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A trust transitivity model for group decision making in social network with intuitionistic fuzzy information

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Abstract. Group decision-making problems within social relationships among experts are called SN-GDM problems and have been widely considered in many fields, such as management, social science, and natural science. In SN-GDM problems, the trust relationships among experts possess uncertainty and attenuation during propagation. However, few studies have focused on the two issues at the same time. This work aims to develop a trust transitivity model for SN-GDM with intuitionistic fuzzy (IF) information. First, to model the trust relationships among experts, we present an intuitionistic fuzzy trust transitivity model that contains a trust propagation operator and an aggregation operator. Second, some concepts of IF trust network centrality analysis are defined and applied to determine expert weights. Third, a new approach is proposed to solve SN-GDM problems with IF information. Finally, comparison analysis is conducted to highlight the advantages of the proposed approach. An example is also provided to show the validity of the proposed method.

1. Introduction

With the rapid development of the Internet technology, online social networks and servers have become an important part of our lives [1]. As emphasized in social psychology [2], the opinions and social influence of ones friends or colleagues with high prestige can affect other peoples decision. In practical applications, many group decision-making (GDM) processes occur in environments where a trust relationship exists among experts who share friendships and common interests [3]. Some websites provide review forums, where people can share their opinions on products and services to help others make decisions. An essential association results from social relationships among members in a group, such as Ciao and Epinions. The trust relationship among members can be built by completing an online form that reflects trust and distrust. In the current work, GDM problems within trust relationships among experts are simply called social network GDM (SN-GDM) problems [5-16].

As a new research branch of GDM, SN-GDM has gained increasing research attention [4]. Existing SN-GDM methods are mostly based on mathematical uncertainty methods. According to different trust

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metrics, existing SN-GDM methods can be roughly divided into the following categories. (1) Continuous value based methods [5, 6]. Using values of membership function to represent fuzzy adjacency relations between experts, Brunelli et al. [5] constructed a fuzzy m-ary adjacency relation and established two optimization rules for solving the problems of majority threshold in GDM problems. Prez et al. [6] built a experts social influence network to handle GDM problems, which include the following three steps: providing information, modeling influence and obtaining the solution. (2) Linguistic values based methods [7, 8]. The work in [7] pointed out that linguistic terms are suitable for the representation of social relationships, e.g., very distrust, distrust, medium, trust, very trust. Prez et al. [8] applied 2-tuple linguistic to describe the relationships among experts on the basis of trustworthiness. In view of the node in-degree, node proximity degree and node rank prestige, three SNA 2-tuple linguistic based IOWA operators are developed and used for GDM problems. (3) Intervals based methods [9-11]. Shakeri and Bafghi [9] investigated a trust propagation algorithm on the basis of trust interval multiplication. Thereafter, they proposed a confidence-aware layer model and applied it to the trust management decision system [10]. In [11], the trust relationships among experts were represented in an interval-valued fuzzy sociomatrix and then used in seclection of GDM problems. (4) Four tuple information based methods [12,13]. Wu et al. [12] measured trust relation with four elements, namely, trust, distrust, inconsistency and hesitancy, and put forward uninorm trust propagation and aggregation methods for SN-GDM. Since experts might have fuzzier and more uncertainty opinions on alternatives, Liu et al. [13] defined the concept and order relation of interval-valued trust functions in which the trust and distrust are then denoted by intervals, and then developed a trust induced recommendation mechanism for GDM, to ensure that the group arrive a higher consensus level.

The above researches have turn out to be the important driving force for the field of GDM. However, these works have overlooked either the inherent uncertainty of trust or the attenuation of trust in propagation, and few have considered them jointly. Naturally, then, there is a question that whether and how the uncertainty and attenuation of trust can be embodied together in trust transitivity model. Intuitionistic fuzzy (IF) sets (IFSs) [14] is a useful tool to describe the uncertainty and fuzziness of trust relationship [15,16]. In this paper, we attempt to address the aforementioned issues by focusing on an IF trust transitivity model (IFTTM) to deal with SN-GDM problems. The main contributions of this work are as follows: (1) an IF trust transitivity model which contains trust propagation and trust aggregation is developed to build indirect trust relationships between experts; (2) an IF degree centrality based method is presented to determine expert weights; (3) a new approach is proposed to solve SN-GDM problems with IF information.

The rest of the paper is structured as follows. In Section 1, we review some relevant concepts of IF trust network. In Section 2, we describe he IFTTM in detail. In Section 3, we propose our method for SN-GDM with IF information. In Section 4, comparision analyses are performed. In Section 5, we present a practical example concerned with SN-GDM to demonstrate the feasibility of the proposed method. Conclusions are likewise provided.

2. Preliminaries

2.1. Intuitionistic fuzzy set

Definition 1 [16]. Let $\tilde{A} = \{< x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x) > | x \in X\}$ be an intuitionistic fuzzy set (IFS) in X, where $\mu_{\tilde{A}}(x) : X \to [0,1]$ and $v_{\tilde{A}}(x) : X \to [0,1]$, with the condition $\mu_{\tilde{A}}(x) + \mu_{\tilde{A}}(x) \in [0,1]$. The parameters $\mu_{\tilde{A}}(x)$ and $v_{\tilde{A}}(x)$ indicate, respectively, the membership degree and non-membership degree of the element x in $\tilde{A}$. The third parameter $\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - v_{\tilde{A}}(x)$, $x \in X$ is called hesitation degree or Atanassov's intuitionistic index of x in $\tilde{A}$ [16]. Obviously, $\pi_{\tilde{A}}(x) \in [0,1], x \in X$.

For convenience, Xu [16] calls $\tilde{\alpha} = (\mu_{\tilde{\alpha}}, v_{\tilde{\alpha}})$ an intuitionistic fuzzy number (IFN), where $\mu_{\tilde{\alpha}} \in [0,1]$, $v_{\tilde{\alpha}} \in [0,1]$, $\mu_{\tilde{\alpha}} + v_{\tilde{\alpha}} \in [0,1]$, $\mu_{\tilde{\alpha}} + v_{\tilde{\alpha}} + \pi_{\tilde{\alpha}} = 1$. To rank IFNs, the score function $s(\tilde{\alpha})$ and accuracy function $h(\tilde{\alpha})$ of α can be represented as follows:

$$s(\tilde{\alpha}) = \mu_{\tilde{\alpha}} - v_{\tilde{\alpha}} \tag{1}$$

$$h(\tilde{\alpha}) = \mu_{\tilde{\alpha}} + v_{\tilde{\alpha}} \tag{2}$$

Definition 2 [15]. If $\tilde{\alpha}_1 = (\mu_{\tilde{\alpha}_1}, v_{\tilde{\alpha}_1})$ and $\tilde{\alpha}_2 = (\mu_{\tilde{\alpha}_2}, v_{\tilde{\alpha}_2})$ are any two IFNs, then:
1) If $s(\tilde{\alpha}_1) > s(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1 > \tilde{\alpha}_2$.
2) If $s(\tilde{\alpha}_1) = s(\tilde{\alpha}_2)$, then: if $h(\tilde{\alpha}_1) > h(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1 > \tilde{\alpha}_2$; if $s(\tilde{\alpha}_1) = s(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1 = \tilde{\alpha}_2$.
Definition 3 [15] For two IFNs $r_1 = (\mu_1, v_1)$ and $r_2 = (\mu_2, v_2)$, the distance can be defined as

$$dis(r_1, r_2) = \frac{1}{2}(|\mu_1 - \mu_2| + |v_1 - v_2|) \tag{3}$$

2.2. Intuitionistic fuzzy trust network centrality analysis

Definition 4 Let an IFN $r =< \mu, v >$ be trust score between experts, where μ is a trust degree, v is a distrust degree, $\pi = 1 - \mu - v$ is hesitation degree. The set of trust score space is denoted by

$$\Delta = \{r =< \mu, v > |\mu, v \in [0, 1]\} \to [0, 1]^2 \tag{4}$$

Definition 5 The fuzzy relationship between experts can be defined as a IF trust network (IFTN) that can be formally represented by a IF or weighted directed graph $\tilde{G} = (E, \tilde{R})$, where $E = (e_1, e_2, \cdots, e_n)$ is a expert set, e_i is the ith expert. $\tilde{R} = \sum_{i=1}^{n} \sum_{j=1}^{n} r_{ij} \subset E \times E$ is a edge set between experts, which can be seen as the degree of connection on E. $r_{ij} =< \mu_{ij}, v_{ij} >\in \tilde{R}$ is the IF relations between expert i and j, which indicates the trust value between expert i and j. Where μ_{ij} and v_{ij} denote trust and distrust information respectively.

Definition 6 Let $\tilde{G} = (E, \tilde{R})$ be a IFTN, $d(e_i) = \sum_{j=1, j\neq i}^{n} r_{ij}$ is the sum of the IF relations between expert e_i and others, then $c(e_i)$ is defined IF degree centrality (IFDC) of e_i, is given as

$$c(e_i) = d(e_i)/(n - 1) \tag{5}$$

Since $c(e_i)$ takes into account the size of the group, it can be used to compare networks of different sizes. Minor and Michael [17] pointed out that an entity with high degree centrality school is the site of the network. It indicates that the entity occupies the center of the network. The larger the $c(e_i)$, the higher the influence of expert in group.

Definition 7 Let $\tilde{G} = (E, \tilde{R})$ be a IFTN composed by an expert group, which the number of experts is n, is degree centrality of , then is defined group IF degree centrality (GIFDC) of the expert group, is given as

$$c(g) = \sum\nolimits_{i=1}^{n} c(e_i) \Big/ n \tag{6}$$

3. Trust transitivity model

3.1. Trust propagation

In real SN-GDM environment, some experts might not have a collaborative relationship in advance, or one might need detailed information about the other. Trust must be propagated to an unknown expert by recommending information from third parties. Zhang and Mao [18] developed a belief propagation algorithm by reducing the propagation operators. Wu et al. [12] constructed a uninorm propagation operator to propagate both trust and distrust simultaneously. However, the results ignored trust reduction and distrust accumulation, because concatenation propagation does not increase trust information and does not reduce distrust information in Axiom 2 given by [19]. The general concepts of the t-conorm S and t-norm T is suitable for defining the trust propagation operator and aggregation operator [20]. S can be denoted by $S(a, b) = 1 - T(1 - a, 1 - b), \forall(a, b) \in [0, 1]^2$. Based on the above issue, a trust propagation operator is defined as follows.

Definition 8 For two trust score $r_1 =< \mu_1, v_1 >$ and $r_2 =< \mu_2, v_2 >$, trust propagation operator on Δ is a mapping $P : \Delta \times \Delta \to \Delta$ can be expressed as follows:

$$P(r_1, r_2) =< T(\mu_1, \mu_2), S(v_1, v_2) > \tag{7}$$

where $T(\mu_1, \mu_2) = \log_2(1 + (2^{\mu_1} - 1)(2^{\mu_2} - 1))$ and $S(v_1, v_2) = 1 - \log_2(1 + (2^{1-v_1} - 1)(2^{1-v_2} - 1))$ are Frank t-norm and t-conorm [21] respectively.

Theorem 1 The trust propagation operator $P(r_1, r_2)$ satisfies the following properties:
(i) (**Trust non-accumulation**) $T(\mu_1, \mu_2) \leq \mu_1$ and $T(\mu_1, \mu_2) \leq \mu_2$;
(ii) (**Distrust non-reduction**) $S(v_1, v_2) \geq v_1$ and $S(v_1, v_2) \geq v_2$;
(iii) (**Double fully trust**) If $r_1 =< 1, 0 >$ and $r_2 =< 1, 0 >$, then $P =< 1, 0 >$;
(iv) (**Single fully trust**) If $r_1 =< 1, 0 >$ (or $r_2 =< 1, 0 >$), then $P = r_2$ (or $P = r_2$);
(v) (**Fully distrust**) If $r_1 \vee r_1 =< 0, 1 >$, then $P =< 0, 1 >$;
(vi) (**Associativity**) $P(P(r_1, r_2), r_3) = P(r_1, P(r_2, r_3))$;
(vii) (**Commutativity**) $P(r_1, r_2) = P(r_2, r_1)$.
Proof. Obviously, the $P(r_1, r_2)$ meets (iii)-(vii) of Theorem 1. We need to prove (i) and (ii).
For (i), we have

$$\begin{aligned}\mu_1 - T(\mu_1, \mu_2) =& \mu_1 - \log_2(1 + (2^{\mu_1} - 1)(2^{\mu_2} - 1)) = \log_2 2^{\mu_1} - \log_2(1 + (2^{\mu_1} - 1)(2^{\mu_2} - 1)) \\ =& \log_2(2^{\mu_1}/(1 + (2^{\mu_1} - 1)(2^{\mu_2} - 1)))\end{aligned}$$

Since $\frac{2^{\mu_1}}{(1+(2^{\mu_1}-1)(2^{\mu_2}-1))} - 1 = \frac{2^{\mu_1+1}+2^{\mu_2}-2^{\mu_1+\mu_2}-2}{(1+(2^{\mu_1}-1)(2^{\mu_2}-1))} = \frac{(2^{\mu_1}-1)(2-2^{\mu_2})}{(1+(2^{\mu_1}-1)(2^{\mu_2}-1))} \geq 0$. We can obtain $\mu_1 - T(\mu_1, \mu_2) \geq 0$.
By the same reason, we can get $\mu_2 - T(\mu_1, \mu_2) \geq 0$. Hence, $T(\mu_1, \mu_2) \leq \mu_1$ and $T(\mu_1, \mu_2) \leq \mu_2$.
For (ii), we have

$$\begin{aligned}S(v_1, v_2) - v_1 =& 1 - \log_2(1 + (2^{1-v_1} - 1)(2^{1-v_2} - 1)) - v_1 = \log_2 2 - \log_2(1 + (2^{1-v_1} - 1)(2^{1-v_2} - 1)) - \log_2 2^{v_1} \\ =& \log_2(2\big/(2^{v_1} + (2 - 2^{v_1})(2^{1-v_2} - 1)))\end{aligned}$$

Since $\frac{2}{(2^{v_1}+(2-2^{v_1})(2^{1-v_2}-1))} - 1 = \frac{(1-2^{-v_2})(2^2-2^{1+v_1})}{(2^{v_1}+(2-2^{v_1})(2^{1-v_2}-1))} \geq 0$, We can obtain $S(v_1, v_2) - v_1 \geq 0$.
By the same reason, we can get $S(v_1, v_2) - v_2 \geq 0$. Hence, $S(v_1, v_2) \geq v_1$ and $S(v_1, v_2) \geq v_2$.

That is to say, the trust propagation operator satisfies (i) and (ii) of Theorem 1, which are consistent with Axiom 2 in [19]. For example 1, by Eq. (7), we get $P(r_1, r_2) =< 0.08, 0.66 >$, which is in agreement with Axiom 2 in [19]. Therefore, the proposed propagation operator is more reasonable.

A propagation path may contain more than three experts in practical GDM problems. For instance, there is a possible path $\rho_1 = (e_4, e_3, e_1, e_2)$ from e_4 to e_2 in Figure 1. According to the associatiity property of Theorem 1, a generalized trust propagation operator can be derived as follows.

$$P(r_1, r_2, \cdots, r_n) =< T(\mu_1, \mu_2 \cdots, \mu_n), S(v_1, v_2 \cdots, v_n) > \tag{8}$$

where $T(\mu_1, \mu_2 \cdots, \mu_n) = \log_2(1 + \prod_{i=1}^{n}(2^{\mu_i} - 1))$ and $S(v_1, v_2 \cdots, v_n) = 1 - \log_2(1 + \prod_{i=1}^{n}(2^{1-v_i} - 1))$.

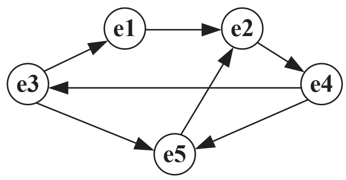

Figure 1: An example of experts' trust network

3.2. *Trust aggregation*

We know about multiple paths between two unknown experts in SN-GDM. To aggregate trust and distrust, Victor et al. [22] investigated some properties of aggregation operators such as trust and distrust boundary preservation. Wu et al. [12] utilized the shortest path to replace the aggregation results. In this section, we focus on the weighted average aggregation operator.

Definition 9 For two trust score $r_1 =< \mu_1, v_1 >$ and $r_2 =< \mu_2, v_2 >$, trust aggregation operator on Δ is a mapping $A : \Delta \times \Delta \to \Delta$ can be expressed as follows:

$$A(r_1, r_2) =< S(\mu_1, \mu_2), T(v_1, v_2) > \tag{9}$$

where $T(v_1, v_2) = \log_2(1 + (2^{v_1} - 1)(2^{v_2} - 1))$ and $S(\mu_1, \mu_2) = 1 - \log_2(1 + (2^{1-\mu_1} - 1)(2^{1-\mu_2} - 1))$ are Frank t-conorm and t-norm [21] respectively.

Considering that there are more than three paths between two experts and longer the path, the less influence, a trust aggregation operator is defined as follows.

Definition 10. For a set of trust score $\lambda_i =< \mu_i, v_i >\in \Delta(i = 1, 2, \cdots, n)$ that have associated an importance weight vector $\mathbf{w} = \{w_1, w_2, \cdots, w_n\}^{\mathrm{T}}$ with $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$. We call:

$$IFTWA(\lambda_1, \lambda_2, \cdots, \lambda_n) =< 1 - \log_2(1 + \prod_{i=1}^{n} (2^{1-\mu_i} - 1)^{w_i}), \log_2(1 + \prod_{i=1}^{n} (2^{v_i} - 1)^{w_i}) > \tag{10}$$

IF trust weighted average (IFTWA) operator. If $\mathbf{w} = \{1/n, 1/n, \cdots, 1/n\}^{\mathrm{T}}$, then the IFTWA operator reduces to an IF trust average (IFTA) operator:

$$IFTA(\lambda_1, \lambda_2, \cdots, \lambda_n) =< 1 - \log_2(1 + \prod_{i=1}^{n} (2^{1-\mu_i} - 1)^{1/n}), \log_2(1 + \prod_{i=1}^{n} (2^{v_i} - 1)^{1/n}) > \tag{11}$$

Let there be n path $\rho_i =< \mu_i, v_i > (i = 1, 2, \cdots, n)$ between the e_i and e_j, the length of ρ_i be $l(\rho_i)$. The longer the path is, the smaller the reliability of its propagation trust is [19, 22]. If $w_i = \theta_i = \frac{1/l(\rho_i)}{\sum_{i=1}^{n} 1/l(\rho_i)}$, then length based $IFTWA$ (L_IFTWA) operator can be showed as follows

$$L\_IFTWA(\rho_1, \rho_2, \cdots, \rho_n) =< 1 - \log_2(1 + \prod_{i=1}^{n} (2^{1-\mu_i} - 1)^{\theta_i}), \log_2(1 + \prod_{i=1}^{n} (2^{v_i} - 1)^{\theta_i}) > \tag{12}$$

Theorem 2 It is easily proved that $L\_IFTWA(\rho_1, \rho_2, \cdots, \rho_n)$ satisfies the following properties:
(i) (fully distrust) If $\forall \rho_i =< 0, 1 >, i = 1, 2, \cdots, n$, then $L\_IFTWA(\rho_1, \rho_2, \cdots, \rho_n) =< 0, 1 >$;
(ii) (Single trust) If $\exists \rho_i =< 1, 0 >, i = 1, 2, \cdots, n$, then $L\_IFTWA(\rho_1, \rho_2, \cdots, \rho_n) =< 1, 0 >$;
(iii) (Idempotency) If $\forall \rho_i = \rho, i = 1, 2, \cdots, n$, then $L\_IFTWA(\rho_1, \rho_2, \cdots, \rho_n) = \rho$;
(iv) (Boundedness) Let $\rho^- =< \min\{\mu_i\}, \max\{v_i\} >$, $\rho^+ =< \max\{\mu_i\}, \min\{v_i\} >$, then

$$\rho^- \le L\_IFTWA(\rho_1, \rho_2, \cdots, \rho_n) \le \rho^+;$$

(v) (Monotonicity) If $\rho_i \le \rho_i^*$, $i = 1, 2, \cdots, n$, then

$$L\_IFTWA(\rho_1, \rho_2, \cdots, \rho_n) \le L\_IFTWA(\rho_1^*, \rho_2^*, \cdots, \rho_n^*);$$

4. The proposed Trust transitivity model for SN-GDM

4.1. Determine expert weights

The work in [23] point out that the average of all individual decisions might be the best decision in GDM process. the individual decision closer to the average of individual preference, the larger the decision-making effect of expert [24, 25]. Analogously, the indicator GDC reflects the overall importance of a group in SN-GDM. So, by computing the closeness between individual IFDC and GIFDC, we can be derive the individual weight.

Definition 11. In a given group trust relationship network $\tilde{G} = (E, \tilde{R})$, the closeness of the DC on individual expert e_i with respect to the DC on expert set E, is defined as

$$\delta_i = 1 - dis(c(e_i) - c(g)) \tag{13}$$

where $dis(c(e_i) - c(g))$ is the distance between $c(e_i)$ and $c(g)$. Clearly, $\delta_i \in [0, 1]$.

Then, individual DC closer to GDC, the larger the importance of expert. Consequently, the expert weight can be caculated as follows:

$$w_i = \delta_i \Big/ \sum_{i=1}^{n} \delta_i \tag{14}$$

4.2. Proposed algorithm for SN-GDM

Based on the aforesaid model and annlysis, the algorithm steps for dealing with SN-GDM under IF environment can be set up as follows:

Step 1. Construct the direct trust sociomatrix by the given direct trust relations between experts.

Step 2. Calculate the indirect trust scores by the trust transitivity model. According to propagation operator Eq. (8), we can calculate the trust score of each path between any two unknown experts. By Eq. (11), the trust score of two unknown experts can be obtained.

Step 3. Determine expert weights. Using Eqs. (5) and (13), the closenesses of each expert with respect to group can be computed. Then, we can obtain the expert weights by Eq. (14).

Step 4. Aggregate the evaluation matrices given by experts into a collective evaluation matrix according to Eq. (10).

Step 5. Derive the collective overall evaluation of each alternative and choose the best one by the Definition 2.

The decision-making process of the proposed algorithm is depicted in Fig. 2.

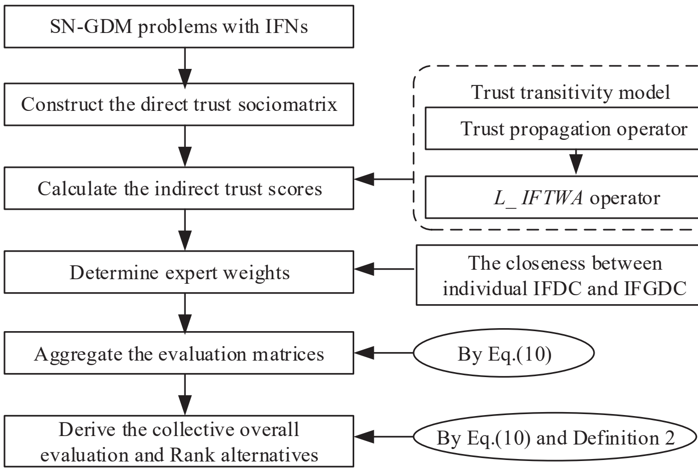

Figure 2: The decision-making process of the proposed algorithm

5. Comparision with existing method

In this section, we make comparision analyses between two existing methods [11,12] and the current method from several aspects. For more details are listed in Table 1.

(1) Both Wu et al. 's method [12] and the current method can describe the trust, distrust and hesitancy information of social relationship between experts by IFNs, whereas the method [11] can only deal with trust information by interval-values. Hence, the proposed method and Wu et al. 's [12] method can deliver more useful information.

(2) Both Wu et al. 's method [12] and the current method can deduce the trust scores between experts who don't know each other through propogation operators, whereas the method [11] can't do it. Since the social matrices constructed by the former two are more dense, they can exploit a more reliable source to calculate the experts weights.

(3) Without considering the attenuation of trust propogation, the method [12] causes some unreasonable results. For example, given $r_1 =< 0.6, 0.2 >$ and $r_2 =< 0.6, 0.2 >$, using current method, we have $P(r_1, r_2) =< 0.34, 0.36 >$. So trust component decreased which is coincident with the Axiom 2 in [19]. However, by the uninorm trust propagation operator [12], we obtain $P(r_1, r_2) =< 0.69, 0.27 >$, which is unreasonable since the trust component increased.

Table 1: Comparisons with existing methods.

Characteristics	Method [11]	Method [12]	The current method
Solved problem	SN-GDM problem	SN-MAGDM problem	SN-MAGDM problem
Expert weight	By linguistic quantifier	By linguistic quantifier	By social network analysis
Trust metric	Trust	Trust, distrust and hesitancy	Trust, distrust and hesitancy
Trust transitivity	Non	Based on cross ratio uninorm	Based on t-norms and t-conorms
Trust propagation	Non	Non-considered trust attenuation	Considered trust attenuation
Trust aggregation	Non	Selected the shortest path	L_IFTWA operator

Table 2: The decision matrix of four companies.

Experts	Companies	a_1	a_2	a_3	a_4
e_1	S_1	$<0.5,0.4>$	$<0.6,0.1>$	$<0.7,0.1>$	$<0.9,0.0>$
	S_2	$<0.7,0.1>$	$<0.8,0.1>$	$<0.4,0.3>$	$<0.6,0.1>$
	S_3	$<0.8,0.2>$	$<0.7,0.1>$	$<0.5,0.5>$	$<0.4,0.1>$
	S_4	$<0.4,0.4>$	$<0.4,0.6>$	$<0.6,0.1>$	$<0.5,0.2>$
e_2	S1	$<0.5,0.3>$	$<0.7,0.1>$	$<0.8,0.2>$	$<0.9,0.1>$
	S_2	$<0.6,0.3>$	$<0.8,0.2>$	$<0.4,0.4>$	$<0.5,0.5>$
	S_3	$<0.7,0.3>$	$<0.6,0.4>$	$<0.5,0.2>$	$<0.6,0.3>$
	S_4	$<0.4,0.2>$	$<0.8,0.1>$	$<0.5,0.3>$	$<0.8,0.1>$
e_3	$S1$	$<0.7,0.2>$	$<0.6,0.1>$	$<0.7,0.3>$	$<0.9,0.1>$
	S_2	$<0.8,0.2>$	$<0.4,0.4>$	$<0.8,0.1>$	$<0.5,0.3>$
	S_3	$<0.7,0.3>$	$<0.5,0.2>$	$<0.7,0.1>$	$<0.6,0.3>$
	S_4	$<0.6,0.3>$	$<0.5,0.3>$	$<0.4,0.6>$	$<0.7,0.3>$
e_4	S_1	$<0.5,0.4>$	$<0.4,0.4>$	$<0.5,0.5>$	$<0.8,0.2>$
	S_2	$<0.4,0.1>$	$<0.5,0.3>$	$<0.5,0.2>$	$<0.7,0.3>$
	S_3	$<0.6,0.2>$	$<0.6,0.3>$	$<0.5,0.3>$	$<0.8,0.2>$
	S_4	$<0.8,0.2>$	$<0.4,0.3>$	$<0.6,0.1>$	$<0.7,0.3>$

(4) We develops a length-based IF weighted average operator which takes account for the length of path and all possible effective paths between unknown experts, whereas the aggregated value just consider the shortest path in [12]. Thus, the method [12] more easily lost information.

6. An example analysis

A person likes to invest a sum of money to an investment company, with four possible companies to select from computer company S_1, food company S_2, car company S_3 and TV company S_4; and the following four attributes (whose weighted vector is $\Omega = (0.32, 0.26, 0.18, 0.24)^T$): growth a_1, environmental impact a_2, risk a_3 and social-political impact index a_4.

Duo to the lack of investment related knowledge, the person wants to consult his circle of friends and treat them as experts. Therefore, the investment company selection can be seen as a SN-GDM, which decision matrices $X^k = (x_{ij}^k)_{4\times4}$ are showed in Table 2.

Step 1. Suppose that a panel of experts is constituted by five persons whose trust relationship network

Table 3: The decision matrix of four companies.

	a_1	a_2	a_3	a_4
S_1	$< 0.563, 0.333 >$	$< 0.693, 0.131 >$	$< 0.639, 0.287 >$	$< 0.854, 0.000 >$
S_2	$< 0.643, 0.191 >$	$< 0.654, 0.234 >$	$< 0.545, 0.187 >$	$< 0.59, 0.261 >$
S_3	$< 0.710, 0.251 >$	$< 0.631, 0.233 >$	$< 0.594, 0.254 >$	$< 0.594, 0.247 >$
S_4	$< 0.690, 0.000 >$	$< 0.520, 0.266 >$	$< 0.527, 0.175 >$	$< 0.666, 0.178 >$

is depicted in Figure 1 with corresponding trust sociomatrix R.

$$R = \begin{pmatrix} - & < 0.6, 0.2 > & & & \\ & - & & < 0.4, 0.5 > & \\ < 0.6, 0.1 > & & - & & < 0.6, 0.3 > \\ & & < 0.5, 0.4 > & - & < 0.6, 0.2 > \\ & < 0.8, 0.1 > & & & - \end{pmatrix}$$

Step 2. Note that this matrix provides trust score based on direct trust, not based on indirect trust. Then, using the trust transitivity model, the trust scores of nonadjacent nodes can be computed. For example, there are three possible paths $\rho_1 = (e_4, e_3, e_1, e_2)$, $\rho_2 = (e_4, e_5, e_2)$ and $\rho_3 = (e_4, e_3, e_5, e_2)$ from e_4 to e_2. According to Eq. (8), we have $r_{\rho 1} =< 0.15, 0.59 >$, $r_{\rho 2} =< 0.47, 0.29 >$ and $r_{\rho 3} =< 0.21, 0.65 >$. Since $l(\rho_1) = 3$, $l(\rho_2) = 2$, $l(\rho_3) = 3$, the weights of the three paths are $\theta_1 = 0.286$, $\theta_2 = 0.429$, $\theta_3 = 0.286$. Using the L_IFTWA operator, we obtain $r_{42} =< 0.32, 0.45 >$. Likewise, the trust sociomatrix R can be completed as follows:

$$R = \begin{pmatrix} - & < 0.60, 0.20 > & < 0.10, 0.79 > & < 0.22, 0.61 > & < 0.10, 0.76 > \\ < 0.10, 0.76 > & - & < 0.18, 0.72 > & < 0.40, 0.50 > & < 0.16, 0.71 > \\ < 0.60, 0.10 > & < 0.41, 0.33 > & - & < 0.14, 0.68 > & < 0.60, 0.30 > \\ < 0.28, 0.47 > & < 0.32, 0.45 > & < 0.50, 0.40 > & - & < 0.60, 0.20 > \\ < 0.07, 0.79 > & < 0.80, 0.10 > & < 0.13, 0.76 > & < 0.31, 0.56 > & - \end{pmatrix}$$

Next, based on the above sociomatrix, by Eq. (5), we have the experts IFDC $c(e_1) =< 0.29, 0.43 >$, $c(e_2) =< 0.57, 0.24 >$, $c(e_3) =< 0.24, 0.65 >$, $c(e_4) =< 0.21, 0.64 >$ and $c(e_5) =< 0.40, 0.43 >$. According to Eq. (6), we have group IFDC $c(g) =< 0.35, 0.45 >$.

Step 3. From Eqs. (5) and (13), the closenesses of each expert with respect to group are found to be $\delta_1 = 0.955$, $\delta_2 = 0.784$, $\delta_3 = 0.845$, $\delta_4 = 0.835$, $\delta_5 = 0.967$. By Eq. (14), we can derive the weights of the five experts as $w_1 = 0.218$, $w_2 = 0.179$, $w_3 = 0.193$, $w_4 = 0.190$, $w_5 = 0.221$.

Step 4. by Applying the IFTWA operator, the evaluation matrices $X^k = (x_{ij}^k)_{4\times 4}$ given by experts can be aggregated into a collective evaluation matrix X. The result is showed in Table 3.

Step 5. According to the attribute weighted vector $\Omega = (0.32, 0.26, 0.18, 0.24)^T$, the collective overall evaluation of each company is derived as $S_1 =< 0.702, 0.000 >$, $S_2 =< 0.608, 0.216 >$, $S_3 =< 0.644, 0.246 >$, $S_4 =< 0.617, 0.000 >$. The application of the Definition 2 results in the following order relation of the four possible companies: $S_1 \succ S_4 \succ S_3 \succ S_2$. Therefore, the best investment company is computer company S_1.

From the above computing process, the proposed approach can obain a complete trust sociomatrix by the trust transitivity model, which give full consideration to the attenuation and uncertainty of trust, to guarantee the completeness of the social matrix. Furthermore, the weights of experts are derived objectively through their IFDC. Therefore, the ranking order is effective and reasonable.

7. conclusion

This article develops a IF trust transitivity model and applies to SN-GDM with IF information. The main features are summed up as follows: (1) In the view of t-norms and t-conorms, we propose the IF trust propagation operator, which propagate trust, distrust and uncertainty simultaneously and have some reasonable properties. (2) The L_IFTWA operator is developed to aggregate all possible effective paths between unknown experts. (3) We can objectively determine the expert weights by the closeness degree of individual IFDC with respect to GIFDC. A new approach is showed to deal with SN-GDM under IF environment. The example demonstated the effectiveness and applicability of the proposed approach. However, the proposed approach just is suitable to handle the SN-GDM with IF information. The future work will investigate the SN-GDM with heterogeneous information [15] or type-2 fuzzy number [25] under unknown weight environment.

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