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Fuzzy adaptive sliding-mode control for hypersonic vehicles

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Abstract. This study focuses on a novel anti-disturbance flight control scheme for the near space hypersonic vehicle (NHV) based on the fuzzy sliding-mode control technique and disturbance observer. First, a fuzzy system is employed to approximate the uncertainty nonlinear dynamics; furthermore, an auxiliary fast dynamic observation system is introduced to tackle the system uncertainty and the time-varying unknown external disturbance of the NHV. Second, the dynamic adaptive terminal sliding-mode control strategy is raised to alleviate the chattering phenomenon. Third, the Lyapunov method is used to prove that the adaptive control system can be guaranteed to be stable and the tracking errors are converged in finite time. Finally, the effectiveness of the proposed method is verified with simulation.

1. Introduction

In the last few years, more people have been paying attention to the NHV technology, because the NHV as a novel aerospace aircraft plays an important role in military and civil applications. Compared with traditional flight vehicles, NHV has many outstanding advantages, such as lower launch cost, rapid response ability, strong penetration ability, difficulty of detection, and superior performance of various tasks. On the other hand, the NHV system presents some negative characteristics: a wide range of flight envelopes, severe nonlinearity, strong uncertainty, highly-coupled control channels, and changeable flight environment with time variations. In addition, the different external disturbance and the state variables always interact with each other and other factors, which further increases the control design difficulty for the NHV. Thus, the robust flight control design for the NHV is a challenging task.

Due to the enormous value of military and civilian, more and more researchers are interested in the NHV[1-6][12-15]. As a powerful tool for designing controllers of the near space vehicle flight control systems, sliding mode control (SMC) as an efficient robust strategy has been extensively studied in [7-8]. Since the remarkable robust control performance, the SMC has been widely applied in many practical systems [9]. However, there are still some disadvantages in SMC. The first is the well-known chattering phenomenon which is harmful to the actuators, especially when large parameter uncertainties or external disturbances appear. To overcome this limitation, a novel flight control scheme is proposed using the fuzzy sliding-mode control method. These proposed dynamic attitude control systems can improve control performance of hypersonic vehicles despite uncertainties and external disturbances.

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Combining the disturbance observer and the fuzzy terminal sliding-mode control, a new control scheme is developed to compensate the disturbances for the NHV in this paper. Using the Lyapunov approach, the proposed controller is provided the uniformly asymptotic stability for the hypersonic vehicles system and achieves perfect disturbance compensation. Compared with the existing work, the highlight of this paper is:

a. A novel anti-disturbance flight control scheme is designed for the near space vehicle (NHV) based on the fuzzy sliding-mode control technique and disturbance observer.

b. An auxiliary fast dynamic observation system is introduced to tackle the system uncertainty and the time-varying unknown external disturbance of the NHV.

c. In order to alleviate the chattering phenomenon, the dynamic adaptive terminal sliding-mode control strategy is raised in the paper.

This work is motivated by the fuzzy terminal sliding-mode control with the disturbance observer for the attitude motion of the NHV. First, the six-DOF hypersonic flight model is presented. Second, the robust controller of the HSV with the disturbance and the uncertainty is designed. Using the Lyapunov approach, the proposed controller is provided the uniformly asymptotic stability for the hypersonic vehicles system. Finally, numerical simulation with a nonlinear the six-DOF hypersonic flight model is carried out to demonstrate the performance.

2. Hypersonic vehicle model

According to the six-degrees-of-freedom of the NHV, the mathematic model of the NHV is given by the NASA Langley Research Center [10] as follows:

$$\dot{\mathbf{\Omega}}(t) = f_s(\mathbf{\Omega}(t)) + g_s(\mathbf{\Omega}(t))\omega(t) + g_{s2}(\mathbf{\Omega}(t))\delta(t) \tag{1}$$

$$\dot{\omega}(t) = g_f(\omega(t))u(t) + f_f(\omega(t)) \tag{2}$$

where $\mathbf{\Omega} = [\alpha, \beta, \mu]^T$ represents the slow-loop state, and $\omega = [p, q, r]^T$ represents the fast-loop state.

In order to design the TSM control for the uncertain attitude motion dynamics (1), (2) of the NHV, we give the following lemma:

Lemma 1 ([11]). For a continuous and positive function , if the following condition is held:

$$\dot{V}(t)+\alpha V(t) - \lambda V(t)^{\gamma} \leq 0 \tag{3}$$

Then, $V(t)$ converges in finite time t_s and t_s is decided by

$$t_s \leq t_0 + \frac{1}{\alpha(1+\gamma)} \ln \frac{\alpha V(t_0)^{1-\gamma} + \lambda}{\lambda} \tag{4}$$

where $\alpha > 0$, $\lambda > 0$, and $0 < \gamma < 1$.

3. Design of the robust controller for NHV

3.1. The design of NHV slow-loop controller with uncertainty

The model of slow-loop is described by (1), and the influence of $g_{s2}(\mathbf{\Omega}(t))$ is omitted. Also, the part of the uncertainty is added to meet the requirement of the project as follows:

$$\dot{\Omega}(t) = f_s(\Omega\,(t)) + \Delta f_s(\Omega\,(t)) + g_s(\Omega\,(t))\omega(t) \tag{5}$$

$\Delta f_s(\Omega\,(t))$ represents the uncertainty of $f_s(\Omega\,(t))$.

For the interference of uncertainty, using the fuzzy logic system to approximate, the expression is as followings:

$$y(\mathbf{\Omega}) = \frac{\sum_{i=1}^{r} y^i \left(\sum_{j=1}^{n} \mu_{A_j^i}\left(\mathbf{\Omega}_j\right) \right)}{\sum_{j=1}^{n} \mu_{A_j^i}\left(\mathbf{\Omega}_j\right)} = \hat{\theta}^T \xi(\mathbf{\Omega})$$

Where $\mu_{A_j^i}(\mathbf{\Omega}_j)$ is the function, $\xi(\mathbf{\Omega})$ is the basis function, $\hat{\theta}^T$ is the adjustable parameter, and $y(\mathbf{\Omega})$ is the output of fuzzy system.

Because the fuzzy system has infinite approximation ability, the unknown uncertainty can be described by a fuzzy system and an error.

$$\Delta f_s(\mathbf{\Omega}(t)) = \theta^{*T} \xi(\mathbf{\Omega}, \omega) + \varepsilon(\Omega, \omega)$$

For the system (5), the following dynamic observation system is designed as follows:

$$\dot{\mu}_s = \sigma(\Omega - \mu_s) + f_s(\Omega(t)) + g_s(\Omega(t))\,\omega(t) + {\theta_0}^T \xi(\Omega, \omega) + \chi_\theta + \chi_\varepsilon \tag{6}$$

Where $\sigma > 0$ is the design parameters, θ_0 is the estimated value of θ^*, and χ_θ, χ_ε are the designed adaptive control inputs, which are satisfied:

$$\chi_\varepsilon = \hat{\pi}^2(\varpi_s) / \left[\hat{\pi}\left(\|\varpi_s\|\right) + a_1\right] \tag{7}$$

$$\chi_\theta = \hat{\upsilon}^2(\varpi_s)\xi(\Omega, \omega) / \left[\hat{\upsilon}\left(\|\varpi_s\|\right) \|\xi(\Omega, \omega)\| + a_2\right] \cdot \xi(\Omega, \omega) \tag{8}$$

Where ϖ_s is the slow-loop disturbance observation error, $a_1, a_2 > 0$ are the design parameters, $\hat{\upsilon}$ and $\hat{\pi}$ are the estimate values of υ, π, which are satisfied in the following equation:

$$\|\varepsilon(\Omega, \omega)\| \leq \pi, \quad \left\|\tilde{\theta}\right\| = \|\theta^* - \theta_0\| \leq \upsilon$$

The slow-loop interference observation error is represented by the following:

$$\varpi_s = \Omega - \mu_s$$

Then

$$\dot{\varpi}_s = -\sigma\varpi_s + \tilde{\theta}^T \xi(\Omega, \omega) + \varepsilon(\Omega, \omega) - \chi_\theta - \chi_\varepsilon \tag{9}$$

In order to design the controller of the slow-loop and track the command signals $\Omega_c(t)$ in a limited time, the terminal sliding mode surface can be designed as:

$$\coprod = \Omega_e + \int_0^t (c_1\Omega_e + c_2\Omega_e^{q_1/p_1}) d\tau = 0 \tag{10}$$

where the tracking error is $\Omega_e = \Omega - \Omega_c$, $c_1, c_2 > 0$, $p_1 > q_1$, and p_1, q_1 are odd.

The robust adaptive control of the hypersonic vehicle based on the terminal sliding mode surface can be given as follows:

Theorem 1. For the given system (5), there exist interference observation error dynamic system (9), control law (11) and adaptive law (12) and (13); under the action of the robust adaptive control, the closed-loop system is stable, and the interference observation error is ultimately bounded.

$$\begin{aligned} \omega(t) = & g_s^{-1}(-f_s(\Omega(t)) - {\theta_0}^T \xi(\Omega, \omega) - c_1\Omega_e - c_2|\Omega_e|^{q_1/p_1} sign(\Omega_e) \\ & + \dot{\Omega}_c - \chi_\theta - \chi_\varepsilon - \zeta_1 \coprod - \zeta_2 \left|\coprod\right|^{\tau_1/\delta_1} sign(\coprod)) \end{aligned} \tag{11}$$

$$\dot{\hat{\pi}} = \gamma_1 \left[\|\varpi_s\| - h_1(\hat{\pi} - \pi_0) \right] \tag{12}$$

$$\dot{\hat{\upsilon}} = \gamma_2 [(\|\varpi_s\|) \ \|\xi(\Omega, \omega)\| \ - h_2(\hat{\upsilon} - \upsilon_0)] \tag{13}$$

where π_0, υ_0 are the initial value of π, υ, $h_1, h_2, \zeta_1, \zeta_2 > 0$ are the designed parameters, $\gamma_1, \gamma_2 > 0$.
Proof: Choose the Lyapunov function as

$$V = \frac{1}{2}\varpi_s{}^T \varpi_s + \frac{1}{2\gamma_1}\tilde{\pi}^2 + \frac{1}{2\gamma_2}\tilde{\upsilon}^2 \tag{14}$$

Invoking (9), (12) and (13) , and differentiating (14) yields

$$\begin{aligned}\dot{V} =& \varpi_s^T \dot{\varpi}_s + \frac{1}{\gamma_1}\tilde{\pi}\dot{\tilde{\pi}} + \frac{1}{\gamma_2}\tilde{\upsilon}\dot{\tilde{\upsilon}} \\ =& \varpi_s^T \{-\sigma\varpi_s + \tilde{\theta}^T \xi(\Omega, \omega) + \varepsilon(\Omega, \omega) - \hat{\pi}^2 \varpi_s / [\hat{\pi}(\|\varpi_s\|) + a_1] \\ & - \hat{\upsilon}^2(\varpi_s) \ \ \xi \ \ ^T(\Omega, \omega) / [\hat{\upsilon}(\|\varpi_s\|) \ \|\xi(\Omega, \omega)\| \ + a_2] \cdot \xi(\Omega, \omega)\} \\ & - \tilde{\pi}[\|\varpi_s\| - h_1(\hat{\pi} - \pi_0)] - \tilde{\upsilon}[(\|\varpi_s\|) \ \|\xi(\Omega, \omega)\| \ - h_2(\hat{\upsilon} - \upsilon_0)]\end{aligned}$$

Then

$$\hat{\pi}^2 (\|\varpi_s\|)^2 / [\hat{\pi}(\|\varpi_s\|) + a_1] > \hat{\pi}(\|\varpi_s\|) - a_1$$

$$\hat{\upsilon}^2 (\|\varpi_s\|)^2 \ \ \|\xi(\Omega, \omega)\| \ \ ^2 / [\hat{\upsilon}(\|\varpi_s\|) \ \|\xi(\Omega, \omega)\| \ + a_2] > \hat{\upsilon}(\|\varpi_s\|) \ \|\xi(\Omega, \omega)\| \ - a_2$$

We have

$$\dot{V} \leq -\sigma\varpi_s{}^T \varpi_s + a_1 + a_2 + \tilde{\pi} h_1(\hat{\pi} - \pi_0) \ + \tilde{\upsilon} h_2(\hat{\upsilon} - \upsilon_0) \tag{15}$$

Due to

$$\tilde{\pi}(\hat{\pi} - \pi_0) \leq -\frac{\tilde{\pi}^2}{2} + \frac{(\pi - \pi_0)^2}{2}$$

$$\tilde{\upsilon}(\hat{\upsilon} - \upsilon_0) \leq -\frac{\tilde{\upsilon}^2}{2} + \frac{(\upsilon - \upsilon_0)^2}{2}$$

From (15), we have

$$\dot{V} \leq -\sigma\varpi_s{}^T \varpi_s + a_1 + a_2 - \frac{h_1 \tilde{\pi}^2}{2} + \frac{h_1(\pi - \pi_0)^2}{2} - \frac{h_2 \tilde{\upsilon}^2}{2} + \frac{h_2(\upsilon - \upsilon_0)^2}{2}$$

Let

$$\phi = a_1 + a_2 + \frac{h_1(\pi - \pi_0)^2}{2} + \frac{h_2(\upsilon - \upsilon_0)^2}{2}$$

Choosing the $\|\varpi_s\| > \sqrt{\frac{\phi}{\sigma}}$, or $|\tilde{\pi}| > \sqrt{\frac{2\phi}{h_1}}$, or $|\tilde{\upsilon}| > \sqrt{\frac{2\phi}{h_2}}$. So $\dot{V} < 0$, the interference observation error is ultimately bounded. Thus, this proof is completed.

The convergence of the tracking error of the whole closed-loop system is given in theorem 2.

Theorem 2. For the given system (7), there exist interference observation error dynamic system (9), control law (16) and adaptive law (12) and (13); then the tracking error of the closed-loop system converges to a very small area.

$$\begin{aligned}\omega(t) =& g_s^{-1}(-f_s(\Omega(t)) - \theta_0{}^T \xi(\Omega, \omega) - c_1\Omega_e - c_2|\Omega_e|^{q_1/p_1} sign(\Omega_e) \\ & + \dot{\Omega}_c - \chi_\theta - \chi_\varepsilon - \zeta_1 \coprod - \zeta_2 \left|\coprod\right|^{\tau_1/\delta_1} sign(\coprod))\end{aligned} \tag{16}$$

Where $\zeta_1, \zeta_2 > 0$, are the designed parameters, $\delta_1 > \tau_1 > 0$,

Proof: We defined the following sliding surface as follows:

$$\coprod = \Omega_e + \int_0^t (c_1\Omega_e + c_2\Omega_e^{q_1/p_1})d\tau = 0 \tag{17}$$

differentiating (17) yields $\coprod = \Omega_e + \int_0^t (c_1\Omega_e + c_2\Omega_e^{q_1/p_1})d\tau = 0$

$$\begin{aligned}\dot{\coprod} &= \dot{\Omega}_e + c_1\Omega_e + c_2\Omega_e^{q_1/p_1} \\ &= \dot{\Omega} - \dot{\Omega}_c + c_1\Omega_e + c_2\Omega_e^{q_1/p_1} \\ &= f_s(\mathbf{\Omega}(t)) + g_s(\mathbf{\Omega}(t))\omega(t) + \Delta f_s(\mathbf{\Omega}(t)) - \dot{\Omega}_c + c_1\Omega_e + c_2\Omega_e^{q_1/p_1} \\ &= \Delta f_s(\mathbf{\Omega}(t)) - {\theta_0}^T\xi(\Omega,\omega) - \chi_\theta - \chi_\varepsilon - \zeta_1\coprod - \zeta_2\left|\coprod\right|^{\tau_1/\delta_1} sign(\coprod)\end{aligned}$$

Choosing the Lyapunov function candidate

$$V = \frac{1}{2}\coprod^T\coprod \tag{18}$$

differentiating (18) yields

$$\begin{aligned}\dot{V} &= \coprod^T\dot{\coprod} \\ &= \coprod^T\left[\Delta f_s(\mathbf{\Omega}(t)) - {\theta_0}^T\xi(\Omega,\omega) - \chi_\theta - \chi_\varepsilon - \zeta_1\coprod - \zeta_2\left|\coprod\right|^{\tau_1/\delta_1} sign\left(\coprod\right)\right] \\ &\le -\zeta_1\left\|\coprod\right\|^2 + \bar{\varepsilon}\left\|\coprod\right\| - \zeta_2\left\|\coprod\right\|\left\|\coprod\right\|^{\tau_1/\delta_1} \\ &\le -\zeta_1\left\|\coprod\right\|^2 - \left(\zeta_2\left\|\coprod\right\|^{\tau_1/\delta_1} - \bar{\varepsilon}\right)\left\|\coprod\right\|\end{aligned} \tag{19}$$

Where $\bar{\varepsilon} \ge \left\|\Delta f_s(\mathbf{\Omega}(t)) - {\theta_0}^T\xi(\Omega,\omega) - \chi_\theta - \chi_\varepsilon\right\|$

Choose $\zeta_2\left\|\coprod^{\tau_1/\delta_1}\right\| - \bar{\varepsilon} \ge p > 0$

Then (19) can be changed into

$$\dot{V} \le -2\zeta_1 V - \sqrt{2}pV^{1/2} \tag{20}$$

According to **Lemma 1**, $\coprod$ will be converged to a small area in a limited time.

$$t_1 = \frac{1}{\zeta_1}\ln\frac{2\zeta_1 V(t_0)^{1/2} + \sqrt{2}p}{\sqrt{2}p} \tag{21}$$

Thus, this proof is completed.

3.2. *The design of NHV fast-loop controller with compound disturbance*

Considering the hypersonic vehicle dynamic modeling of the fast-loop equation (2), which includes the uncertainty and external disturbance part:

$$\begin{aligned}\dot{\omega}(t) &= f_f(\omega(t)) + \Delta f_f(\omega(t)) + (g_f(\omega(t)) + \Delta g_f(\omega(t)))u(t) + d_f(t) \\ y_1 &= \omega(t)\end{aligned} \tag{22}$$

where $d_f(t)$ is the external disturbance of the hypersonic vehicle, and $\Delta f_f(\omega(t))$, $\Delta g_f(\omega(t))$ are the system uncertainties, $\psi(\omega,u) = \Delta f_f(\omega(t)) + \Delta g_f(\omega(t))u(t) + d_f(t)$ is the compound disturbance.

In order to eliminate the influence of the uncertainty and external disturbance, the robust terminal sliding mode control law is designed based on adaptive disturbance observer for the compensative control.

The fast-loop dynamic observation system (23) is designed to observe the compound disturbance.

$$\dot{\mu}_f = f_f(\omega, t) + g_f(\omega, t)u(t) + \alpha(\omega - \mu_f) + \beta(\omega - \mu_f)^{\tau/\delta} + \vartheta_\theta + \vartheta_\varepsilon + \theta^T \xi(\omega, u) \tag{23}$$

Where $\varpi = \omega - \mu_f$ is the observer error, ϑ_θ, ϑ_ε are the designed adaptive law, $\alpha, \beta > 0$ are the designed parameters, and $\delta > \tau > 0$ is odd.

Then

$$\dot{\varpi} = -\alpha\varpi - \beta\varpi^{\tau/\delta} + \tilde{\theta}^T \xi(\omega, u) + \varepsilon(\omega, u) - \vartheta_\theta - \vartheta_\varepsilon \tag{24}$$

Where

$$\|\varepsilon(\omega, u)\| \le \lambda, \quad \|\tilde{\theta}\| = \|\theta^* - \theta\| \le \rho \tag{25}$$

$\hat{\lambda}, \hat{\rho}$ are estimates of the λ, ρ and $\tilde{\lambda} = \lambda - \hat{\lambda}$, $\tilde{\rho} = \rho - \hat{\rho}$

The design and analysis of the terminal sliding mode control laws based on adaptive disturbance observer as compensation control are given in theorem 3.

Theorem 3. For the given system (22), there exist interference observation error dynamic system (24), adaptive regulation law (26), (27), (28), (29); then, the observed interference error is ultimately bounded.

$$\dot{\hat{\lambda}} = w_1[\|\varpi\| + \|\varpi^{\tau/\delta}\| - k_1(\hat{\lambda} - \lambda_0)] \tag{26}$$

$$\dot{\hat{\rho}} = w_2[(\|\varpi\| + \|\varpi^{\tau/\delta}\|)\ \|\xi(\omega, u)\| - k_2(\hat{\rho} - \rho_0)] \tag{27}$$

$$\vartheta_\varepsilon = \hat{\lambda}^2(\varpi + \varpi^{\tau/\delta})/[\hat{\lambda}(\|\varpi\| + \|\varpi^{\tau/\delta}\|) + r_1] \tag{28}$$

$$\vartheta_\theta = \hat{\rho}^2(\varpi + \varpi^{\tau/\delta})\ \xi^T(\omega, u)/[\hat{\rho}(\|\varpi\| + \|\varpi^{\tau/\delta}\|)\ \|\xi(\omega, u)\| + r_2] \cdot \xi(\omega, u) \tag{29}$$

Where λ_0, ρ_0 is the initial value of λ, ρ, $r_1, r_2 > 0$, $k_1, k_2 > 0$ are the designed parameters, and $w_1, w_2 > 0$ is the learning rate.

Proof: Defining the Lyapunov function as

$$V = \frac{1}{2}\varpi^T\varpi + \frac{\delta}{\tau + \delta}(\varpi^{(\tau+\delta)/(2\delta)})^T\varpi^{(\tau+\delta)/(2\delta)} + \frac{1}{2w_1}\tilde{\lambda}^2 + \frac{1}{2w_2}\tilde{\rho}^2 \tag{30}$$

Invoking (22), (26) and (27), and differentiating (30) yields

$$\begin{aligned}
\dot{V} &= (\varpi + \varpi^{\tau/\delta})^T\dot{\varpi} + \frac{1}{w_1}\tilde{\lambda}\dot{\hat{\lambda}} + \frac{1}{w_2}\tilde{\rho}\dot{\hat{\rho}} \\
&= (\varpi + \varpi^{\tau/\delta})^T\{-\alpha\varpi - \beta\varpi^{\tau/\delta} + \tilde{\theta}^T\xi(\omega, u) + \varepsilon(\omega, u) - \hat{\lambda}^2(\varpi + \varpi^{\tau/\delta})/[\hat{\lambda}(\|\varpi\| + \|\varpi^{\tau/\delta}\|) + r_1] \\
&\quad - \hat{\rho}^2(\varpi + \varpi^{\tau/\delta})\ \xi^T(\omega, u)/[\hat{\rho}(\|\varpi\| + \|\varpi^{\tau/\delta}\|)\ \|\xi(\omega, u)\| + r_2] \cdot \xi(\omega, u)\} \\
&\quad - \tilde{\lambda}[\|\varpi\| + \|\varpi^{\tau/\delta}\| - k_1(\hat{\lambda} - \lambda_0)] - \tilde{\rho}[(\|\varpi\| + \|\varpi^{\tau/\delta}\|)\ \|\xi(\omega, u)\| - k_2(\hat{\rho} - \rho_0)] \\
&\le -\alpha\|\varpi\|^2 - \beta\|\varpi^{\tau/\delta}\|^2 - \hat{\lambda}^2(\|\varpi\| + \|\varpi^{\tau/\delta}\|)^2/[\hat{\lambda}(\|\varpi\| + \|\varpi^{\tau/\delta}\|) + r_1] \\
&\quad - \hat{\rho}^2(\|\varpi\| + \|\varpi^{\tau/\delta}\|)^2\ \|\xi(\omega, u)\|^2/[\hat{\rho}(\|\varpi\| + \|\varpi^{\tau/\delta}\|)\ \|\xi(\omega, u)\| + r_2] \\
&\quad + \hat{\lambda}(\|\varpi\| + \|\varpi^{\tau/\delta}\|) + \tilde{\lambda}k_1(\hat{\lambda} - \lambda_0) + \hat{\rho}(\|\varpi\| + \|\varpi^{\tau/\delta}\|)\ \|\xi(\omega, u)\| + \tilde{\rho}k_2(\hat{\rho} - \rho_0)
\end{aligned} \tag{31}$$

Considering

$$\hat{\lambda}^2\Big(\|\varpi\| + \big\|\varpi^{\tau/\delta}\big\|\Big)^2 / \left[\hat{\lambda}\left(\|\varpi\| + \big\|\varpi^{\tau/\delta}\big\|\right) + r_1\right] > \hat{\lambda}\left(\|\varpi\| + \big\|\varpi^{\tau/\delta}\big\|\right) - r_1 \tag{32}$$

$$\hat{\rho}^2\Big(\|\varpi\| + \big\|\varpi^{\tau/\delta}\big\|\Big)^2 \|\xi(\omega,u)\|^2 / \left[\hat{\rho}\left(\|\varpi\| + \big\|\varpi^{\tau/\delta}\big\|\right)\|\xi(\omega,u)\| + r_2\right] > \hat{\rho}\left(\|\varpi\| + \big\|\varpi^{\tau/\delta}\big\|\right)\|\xi(\omega,u)\| - r_2 \tag{33}$$

Then

$$\dot{V} \le -\alpha\|\varpi\|^2 + r_1 + r_2 + \tilde{\lambda}k_1(\hat{\lambda} - \lambda_0) \ +\tilde{\rho}k_2(\hat{\rho} - \rho_0) \tag{34}$$

Thus

$$\tilde{\lambda}(\hat{\lambda} - \lambda_0) \le -\frac{\tilde{\lambda}^2}{2} + \frac{(\lambda - \lambda_0)^2}{2} \tag{35}$$

$$\tilde{\rho}(\hat{\rho} - \rho_0) \le -\frac{\tilde{\rho}^2}{2} + \frac{(\rho - \rho_0)^2}{2} \tag{36}$$

From (33)

$$\dot{V} \le -\alpha\|\varpi\|^2 + r_1 + r_2 - \frac{k_1\tilde{\lambda}^2}{2} + \frac{k_1(\lambda - \lambda_0)^2}{2} - \frac{k_2\tilde{\rho}^2}{2} + \frac{k_2(\rho - \rho_0)^2}{2} \tag{37}$$

Let

$$\gamma = r_1 + r_2 + \frac{k_1(\lambda - \lambda_0)^2}{2} + \frac{k_2(\rho - \rho_0)^2}{2}$$

Choosing $\|\varpi\| > \sqrt{\frac{\gamma}{\alpha}}$, or $\left|\tilde{\lambda}\right| > \sqrt{\frac{2\gamma}{k_1}}$, or $\left|\tilde{\rho}\right| > \sqrt{\frac{2\gamma}{k_2}}$. So $\dot{V} < 0$, the interference observation error is ultimately bounded. Thus, this proof is completed.

4. Conclusion

A novel fuzzy robust sliding mode control scheme has been proposed to cope with the convergence speed and control precision for the NHV attitude system. To deal with distinct characteristics of the complex near space vehicle flight control systems, an auxiliary fast dynamic observation system is introduced to tackle the system uncertainty and the time-varying unknown external disturbance of the NHV . Using the proposed controller, the compound interference can be effectively compensated as well as the closed-loop stability is guaranteed.

References

[1] Famularo Douglas, Valasek, John; Muse, Jonathan A.; Bolender, Michael A. Observer-based feedback adaptive control for nonlinear hypersonic vehicles [C]. AIAA Guidance, Navigation, and Control Conference, 2017, 299–305.

[2] Hu Chaofang, Yang Na, Ren, Yanli. Polytopic linear parameter varying model-based tube model predictive control for hypersonic vehicles [J], International Journal of Advanced Robotic Systems, 14(3) (2017) 1–13.

[3] Hao Liu; Deyuan Liu; and Yao Yu, Robust Optimal Controller Design for Generic Hypersonic Vehicles [J], Journal of Aerospace Engineering , 30(4) (2017) 1833–1853.

[4] Klock, Ryan J. Cesnik, Carlos E. S. Nonlinear thermal reduced-order modeling for hypersonic vehicles[J], AIAA Journal, 55(7) (2017) 2358–2368.

[5] Hu Chaofang, Gao Zhifei, Liu Yunbing, Wang Na.Fuzzy Adaptive Dynamic Surface Fault-Tolerant Control for Hypersonic Vehicles [J], Journal of Tianjin University Science and Technology , 50(5) (2017) 491–499.

[6] Naibao He, Gutierrez, HectorQian Gao, and Changsheng Jiang. Fuzzy terminal sliding-mode control for hypersonic vehicles [J] . Journal of Intelligent & Fuzzy Systems, 33(3) (2017) 1831–1839.

[7] Hu Xiaoxiang, Guo Yang, Cao Lijia. Stable inversion based sliding mode control of flexible air-breathing hypersonic vehicles[C], Proceedings-2014 International Conference on Mechatronics and Control, ICMC 2014, 966–970.

[8] Bu Xiang-Wei, Wu Xiao-Yan, Ma Zhen, Zhang Rui. Second-Order Dynamic Sliding-Mode Control for Nonminimum Phase Underactuated Hypersonic Vehicles [J], Journal of Solid Rocket Technology, 27(3) (2010) 310–316.

[9] Yin Qunya, Yi Jianqiang, Tan Xiangmin. Time-varying sliding mode control for air-breathing hypersonic vehicles [J], ICIC Express Letters, 5(4A) (2011) 1031–1036.

[10] Luo Wenli, Li Daochun; Xiang Jinwu. Aerodynamic characteristics analysis of air-breathing hypersonic vehicles at high angle of attack [J], Acta Aeronautica et Astronautica Sinica, 36(1) (2015) 223–231.

[11] Frendreis, Scott G. V. Skujins, Torstens; Cesnik, Carlos E. S. Six-degree-of-freedom simulation of hypersonic vehicles [C] , 2009, AIAA Atmospheric Flight Mechanics Conference.

[12] Dillsaver, Matthew J. Cesnik, Carlos E.S.; Kolmanovsky, Ilya V. Trajectory control of very flexible aircraft with gust disturbance [C]. AIAA Atmospheric Flight Mechanics (AFM) Conference, 2013

[13] Huang, Xiyuan, Wang, Qing; Hou, Delong; Dong, Chaoyang. Model reference adaptation based fault-tolerant control for hypersonic vehicles [J]. Journal of Nanjing University of Aeronautics and Astronautics, 43(S1) (2011) 50–54.

[14] Falkiewicz, Nathan J. Cesnik, Carlos E. S.; Crowell, Andrew R.; McNamara, Jack J. Reduced-order aerothermoelastic framework for hypersonic vehicle control simulation [J]. AIAA Journal, 49(8) (2011) 1625–1646.

[15] Wan, Zhiqiang. Cesnik, Carlos E.S. Geometrically nonlinear aeroelastic scaling for very flexible aircraft [J]. AIAA Journal, 52(10) (2014) 2251–2260.