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Generalized Hyers-Ulam Stability for General Additive Functional Equations on Non-Archimedean Random Lie C^*-Algebras

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Abstract. In this paper, using the fixed point method, we prove some results related to the generalized Hyers-Ulam stability of homomorphisms and derivations in non-Archimedean random C^*-algebras and non-Archimedean random Lie C^*-algebras for the generalized additive functional equation

$$\sum_{1\le i<j\le n} f\Big(\frac{x_i+x_j}{2}+\sum_{l=1,\,k_l\ne i,j}^{n-2} x_{k_l}\Big)=\frac{(n-1)^2}{2}\sum_{i=1}^{n} f(x_i)$$

where $n \in \mathbb{N}$ is a fixed integer with $n \ge 3$.

1. Introduction

The study of the stability problem for functional equations is related to a question of Ulam [39] in 1940 concerning the stability of group homomorphisms. In 1941, Hyers [10] affirmatively answered Ulam's question for Banach spaces. Subsequently, Hyers' result was generalized by Aoki [1] for additive mappings and by Rassias [30] for linear mappings by considering an unbounded Cauchy difference. The paper [30] of Rassias has provided a lot of influence in the development of what we now call the *generalized Hyers-Ulam stability (or Hyers-Ulam-Rassias stability)* of functional equations. In 1994, Găvruţă [7] obtained a generalized result of Rassias' theorem which allow the Cauchy difference to be controlled by a general unbounded function. We refer the interested reader to [9, 11, 13, 15, 21, 22, 31, 35] for more information.

In [34], Rassias and Kim introduced and investigated the following functional equation:

$$\sum_{1\le i<j\le n} f\Big(\frac{x_i+x_j}{2}+\sum_{l=1,\,k_l\ne i,j}^{n-2} x_{k_l}\Big)=\frac{(n-1)^2}{2}\sum_{i=1}^{n} f(x_i) \tag{1}$$

where n is a fixed integer with $n \ge 2$. We observe that in the case $n = 2$, the functional equation (1) yields the Jensen functional equation $2f((x+y)/2) = f(x) + f(y)$ and there are many interesting results concerning the

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stability problems of the Jensen equation [19, 32, 33]. In [12], Jang and Saadati proved the generalized Hyers-Ulam stability of homomorphisms and derivations in non-Archimedean C^*-algebras and non-Archimedean Lie C^*-algebras for the Jensen type functional equation $f((x+y)/2) + f((x-y)/2) = f(x)$. For the case $n = 3$, Najati and Ranjbari [25] investigated homomorphisms between C^*-ternary algebras, and derivations on C^*-ternary algebras. In fact, in [34], the authors established the general solution of the functional equation (1) and investigated the generalized Hyers-Ulam stability problem of the functional equation (1) with $n \geq 3$ in quasi-β-normed spaces. In 2013, Kim et al. [18] proved some new Hyers-Ulam-Rassias stability results of n-Lie homomorphisms and Jordan n-Lie homomorphisms on n-Lie Banach algebras associated to the functional equation (1) using the fixed point method.

In this paper, using the fixed point method, we will investigate the generalized Hyers-Ulam stability results of homomorphisms and derivations in non-Archimedean random C^*-algebras and on non-Archimedean random Lie C^*-algebras for the additive functional equation (1) with $n \geq 3$.

2. Preliminaries

In this section, we adopt the usual terminology, notions and conventions of the theory of non-Archimedean random normed space as in [3–5, 16, 17, 20, 27, 29, 36, 37]. Throughout this paper, Δ^+ is the space of all probability distribution functions, i.e., the space of all mappings $F : \mathbb{R} \cup \{-\infty, \infty\} \to [0,1]$ such that F is left-continuous and non-decreasing on $\mathbb{R}$, $F(0) = 0$ and $F(+\infty) = 1$. D^+ is a subset of Δ^+ consisting of all functions $F \in \Delta^+$ for which $l^- F(+\infty) = 1$, where $l^- f(x)$ denotes the left limit of the function f at the point x, that is, $l^- f(x) = \lim_{t \to x^-} f(t)$. The space Δ^+ is partially ordered by the usual point-wise ordered of functions, i.e., $F \leq G$ if and only if $F(t) \leq G(t)$ for all $t \in \mathbb{R}$. The maximal element for Δ^+ in this order is the distribution function ε_0 given by

$$\varepsilon_0(t) = \begin{cases} 0, & \text{if } t \leq 0, \\ 1, & \text{if } t > 0. \end{cases}$$

Definition 2.1. (cf. [36]). *A mapping $T : [0,1] \times [0,1] \to [0,1]$ is a continuous triangular norm (briefly, a continuous t-norm) if T satisfies the following conditions:*
(1) *T is commutative and associative;*
(2) *T is continuous;*
(3) *$T(a, 1) = a$ for all $a \in [0,1]$;*
(4) *$T(a,b) \leq T(c,d)$ whenever $a \leq c$ and $b \leq d$ for all $a,b,c,d \in [0,1]$.*

Typical examples of continuous t-norms are the Lukasiewicz t-norm T_L, where $T_L(a,b) = \max(a+b-1, 0)$, $\forall a, b \in [0,1]$ and the t-norms T_P, T_M, T_D, where $T_P(a,b) := ab$, $T_M(a,b) := \min(a,b)$,

$$T_D(a,b) := \begin{cases} \min(a,b), & \text{if } \max(a,b) = 1, \\ 0, & \text{otherwise.} \end{cases}$$

By a non-Archimedean field we mean a field $\mathbb{K}$ equipped with a function (valuation) $|\cdot|$ from $\mathbb{K}$ into $[0,\infty)$ such that $|r| = 0$ if and only if $r = 0$, $|rs| = |r||s|$, and $|r+s| \leq \max\{|r|, |s|\}$ for $r, s \in \mathbb{K}$. Clearly $|1| = |-1| = 1$ and $|n| \leq 1$ for all $n \in \mathbb{N}$. By the trivial valuation we mean the function $|\cdot|$ taking everything but 0 into 1 and $|0| = 0$ (i.e., the function $|\cdot|$ is called the trivial valuation if $|r| = 1, \forall r \in \mathbb{K}, r \neq 0$, and $|0| = 0$).

Let X be a vector space over a field $\mathbb{K}$ with a non-Archimedean non-trivial valuation $|\cdot|$. A function $\|\cdot\| : X \to [0,\infty)$ is called a non-Archimedean norm if it satisfies the following conditions:
(i) $\|x\| = 0$ if and only if $x = 0$;
(ii) For any $r \in \mathbb{K}$ and $x \in X$, $\|rx\| = |r|\|x\|$;
(iii) For all $x, y \in X$, $\|x+y\| \leq \max\{\|x\|, \|y\|\}$ (the strong triangle inequality).
Then $(X, \|\cdot\|)$ is called a non-Archimedean normed space. Due to the fact that

$$\|x_n - x_m\| \leq \max\{\|x_{j+1} - x_j\| : m \leq j \leq n-1\}, \quad (n > m),$$

a sequence $\{x_n\}$ is Cauchy if and only if $\{x_{n+1}-x_n\}$ converges to zero in a non-Archimedean normed space. By a complete non-Archimedean normed space we mean one in which every Cauchy sequence is convergent.

Example 2.2. (cf. [14]). *For any non-zero rational number x, there exists a unique integer $n_x \in \mathbb{Z}$ such that $x = \frac{a}{b}p^{n_x}$, where a and b are integers not divisible by p. Then $|x|_p := p^{-n_x}$ defines a non-Archimedean norm on $\mathbb{Q}$. The completion of $\mathbb{Q}$ with respect to the metric $d(x,y) = |x-y|_p$ is denoted by $\mathbb{Q}_p$, which is called the p-adic number field.*

A non-Archimedean Banach algebra is a complete non-Archimedean algebra $\mathcal{A}$ which satisfies $\|ab\| \leq \|a\|\|b\|$ for all $a, b \in \mathcal{A}$. For more detailed definitions of non-Archimedean Banach algebras, we refer the reader to [8, 38].

If $\mathcal{U}$ is a non-Archimedean Banach algebra, then an involution on $\mathcal{U}$ is a mapping $t \to t^*$ from $\mathcal{U}$ into $\mathcal{U}$ which satisfies
(I) $t^{**} = t$ for $t \in \mathcal{U}$;
(II) $(\alpha s + \beta t)^* = \bar{\alpha}s^* + \bar{\beta}t^*$;
(III) $(st)^* = t^*s^*$ for $s, t \in \mathcal{U}$.
If, in addition, $\|t^*t\| = \|t\|^2$ for $t \in \mathcal{U}$, then $\mathcal{U}$ is a non-Archimedean C^*-algebra.

Definition 2.3. (cf. [14, 37]). *A non-Archimedean random normed space (briefly, NA-RN-space) is a triple (X, μ, T), where X is a linear space over a non-Archimedean field $\mathbb{K}$, T is a continuous t-norm, and μ is a mapping from X into D^+ such that the following conditions hold:*
(NA-RN1) $\mu_x(t) = \varepsilon_0(t)$ *for all $t > 0$ if and only if $x = 0$;*
(NA-RN2) $\mu_{\alpha x}(t) = \mu_x(\frac{t}{|\alpha|})$ *for all $x \in X$, $t > 0$, and $\alpha \neq 0$;*
(NA-RN3) $\mu_{x+y}(\max(t,s)) \geq T(\mu_x(t), \mu_y(s))$ *for all $x, y \in X$ and $t, s \geq 0$;*
It is easy to see that if (NA-RN3) *holds, then*
(RN3) $\mu_{x+y}(t+s) \geq T(\mu_x(t), \mu_y(s))$.

Example 2.4. (cf. [26]). *Let $(X, \|\cdot\|)$ be a non-Archimedean normed linear space, and $\alpha, \beta > 0$. Define*

$$\mu_x(t) = \frac{\alpha t}{\alpha t + \beta\|x\|}$$

for all $x \in X$ and $t > 0$. Then (X, μ, T_M) is a non-Archimedean RN-space.

Proof. (NA − RN1) is obviously true. Notice that for any $t \in \mathbb{R}$, $t > 0$ and $c \neq 0$

$$\mu_{cx}(t) = \frac{\alpha t}{\alpha t + \beta\|cx\|} = \frac{\alpha t}{\alpha t + \beta|c|\|x\|} = \frac{\alpha \cdot \frac{t}{|c|}}{\alpha \cdot \frac{t}{|c|} + \beta\|x\|} = \mu_x(\frac{t}{|c|}),$$

which implies that (NA − RN2) holds.
To prove (NA − RN3). We assume that $\mu_x(t) \leq \mu_y(s)$, thus we have

$$\frac{\|y\|}{s} \leq \frac{\|x\|}{t}.$$

Now, if $\|x\| \geq \|y\|$ for all $x, y \in X$, then we have by the strong triangle inequality

$$t\|x+y\| \leq t\|x\| \leq (\max(t,s))\|x\|.$$

Therefore,

$$\frac{\beta\|x+y\|}{\alpha(\max(t,s))} \leq \frac{\beta\|x\|}{\alpha t}$$

and so

$$1 + \frac{\beta\|x+y\|}{\alpha(\max(t,s))} \leq 1 + \frac{\beta\|x\|}{\alpha t},$$

which implies that $\mu_{x+y}(\max(t,s)) \geq \mu_x(t)$.

if $\|x\| \leq \|y\|$ for all $x, y \in X$, then we also have

$$t\|x+y\| \leq t\|y\| \leq t \cdot \frac{s}{t}\|x\| \leq (\max(t,s))\|x\|.$$

By the same way to the above, we can also get $\mu_{x+y}(\max(t,s)) \geq \mu_x(t)$. Hence, $\mu_{x+y}(\max(t,s)) \geq T_M(\mu_x(t), \mu_y(s))$ for all $x, y \in X$ and $t, s \geq 0$. Then (X, μ, T_M) is a non-Archimedean RN-space. □

Example 2.5. (cf. [26]). *Let $(X, \|\cdot\|)$ be a non-Archimedean normed linear space, let $\beta > \alpha > 0$ and*

$$\mu_x(t) = \begin{cases} 0, & t \leq \alpha\|x\|, \\ \frac{t}{t+(\beta-\alpha)\|x\|}, & \alpha\|x\| < t \leq \beta\|x\|, \\ 1, & t > \beta\|x\|. \end{cases}$$

Then (X, μ, T_M) is a non-Archimedean RN-space.

Proof. (NA − RN1) is obviously true. Notice that for $c \neq 0$, if $\mu_{cx}(t) = 1$, then $t > \beta\|cx\|$, i.e. $\frac{t}{|c|} > \beta\|x\|$ $\Rightarrow \mu_x(\frac{t}{|c|}) = 1$
thus $\mu_{cx}(t) = \mu_x(\frac{t}{|c|})$.

Again if $\mu_{cx}(t) = \frac{t}{t+(\beta-\alpha)\|cx\|}$, then $\alpha\|cx\| < t \leq \beta\|cx\|$, i.e. $\alpha\|x\| < \frac{t}{|c|} \leq \beta\|x\|$, so we have

$$\mu_x(\frac{t}{|c|}) = \frac{t}{t+(\beta-\alpha)\|cx\|},$$

therefore, $\mu_{cx}(t) = \mu_x(\frac{t}{|c|})$. Similarly, when $\mu_{cx}(t) = 0$, then $\mu_{cx}(t) = \mu_x(\frac{t}{|c|}) = 0$. Thus for $c \neq 0$, $\mu_{cx}(t) = \mu_x(\frac{t}{|c|})$ which implies that (NA − RN2) holds.

Next, we have to show that

$$\mu_{x+y}(\max(t,s)) \geq T_M(\mu_x(t), \mu_y(s)).$$

If $s = t = 0$, then in this case the relation is obvious. So we consider the case when $t > 0, s > 0$.

If $t > \beta\|x\|, s > \beta\|y\|$, then $\max(t,s) > \beta\|x\|, \max(t,s) > \beta\|y\|$, and $\mu_x(t) = 1, \mu_y(s) = 1$. Now, we have

$$\max(t,s) \geq \beta\|x\|(\text{ or } \beta\|y\|) = \max(\beta\|x\|, \beta\|y\|) \geq \beta(\|x+y\|)$$

Hence, we get

$$\mu_{x+y}(\max(t,s)) = 1 \Rightarrow \mu_{x+y}(\max(t,s)) \geq T_M(\mu_x(t), \mu_y(s)).$$

If $t > \beta\|x\|$, and $\alpha\|y\| < s \leq \beta\|y\|$, then $\mu_x(t) = 1, \mu_y(s) = \frac{s}{s+(\beta-\alpha)\|y\|}$. Now, if $\|x\| \geq \|y\|$, then we obtain

$$\max(t,s) \geq \beta\|x\| = \max(\beta\|x\|, \beta\|y\|) \geq \beta(\|x+y\|)$$

Hence, we have

$$\mu_{x+y}(\max(t,s)) = 1 \Rightarrow \mu_{x+y}(\max(t,s)) \geq T_M(\mu_x(t), \mu_y(s)).$$

Next, if $\|y\| \geq \|x\|$. So we get

$$\max(t,s) \geq \alpha\|y\| = \max(\alpha\|x\|, \alpha\|y\|) \geq \alpha(\|x+y\|)$$

Hence, we get

$$\mu_{x+y}(\max(t,s)) = \frac{\max(t,s)}{\max(t,s)+(\beta-\alpha)\|x+y\|} \Rightarrow \mu_{x+y}(\max(t,s)) \geq T_M(\mu_x(t), \mu_y(s)).$$

If $\alpha\|x\| < t \leq \beta\|x\|$, and $\alpha\|y\| < s \leq \beta\|y\|$, then in this case the relation is similar to the proof of Example 2.4, and thus it is omitted. This completes the proof of the example. □

Definition 2.6. (cf. [14, 23]). *A non-Archimedean random normed algebra* (X, μ, T, T') *is a non-Archimedean random normed space* (X, μ, T) *with an algebraic structure such that*
(NA-RN4) $\mu_{xy}(t) \geq T'(\mu_x(t), \mu_y(t))$ *for all* $x, y \in X$ *and all* $t > 0$, *in which* T' *is a continuous t-norm.*

Example 2.7. (cf. [23]). *Let* $(X, \|\cdot\|)$ *be a non-Archimedean normed algebra. Define*

$$\mu_x(t) = \begin{cases} 0, & x \neq 0, t \leq 0, \\ \frac{t}{t+\|x\|}, & x \neq 0, t > 0, \\ 1, & x = 0 \end{cases}$$

Then (X, μ, T_M) *is a non-Archimedean RN-space. An easy computation shows that* $\mu_{xy}(t) \geq \mu_x(t)\mu_y(t)$ *if and only if*

$$\|xy\| \leq \|x\|\|y\| + t\|y\| + t\|x\|$$

for all $x, y \in X$ *and* $t > 0$. *It follows that* (X, μ, T_M, T_P) *is a non-Archimedean random normed algebra.*

Definition 2.8. (cf. [14]). *Let* (X, μ, T, T') *and* (Y, μ, T, T') *be non-Archimedean random normed algebras.*
(a) *An* $\mathbb{R}$*-linear mapping* $f : X \to Y$ *is called a homomorphism if* $f(xy) = f(x)f(y)$ *for all* $x, y \in X$.
(b) *An* $\mathbb{R}$*-linear mapping* $f : X \to Y$ *is called a derivation if* $f(xy) = f(x)y + xf(y)$ *for all* $x, y \in X$.

Definition 2.9. (cf. [14]). *Let* $(\mathcal{U}, \mu, T, T')$ *be non-Archimedean random Banach algebra, then an involution on* $\mathcal{U}$ *is a mapping* $u \to u^*$ *from* $\mathcal{U}$ *into* $\mathcal{U}$ *which satisfies*
(I′) $u^{**} = u$ *for* $u \in \mathcal{U}$;
(II′) $(\alpha u + \beta v)^* = \bar{\alpha}u^* + \bar{\beta}v^*$;
(III′) $(uv)^* = v^*u^*$ *for* $u, v \in \mathcal{U}$.
If, in addition, $\mu_{u^*u}(t) = T'(\mu_u(t), \mu_u(t))$ *for* $u \in \mathcal{U}$ *and* $t > 0$, *then* $\mathcal{U}$ *is a non-Archimedean random* C^**-algebra.*

Definition 2.10. (cf. [14]) *Let* (X, μ, T) *be a non-Archimedean RN-space. Let* $\{x_n\}$ *be a sequence in X. Then* $\{x_n\}$ *is said to be convergent if there exists* $x \in X$ *such that*

$$\lim_{n\to\infty} \mu_{x_n - x}(t) = 1,$$

for all $t > 0$. *In this case, x is called the limit of the sequence* $\{x_n\}$.

A sequence $\{x_n\}$ in X is called Cauchy if for each $\varepsilon > 0$ and $t > 0$, there exists n_0 such that for all $n \geq n_0$ and all $p > 0$ we have $\mu_{x_{n+p}-x_n}(t) > 1 - \varepsilon$. Due to

$$\mu_{x_{n+p}-x_n}(t) \geq \min\{\mu_{x_{n+p}-x_{n+p-1}}(t), \ldots, \mu_{x_{n+1}-x_n}(t)\}.$$

Therefore, the sequence $\{x_n\}$ is Cauchy if for each $\varepsilon \geq 0$ and $t > 0$ there exists n_0 such that for all $n \geq n_0$, we have $\mu_{x_{n+1}-x_n}(t) > 1 - \varepsilon$.

If each Cauchy sequence is convergent, then the random norm is said to be complete, and the non-Archimedean RN-space is called a non-Archimedean random Banach space.

Definition 2.11. *Let S be a set. A function* $d : S \times S \to [0, \infty]$ *is called a generalized metric on S if d satisfies*
(1) $d(x, y) = 0$ if and only if $x = y$;
(2) $d(x, y) = d(y, x)$, $\forall x, y \in S$;
(3) $d(x, z) \leq d(x, y) + d(y, z)$, $\forall x, y, z \in S$.

The next Lemma 2.12 is due to Diaz and Margolis [6], which is extensively applied to the stability theory of functional equations.

Lemma 2.12. ([6]). *Let (S, d) be a complete generalized metric space and $J : S \to S$ be a strictly contractive mapping with Lipschitz constant $L < 1$. Then for each fixed element $x \in S$, either*

$$d(J^n x, J^{n+1} x) = \infty$$

for all nonnegative integers n or there exists a positive integer n_0 such that
(i) $d(J^n x, J^{n+1} x) < \infty, \quad \forall n \geq n_0$;
(ii) *the sequence $\{J^n x\}$ is convergent to a fixed point y^* of J;*
(iii) *y^* is the unique fixed point of J in the set $S^* := \{y \in S \mid d(J^{n_0} x, y) < +\infty\}$;*
(iv) $d(y, y^*) \leq \frac{1}{1-L} d(y, Jy), \quad \forall y \in S^*$.

3. Stability of homomorphisms and derivations in non-Archimedean random C^*-algebras

In this section, assume that $\mathcal{A}$ is a non-Archimedean random C^*-algebra with the norm $\mu^{\mathcal{A}}_{\cdot}$ and that $\mathcal{B}$ is a non-Archimedean random C^*-algebra with the norm $\mu^{\mathcal{B}}_{\cdot}$. For a given mapping $f : \mathcal{A} \to \mathcal{B}$, we define

$$\mathcal{D}_{\lambda,f}(x_1, \ldots, x_n) = \sum_{1 \leq i < j \leq n} f\Big(\frac{\lambda x_i + \lambda x_j}{2} + \sum_{l=1, k_l \neq i,j}^{n-2} \lambda x_{k_l}\Big) - \frac{(n-1)^2}{2} \sum_{i=1}^{n} \lambda f(x_i)$$

for all $x_1, \ldots, x_n \in \mathcal{A} (n \geq 3)$ and $\lambda \in \mathbb{T}^1 := \{\lambda \in \mathbb{C} : |\lambda| = 1\}$.

We need the following lemmas to prove the main results.

Lemma 3.1. (cf. [24]). *Let V and W be linear spaces and let $n \geq 3$ be a fixed positive integer. A mapping $f : V \to W$ satisfies the functional equation (1) for all $x_1, \ldots, x_n \in V$ if and only if f is an additive mapping.*

Lemma 3.2. (cf. [28]). *Let $f : \mathcal{A} \to \mathcal{A}$ be an additive mapping such that $f(\lambda x) = \lambda f(x)$ for all $\lambda \in \mathbb{T}^1$ and all $x \in \mathcal{A}$. Then the mapping f is $\mathbb{C}$-linear.*

Note that a $\mathbb{C}$-linear mapping $H : \mathcal{A} \to \mathcal{B}$ is called homomorphism in non-Archimedean random C^*-algebras if H satisfies $H(xy) = H(x)H(y)$ and $H(x^*) = H(x)^*$ for all $x, y \in \mathcal{A}$.

Now we are going to prove the generalized Hyers-Ulam stability of homomorphisms in non-Archimedean random C^*-algebras for the functional equation $\mathcal{D}_{\lambda,f}(x_1, \ldots, x_n) = 0$.

Theorem 3.3. *Let $f : \mathcal{A} \to \mathcal{B}$ be a mapping for which there are functions $\varphi : \mathcal{A}^n \to D^+$, $\psi : \mathcal{A}^2 \to D^+$ and $\eta : \mathcal{A} \to D^+$ such that $|\rho| < 1$ is far from zero and*

$$\mu^{\mathcal{B}}_{\mathcal{D}_{\lambda,f}(x_1,\ldots,x_n)}(t) \geq \varphi_{x_1,\ldots,x_n}(t) \tag{2}$$
$$\mu^{\mathcal{B}}_{f(xy)-f(x)f(y)}(t) \geq \psi_{x,y}(t) \tag{3}$$
$$\mu^{\mathcal{B}}_{f(x^*)-f(x)^*}(t) \geq \eta_x(t) \tag{4}$$

for all $\lambda \in \mathbb{T}^1$, $x_1, \ldots, x_n, x, y \in \mathcal{A}$ and $t > 0$. If there exits a constant $0 < L < 1$ such that

$$\varphi_{\rho x_1,\ldots,\rho x_n}(|\rho| L t) \geq \varphi_{x_1,\ldots,x_n}(t) \tag{5}$$
$$\psi_{\rho x,\rho y}(|\rho|^2 L t) \geq \psi_{x,y}(t) \tag{6}$$
$$\eta_{\rho x}(|\rho| L t) \geq \eta_x(t) \tag{7}$$

for all $x, y, x_1, \ldots, x_n \in \mathcal{A}$ and $t > 0$, then there exists a unique homomorphism $H : \mathcal{A} \to \mathcal{B}$ such that

$$\mu^{\mathcal{B}}_{f(x)-H(x)}(t) \geq \varphi_{x,\ldots,x}\left(\frac{|n||\rho|^2(1-L)}{|2|} t\right) \tag{8}$$

for all $x \in \mathcal{A}$ and $t > 0$, where $\rho := n - 1$.

Proof. Letting $\lambda = 1$, and $x_1 = \cdots = x_n = x$ in (2), we obtain

$$\mu^{\mathcal{B}}_{\binom{n}{2} f((n-1)x) - \frac{n(n-1)^2}{2} f(x)}(t) \geq \varphi_{x,\ldots,x}(t) \tag{9}$$

for all $x \in \mathcal{A}$ and $t > 0$. Then

$$\mu^{\mathcal{B}}_{f(x) - \frac{f(\rho x)}{\rho}}\left(\frac{|2|}{|n||\rho|^2} t\right) \geq \varphi_{x,\ldots,x}(t) \tag{10}$$

for all $x \in \mathcal{A}$ and $t > 0$.

Let us define Ω to be the set of all mappings $g : \mathcal{A} \to \mathcal{B}$ and introduce a generalized metric on Ω as follows:

$$d(g,h) := \inf\left\{\delta \in \mathbb{R}_+ \middle| \mu^{\mathcal{B}}_{g(x)-h(x)}(\delta t) \geq \varphi_{x,\ldots,x}(t), \forall x \in \mathcal{A}, t > 0\right\}.$$

It is easy to see that (Ω, d) is a complete generalized metric space [2, 20]. Now, we consider the mapping $\mathcal{J} : \Omega \to \Omega$ defined by

$$\mathcal{J}g(x) := \frac{1}{\rho} g(\rho x) \tag{11}$$

for all $g \in \Omega$ and $x \in \mathcal{A}$. Note that for all $g, h \in \Omega$, we have

$$\begin{aligned} \mu^{\mathcal{B}}_{\mathcal{J}g(x)-\mathcal{J}h(x)}(L\delta t) &= \mu^{\mathcal{B}}_{\frac{1}{\rho}g(\rho x) - \frac{1}{\rho}h(\rho x)}(L\delta t) = \mu^{\mathcal{B}}_{g(\rho x)-h(\rho x)}(|\rho| L\delta t) \\ &\geq \varphi_{\rho x,\ldots,\rho x}(|\rho| Lt) \geq \varphi_{x,\ldots,x}(t) \end{aligned} \tag{12}$$

for all $x \in \mathcal{A}$ and $t > 0$. So $d(\mathcal{J}g, \mathcal{J}h) \leq Ld(g,h)$ holds for all $g, h \in \Omega$.

By (10), we have $d(f, \mathcal{J}f) \leq \frac{|2|}{|n||\rho|^2}$. Hence according to Lemma 2.12, the sequence $\mathcal{J}^m f$ converges to a fixed point H of $\mathcal{J}$, that is,

$$\lim_{m\to\infty} \frac{1}{|\rho|^m} f(\rho^m x) = H(x) \tag{13}$$

and

$$H(\rho x) = \rho H(x) \tag{14}$$

for all $x \in \mathcal{A}$. Also H is the unique fixed point of $\mathcal{J}$ in the set $\Omega^* = \{g \in \Omega : d(f,g) < \infty\}$. This implies that H is a unique mapping satisfying (14) such that there exists a $\delta \in \mathbb{R}_+$ such that

$$\mu^{\mathcal{B}}_{f(x)-H(x)}(\delta t) \geq \varphi_{x,\ldots,x}(t)$$

for all $x \in \mathcal{A}$ and $t > 0$. Also,

$$d(f, H) \leq \frac{1}{1-L} d(f, \mathcal{J}f) \leq \frac{|2|}{|n||\rho|^2(1-L)}.$$

This implies that the inequality (8) holds. It follows from (2), (5) and (13) that

$$\begin{aligned} \mu^{\mathcal{B}}_{\mathcal{D}_{\lambda,H}}(x_1, \ldots, x_n)(t) &= \lim_{m\to\infty} \mu^{\mathcal{B}}_{\frac{1}{\rho^m}\mathcal{D}_{\lambda,f}(\rho^m x_1,\ldots,\rho^m x_n)}(t) \\ &\geq \lim_{m\to\infty} \varphi_{\rho^m x_1,\ldots,\rho^m x_n}(|\rho|^m t) = 1 \end{aligned}$$

for all $\lambda \in \mathbb{T}^1$, $x_1, \ldots, x_n \in \mathcal{A}$ and $t > 0$. Hence, we obtain

$$\mathcal{D}_{\lambda,H}(x_1, \ldots, x_n) = 0 \tag{15}$$

for all $x_1, \ldots, x_n \in \mathcal{A}$. If we put $\lambda = 1$ in (15), then H is additive by Lemma 3.1. Also, letting $x_1 = \cdots = x_n = x$ in the last equality, we obtain $H(\lambda x) = \lambda H(x)$. Now by using Lemma 3.2, we infer that the mapping H is $\mathbb{C}$-linear. On the other hand, it follows from (3), (6) and (13) that

$$\begin{aligned}\mu^{\mathcal{B}}_{H(xy)-H(x)H(y)}(t) &= \lim_{m\to\infty} \mu^{\mathcal{B}}_{f(\rho^{2m}xy)-f(\rho^m x)f(\rho^m y)}(|\rho|^{2m}t) \\ &\geq \lim_{m\to\infty} \psi_{\rho^m x,\rho^m y}(|\rho|^{2m}t) = 1\end{aligned}$$

for all $x, y \in \mathcal{A}$. So, $H(xy) = H(x)H(y)$ for all $x, y \in \mathcal{A}$. Thus $H : \mathcal{A} \to \mathcal{B}$ is a homomorphism satisfying (8), as desired. Also, by (4), (7) and (13) and by a similar method, we have $H(x^*) = H(x)^*$. This completes the proof of the theorem. □

Theorem 3.4. *Let $f : \mathcal{A} \to \mathcal{B}$ be a mapping for which there are functions $\varphi : \mathcal{A}^n \to D^+$, $\psi : \mathcal{A}^2 \to D^+$ and $\eta : \mathcal{A} \to D^+$ such that $|\rho| < 1$ is far from zero, and (2), (3) and (4) hold for all $\lambda \in \mathbb{T}^1$, $x_1, \ldots, x_n, x, y \in \mathcal{A}$ and $t > 0$. If there exits a constant $0 < L < 1$ such that*

$$\varphi_{\frac{x_1}{\rho},\ldots,\frac{x_n}{\rho}}\left(\frac{L}{|\rho|}t\right) \geq \varphi_{x_1,\ldots,x_n}(t) \tag{16}$$

$$\psi_{\frac{x}{\rho},\frac{y}{\rho}}\left(\frac{L}{|\rho|^2}t\right) \geq \psi_{x,y}(t) \tag{17}$$

$$\eta_{\frac{x}{\rho}}\left(\frac{L}{|\rho|}t\right) \geq \eta_x(t) \tag{18}$$

for all $x, y, x_1, \ldots, x_n \in \mathcal{A}$ and $t > 0$, then there exists a unique homomorphism $H : \mathcal{A} \to \mathcal{B}$ such that

$$\mu^{\mathcal{B}}_{f(x)-H(x)}(t) \geq \varphi_{x,\ldots,x}\left(\frac{|n||\rho|^2(1-L)}{|2|L}t\right) \tag{19}$$

for all $x \in \mathcal{A}$ and $t > 0$, where $\rho := n - 1$.

Proof. Let Ω and d be as in the proof of Theorem 3.3. Then (Ω, d) becomes complete generalized metric space and the mapping $\mathcal{J} : \Omega \to \Omega$ defined by

$$\mathcal{J}g(x) := \rho g\left(\frac{x}{\rho}\right), \quad \text{for all } g \in \Omega \text{ and } x \in \mathcal{A}.$$

Then, it is easy to see that $d(\mathcal{J}g, \mathcal{J}h) \leq Ld(g, h)$ for all $g, h \in S$. By (9) and (16), we obtain

$$\mu^{\mathcal{B}}_{f(x)-\rho f(\frac{x}{\rho})}\left(\frac{|2|L}{|n||\rho|^2}t\right) \geq \varphi_{\frac{x}{\rho},\ldots,\frac{x}{\rho}}\left(\frac{L}{|\rho|}t\right) \geq \varphi_{x,\ldots,x}(t)$$

for all $x \in \mathcal{A}$ and $t > 0$. So, we have $d(f, \mathcal{J}f) \leq \frac{|2|L}{|n||\rho|^2}$.

The remaining assertion is similar to the corresponding part of Theorem 3.3. This completes the proof. □

Corollary 3.5. *Let $\ell \in \{-1, 1\}$, $r \neq 1$ and θ be nonnegative real numbers. Suppose that $f : \mathcal{A} \to \mathcal{B}$ be a mapping such that*

$$\mu^{\mathcal{B}}_{\mathcal{D}_{\lambda,f}(x_1,\ldots,x_n)}(t) \geq \frac{t}{t + \theta(\|x_1\|^r_{\mathcal{A}} + \|x_2\|^r_{\mathcal{A}} + \cdots + \|x_n\|^r_{\mathcal{A}})}$$

$$\mu^{\mathcal{B}}_{f(xy)-f(x)f(y)}(t) \geq \frac{t}{t + \theta \cdot (\|x\|^r_{\mathcal{A}} \cdot \|y\|^r_{\mathcal{A}})}$$

$$\mu^{\mathcal{B}}_{f(x^*)-f(x)^*}(t) \geq \frac{t}{t + \theta \cdot \|x\|^r_{\mathcal{A}}}$$

for all $\lambda \in \mathbb{T}^1$, $x_1, \dots, x_n, x, y \in \mathcal{A}$ and $t > 0$. Then there exists a unique homomorphism $H : \mathcal{A} \to \mathcal{B}$ such that, if $\ell r > \ell$,

$$\mu^{\mathcal{B}}_{f(x)-H(x)}(t) \geq \frac{\ell|n||\rho|(|\rho| - |\rho|^r)t}{\ell|n||\rho|(|\rho| - |\rho|^r)t + \theta|2||n|\ \|x\|^r_{\mathcal{A}}} \tag{20}$$

for all $x \in \mathcal{A}$ and $t > 0$, where $\rho := n - 1$.

Proof. The proof follows from Theorems 3.3 and 3.4 by taking

$$\varphi_{x_1,\dots,x_n}(t) = \frac{t}{t + \theta(\|x_1\|^r_{\mathcal{A}} + \|x_2\|^r_{\mathcal{A}} + \cdots + \|x_n\|^r_{\mathcal{A}})}$$
$$\psi_{x,y}(t) = \frac{t}{t + \theta \cdot (\|x\|^r_{\mathcal{A}} \cdot \|y\|^r_{\mathcal{A}})}, \qquad \eta_x(t) = \frac{t}{t + \theta \cdot \|x\|^r_{\mathcal{A}}}$$

for all $x_1, \dots, x_n, x, y \in \mathcal{A}$ and $t > 0$. We can choose $L = |\rho|^{\ell(r-1)}$, we obtain the desired result. □

Note that a $\mathbb{C}$-linear mapping $\delta : \mathcal{A} \to \mathcal{A}$ is called derivation on $\mathcal{A}$ if δ satisfies $\delta(xy) = \delta(x)y + x\delta(y)$ for all $x, y \in \mathcal{A}$.

We prove the generalized Hyers-Ulam stability of derivations on non-Archimedean random C^*-algebras for the functional equation $\mathcal{D}_{\lambda,f}(x_1, \dots, x_n) = 0$.

Theorem 3.6. *Let $f : \mathcal{A} \to \mathcal{A}$ be a mapping for which there are functions $\varphi : \mathcal{A}^n \to D^+$, $\psi : \mathcal{A}^2 \to D^+$ and $\eta : \mathcal{A} \to D^+$ such that $|\rho| < 1$ is far from zero and*

$$\mu^{\mathcal{A}}_{\mathcal{D}_{\lambda,f}(x_1,\dots,x_n)}(t) \geq \varphi_{x_1,\dots,x_n}(t) \tag{21}$$
$$\mu^{\mathcal{A}}_{f(xy)-f(x)y-xf(y)}(t) \geq \psi_{x,y}(t) \tag{22}$$
$$\mu^{\mathcal{A}}_{f(x^*)-f(x)^*}(t) \geq \eta_x(t) \tag{23}$$

for all $\lambda \in \mathbb{T}^1$, $x_1, \dots, x_n, x, y \in \mathcal{A}$ and $t > 0$. If there exits a constant $0 < L < 1$ such that (5), (6) and (7) hold, then there exists a unique derivation $\delta : \mathcal{A} \to \mathcal{A}$ such that

$$\mu^{\mathcal{A}}_{f(x)-\delta(x)}(t) \geq \varphi_{x,\dots,x}\left(\frac{|n||\rho|^2(1-L)}{|2|}t\right) \tag{24}$$

for all $x \in \mathcal{A}$ and $t > 0$, where $\rho := n - 1$.

Proof. By the same reasoning as in the proof of Theorem 3.3, the mapping $\delta : \mathcal{A} \to \mathcal{A}$ defined by

$$\delta(x) := \lim_{m\to\infty} \frac{1}{|\rho|^m} f(\rho^m x) \quad \forall x \in \mathcal{A} \tag{25}$$

is a unique $\mathbb{C}$-linear mapping which satisfies (24). We show that δ is a derivation. By (22) and (25), we have

$$\begin{aligned}\mu^{\mathcal{A}}_{\delta(xy)-\delta(x)y-x\delta(y)}(t) &= \lim_{m\to\infty} \mu^{\mathcal{A}}_{f(\rho^{2m}xy)-f(\rho^m x)\rho^m y-\rho^m x\delta(\rho^m y)}(|\rho|^{2m}t) \\ &\geq \lim_{m\to\infty} \psi_{\rho^m x,\rho^m y}(|\rho|^{2m}t) = 1\end{aligned}$$

for all $x, y \in \mathcal{A}$ and all $t > 0$. Hence we have $\delta(xy) = \delta(x)y + x\delta(y)$ for all $x, y \in \mathcal{A}$. This means that δ is a derivation satisfying (24). This completes the proof. □

4. Stability of homomorphisms and derivations in non-Archimedean random Lie C^*-algebras

A non-Archimedean random C^*-algebra C, endowed with the Lie product $[x, y] = \frac{xy-yx}{2}$ on C, is called a non-Archimedean random Lie C^*-algebra.

Definition 4.1. *Let $\mathcal{A}$ and $\mathcal{B}$ be non-Archimedean random Lie C^*-algebras. A $\mathbb{C}$-linear mapping $H : \mathcal{A} \to \mathcal{B}$ is called a non-Archimedean random Lie C^*-algebra homomorphism if $H([x, y]) = [H(x), H(y)]$ for all $x, y \in \mathcal{A}$.*

In this section, assume that $\mathcal{A}$ is a non-Archimedean random Lie C^*-algebra with the norm $\mu^{\mathcal{A}}_{\cdot}$ and that $\mathcal{B}$ is a non-Archimedean random Lie C^*-algebra with the norm $\mu^{\mathcal{B}}_{\cdot}$.

Now, we prove the generalized Hyers-Ulam stability of homomorphisms in non-Archimedean random Lie C^*-algebras for the equation $\mathcal{D}_{\lambda,f}(x_1, \ldots, x_n) = 0$.

Theorem 4.2. *Let $f : \mathcal{A} \to \mathcal{B}$ be a mapping for which there are functions $\varphi : \mathcal{A}^n \to D^+$, $\psi : \mathcal{A}^2 \to D^+$ and $\eta : \mathcal{A} \to D^+$ such that $|\rho| < 1$ is far from zero, (2) and (4) hold and*

$$\mu^{\mathcal{B}}_{f([x,y])-[f(x),f(y)]}(t) \geq \psi_{x,y}(t) \tag{26}$$

for all $x, y \in \mathcal{A}$ and $t > 0$. If there exits a constant $0 < L < 1$ and (5), (6) and (7) hold, then there exists a unique homomorphism $H : \mathcal{A} \to \mathcal{B}$ such that (8) holds for all $x \in \mathcal{A}$ and $t > 0$, where $\rho := n - 1$.

Proof. By the same reasoning as in the proof of Theorem 3.3, we can find the mapping $H : \mathcal{A} \to \mathcal{B}$ given by

$$H(x) := \lim_{m\to\infty} \frac{1}{|\rho|^m} f(\rho^m x) \tag{27}$$

for all $x \in \mathcal{A}$. It follows from (6), (26) and (27) that

$$\begin{aligned}\mu^{\mathcal{B}}_{H([x,y])-[H(x),H(y)]}(t) &= \lim_{m\to\infty} \mu^{\mathcal{B}}_{f(\rho^{2m}[x,y])-[f(\rho^m x),f(\rho^m y)]}(|\rho|^{2m} t)\\ &\geq \lim_{m\to\infty} \psi_{\rho^m x,\rho^m y}(|\rho|^{2m} t) = 1\end{aligned}$$

for all $x, y \in \mathcal{A}$ and $t > 0$, then

$$H([x, y]) = [H(x), H(y)]$$

for all $x, y \in \mathcal{A}$. Thus, $H : \mathcal{A} \to \mathcal{B}$ is a Lie C^*-algebra homomorphism satisfying (8), as desired. □

Theorem 4.3. *Let $f : \mathcal{A} \to \mathcal{B}$ be a mapping for which there are functions $\varphi : \mathcal{A}^n \to D^+$, $\psi : \mathcal{A}^2 \to D^+$ and $\eta : \mathcal{A} \to D^+$ such that $|\rho| < 1$ is far from zero, and (2), (4) and (26) hold for all $\lambda \in \mathbb{T}^1$, $x_1, \ldots, x_n, x, y \in \mathcal{A}$ and $t > 0$. If there exits a constant $0 < L < 1$ and (16), (17) and (18) hold, then there exists a unique homomorphism $H : \mathcal{A} \to \mathcal{B}$ such that (19) holds for all $x \in \mathcal{A}$ and $t > 0$, where $\rho := n - 1$.*

Proof. The proof follows from Theorem 3.4 and a method similar to Theorem 4.2. □

Corollary 4.4. *Let $\ell \in \{-1, 1\}$, $r =\neq 1$ and θ be nonnegative real numbers. Suppose that $f : \mathcal{A} \to \mathcal{B}$ be a mapping such that*

$$\mu^{\mathcal{B}}_{\mathcal{D}_{\lambda,f}(x_1,\ldots,x_n)}(t) \geq \frac{t}{t + \theta(\|x_1\|^r_{\mathcal{A}} + \|x_2\|^r_{\mathcal{A}} + \cdots + \|x_n\|^r_{\mathcal{A}})}$$

$$\mu^{\mathcal{B}}_{f([x,y])-[f(x),f(y)]}(t) \geq \frac{t}{t + \theta \cdot (\|x\|^r_{\mathcal{A}} \cdot \|y\|^r_{\mathcal{A}})}$$

$$\mu^{\mathcal{B}}_{f(x^*)-f(x)^*}(t) \geq \frac{t}{t + \theta \cdot \|x\|^r_{\mathcal{A}}}$$

for all $\lambda \in \mathbb{T}^1$, $x_1, \ldots, x_n, x, y \in \mathcal{A}$ and $t > 0$. Then there exists a unique homomorphism $H : \mathcal{A} \to \mathcal{B}$ such that (20) holds.

Proof. The proof follows from Theorems 4.2 and 4.3, and a method similar to Corollary 3.5. □

Definition 4.5. *Let $\mathcal{A}$ be non-Archimedean random Lie C^*-algebra. A $\mathbb{C}$-linear mapping $\delta : \mathcal{A} \to \mathcal{A}$ is called a Lie derivation if $\delta([x, y]) = [\delta(x), y] + [x, \delta(y)]$ for all $x, y \in \mathcal{A}$.*

We prove the generalized Hyers-Ulam stability of derivations on non-Archimedean random Lie C^*-algebras for the functional equation $\mathcal{D}_{\lambda,f}(x_1, \ldots, x_n) = 0$.

Theorem 4.6. *Let $f : \mathcal{A} \to \mathcal{A}$ be a mapping for which there are functions $\varphi : \mathcal{A}^n \to D^+$, $\psi : \mathcal{A}^2 \to D^+$ and $\eta : \mathcal{A} \to D^+$ such that $|\rho| < 1$ is far from zero, and (21) and (23) hold and*

$$\mu^{\mathcal{A}}_{f([x,y])-[f(x),y]-[x,f(y)]}(t) \geq \psi_{x,y}(t) \tag{28}$$

for all $x, y \in \mathcal{A}$ and $t > 0$. If there exits a constant $0 < L < 1$ such that (5), (6) and (7) hold, then there exists a unique derivation $\delta : \mathcal{A} \to \mathcal{A}$ such that (24) holds for all $x \in \mathcal{A}$ and $t > 0$, where $\rho := n - 1$.

Proof. By the same reasoning as in the proof of Theorem 4.2, we can find the mapping $\delta : \mathcal{A} \to \mathcal{B}$ given by

$$\delta(x) := \lim_{m\to\infty} \frac{1}{|\rho|^m} f(\rho^m x) \tag{29}$$

for all $x \in \mathcal{A}$. It follows from (6), (28) and (29) that

$$\begin{aligned}\mu^{\mathcal{A}}_{\delta([x,y])-[\delta(x),y]-[x,\delta(y)]}(t) &= \lim_{m\to\infty} \mu^{\mathcal{A}}_{f(\rho^{2m}[x,y])-[f(\rho^m x),\rho^m y]-[x,f(\rho^m)]}(|\rho|^{2m} t)\\ &\geq \lim_{m\to\infty} \psi_{\rho^m x,\rho^m y}(|\rho|^{2m} t) = 1\end{aligned}$$

for all $x, y \in \mathcal{A}$ and $t > 0$, then

$$\delta([x, y]) = [\delta(x), y] + [x, \delta(y)]$$

for all $x, y \in \mathcal{A}$. Thus, $\delta : \mathcal{A} \to \mathcal{A}$ is a Lie derivation satisfying (24), as desired. □

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