applications, 28 (2017) 1011-1022.
[30] J.-j. Peng, J.-q. Wang, W.-E. Yang, A multi-valued neutrosophic qualitative flexible approach based on likelihood for multi-criteria decision-making problems, International Journal of Systems Science, 48 (2017) 425-435.
[31] Z.h. Xu, M.m. Xia, Hesitant fuzzy entropy and cross-entropy and their use in multiattribute decision-making, International Journal of Intelligent Systems, 27 (2012) 799-822.
[32] P. Burillo, H. Bustince, Entropy on intuitionistic fuzzy sets and on interval-valued fuzzy sets, Fuzzy Sets & Systems, 78 (1996) 305-316.
[33] C. Wei, F. Yan, R.M. Rodrguez, Entropy measures for hesitant fuzzy sets and their application in multi-criteria decision-making, Journal of Intelligent & Fuzzy Systems, 31 (2016) 673-685.
[34] P. Majumdar, S.K. Samanta, On similarity and entropy of neutrosophic sets, Journal of Intelligent & Fuzzy Systems, 26 (2014) 1245-1252.
[35] M. Li, C. Wu, A distance model of intuitionistic fuzzy cross entropy to solve preference problem on alternatives, Mathematical Problems in Engineering, 2016 (2016). doi:10.1155/2016/8324124
[36] X.-h. Wu, J.-q. Wang, J.-j. Peng, X.-h. Chen, Cross-entropy and prioritized aggregation operator with simplified neutrosophic sets and their application in multi-criteria decision-making problems, International Journal of Fuzzy Systems, 18 (2016) 1104-1116.
[37] S. Opricovic, Multicriteria optimization of civil engineering systems, Faculty of Civil Engineering, Belgrade, 2 (1998) 5-21.
[38] S. Opricovic, G.-H. Tzeng, Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS, European Journal of Operational Research, 156 (2004) 445-455.
[39] X. Zhao, S. Tang, S. Yang, K. Huang, Extended VIKOR method based on cross-entropy for interval-valued intuitionistic fuzzy multiple criteria group decision making, Journal of Intelligent & Fuzzy Systems, 25 (2013) 1053-1066.
[40] R. Bausys, E.-K. Zavadskas, Multicriteria decision making approach by VIKOR under interval neutrosophic set environment, Economic Computation & Economic Cybernetics Studies & Research, 49 (2015) 33-48.
[41] S. Opricovic, G.-H. Tzeng, Extended VIKOR method in comparison with outranking methods, European Journal of Operational Research, 178 (2007) 514-529.
[42] X.-h. Wu, J.-q. Wang, Cross-entropy measures of multi-valued neutrosophic sets and its application in selecting middle-level manager, International Journal for Uncertainty Quantification, 7 (2017) 155-176.
[43] J. Ye, Single valued neutrosophic cross-entropy for multicriteria decision making problems, Applied Mathematical Modelling, 38 (2014) 1170-1175.
[44] F. Meng, X. Chen, Interval-valued intuitionistic fuzzy multi-criteria group decision making based on cross entropy and 2-additive measures, Soft Computing, 19 (2014) 2071-2082.
[45] D. Yao, X. Liu, X. Zhang, C. Wang, Type-2 fuzzy cross-entropy and entropy measures and their applications, Journal of Intelligent & Fuzzy Systems, 30 (2016) 2169-2180.