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# Bi-level chance constraint programming model for hazardous materials distribution

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**Abstract.** With the growing presence of hazardous materials in daily life, a large number of institutions and scholars have been paying close attention to this field, providing new directions for exploring hazardous materials distribution patterns. This paper employs two fuzzy random variables, transportation cost and risk, to put forward a bi-level minimum objective programming model with a chance measure constraint within a specified chance level. The lower level is to seek minimum transportation costs and the upper level is for minimum risk. The model presented in this article simultaneously designs the hybrid algorithm, which is the combination of the fuzzy random simulation with the genetic algorithm. In the end, a small-scale instance is given to account for the efficiency of the presented model and algorithm, and the best distribution solution is presented.

## 1. Introduction

In recent years, hazardous material transportation has played a gradually increasing role in the economy. Hazardous materials have a unique character, which means they may create bad consequences for citizens, the environment, and property. This is why more and more scholars have devoted themselves to this field and have made great contributions. Fuzzy theory can explain the situation in that the consequence cannot be calculated using a mathematical formula; therefore, filling in this significant gap is no other than fuzzy language. As far as we know, the direction of the original is the problem of fuzzy location programming. After discussing fuzzy accessibility by Darentas [1], Zhou and Liu proposed a maximization model in order to minimize the total transportation cost [2]. Chen designed the location of distribution centers using a fuzzy preference relation matrix [3]. To move forward a single step, the time window was considered. Zheng and Liu were no longer confined to consider the transportation cost to be a fuzzy variable, but also thought of the travel time as a fuzzy variable to establish a model in order to improve customer satisfying degree [4]. Wang and Zhao put up a modified model with three layer constraints [5] based on [6]. The previously mentioned models all considered the transportation cost to be one of the important objectives. However, Li and Jiang constructed a multi-objective optimization model for the hazardous materials road transport [7].

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The risk of hazardous material distribution not only has fuzzy attribute, but also random attribute. The recognition of fuzzy random variables was put forward by Puri [8], Kruse and Meyer [9], Liu and Liu [10], and Li and Liu [11]. Meiyi set a scene for the distribution of hazardous materials and presumed the risk was a time-dependent fuzzy random variable. They then explored the optimal departure time and dwell times for each depotCcustomer pair [12]. This inspired us to speculate new distribution methods considering the fuzzy random of risk. Different from Meiyis work with chance-constrained programming (CCP) [13], we built a two-level minimum objective programming model with chance measure constraints for solving the distribution on the hypothesis that every route between each depot-customer pair is different, with the purpose of servicing customers using minimal cost and facing minimal risk within the acceptance range of decision makers.

In this paper, the context is as follows. Section 2 analyses the two-level minimum objective programming model under certain constraints. In Section 3, the design of the hybrid algorithm integrating fuzzy random algorithm and genetic algorithm (GA) is discussed. Section 4 provides a small-scale example to show the best solution. In the end, a brief conclusion is summarized.

## 2. Mathematic model

Next, we formulate a bi-level chance model minimize transportation cost and risk under certain constraints.

### 2.1. Risk quantitative model

Risk is difficult to calculate. Once an accident occurs, we can say there is a certain number of deaths, injured persons, or there is a certain loss of property.As there is no united criterion, using population exposure to model the risk occurring on the transporting path is prevalent. This paper also adopts the model developed by Erkut and Ingolfsson [14], which is

$$Risk = p * POP$$

which means the transportation risk is defined the result of the accident frequency $p$ multiply by the number of people $POP$ that may be at risk. we can know $POP$ is a fuzzy variable while the accident frequency $p$ is a random variable, so the risk is a fuzzy random variable.

Similarity, the transportation cost is a fuzzy random variable. Then, we use these two variables to set up our model.

### 2.2. Bi-level chance constraint model

In this section, we consider a simple two-stage supply chain distribution problem under an uncertain environment. The two-stage chain includes several depots and a certain number of customers. The uncertain environment means the environment has many factors influencing the objectives, such as the traffic congestion status, road-surface behavior, weather situation, or the experience of a trucker and so on. Different from the conventional distribution that customers demand comes from the same depot, several depots can distribute the hazardous material to customers at the same time, and the sum of the distribution these depots can meet the customer demand. The quantity of hazardous materials delivered may vary or the route from the depot to the customer may vary; hence, the risk may be different. The distribution process can be seen in Figure 1.

To set up the bi-level chance model, some symbols need to be defined, as follows:

Index

$i$ depot index, $j$ customer index.

Parameter

$D_i$ the capacity of the depot, $i = 1, 2, ..., I$ .

$x_i$ the sum of all the hazardous material that needs to be distributed by the depot $i$ .

$t_j$ the demand amount of customer $j$ ,$j = 1, 2, ..., J$ .

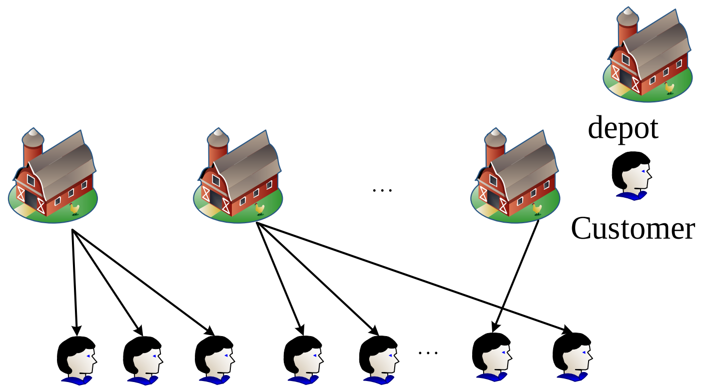


Figure 1: Figure 1 Illustration of the hazardous materials distribution pattern

$y_{ij}$ the amount of hazardous materials transport from depot $i$ to customer $j$ .
$\delta_i$ if it uses depot $i$ ,it takes the value 1, otherwise, it takes 0.
$a_i$ the fixed cost of using depot $i$ .
$b_i$ the unit cost of using the vehicle in depot $i$ .
Random variable
$u_i$ the accident frequency in depot $i$ ; it obeys uniform distribution.
$w_{ij}$ the accident frequency on the route from the depot $i$ to customer $j$,it obeys uniform distribution.
Fuzzy variable
$\widetilde{\zeta}_{ij}$ the unit transportation cost from the depot $i$ to customer $j$ .
$\widetilde{\eta}_i$ the number of influenced people once an accident happens in the depot $i$ .
$\widetilde{\tau}_{ij}$ the number of influenced people once an accident happens on the route from the depot $i$ to customer $j$ .
Fuzzy random variable
$R$ is transportation risk; the risk is denoted as a fuzzy variable multiply a random variable, which is the number of the influenced people multiplied by the accident frequency. According to the attribute of the fuzzy random variable, the risk is a fuzzy random variable.
$C$ is transportation cost; the cost is also a fuzzy random variable, as there are lots of random factors, such as the weather influence, road conditions, traffic condition and so on.

In what follows, we give the expression formula of the risk and cost of the hazardous materials transportation. The total risk mainly consists of two sections, which is distinguished from the place where the accident happens. The first kind is the risk happening in the depot and the second risk is happening on the route between the depot and the customer. This can be expressed as

$$R = \sum_{i=1}^{I} \widetilde{\eta}_i u_i x_i + \sum_{i=1}^{I} \sum_{j=1}^{J} \widetilde{\tau}_{ij} w_{ij} y_{ij}$$

The cost can be referred to the sum of the fixed cost, the total vehicle cost, and the transportation cost between depots and customers:

$$C = \sum_{i=1}^{I} (a_i \delta_i + b_i x_i) + \sum_{i=1}^{I} \sum_{j=1}^{J} \widetilde{\zeta}_{ij} y_{ij}$$

After the above analysis, then the bi-level chance programming model is proposed:

$$\begin{cases} min\ \overline{R} \\ subject\ to: \\ \quad Ch\{\sum_{i=1}^{I}\widetilde{\eta_i}u_ix_i + \sum_{i=1}^{I}\sum_{j=1}^{J}\widetilde{\tau_{ij}}w_{ij}y_{ij} \le \overline{R}\}(\alpha) \ge \beta \\ \quad \begin{cases} min\ \overline{C} \\ subject\ to: \\ \quad Ch\{\sum_{i=1}^{I}(a_i\delta_i + b_ix_i) + \sum_{i=1}^{I}\sum_{j=1}^{J}\widetilde{\zeta_{ij}}y_{ij} \le \overline{C}\}(\chi) \ge \gamma \end{cases} \\ \quad \sum_{j=1}^{j} y_{ij} = x_i\ , i = 1,2,...,I \\ \quad x_i\ \le D_i\delta_i\ , i = 1,2,...,I \\ \quad \sum_{i=1}^{I} y_{ij} = t_j, j = 1,2,...,J \\ \quad x_i\ ,\ y_{ij} \in\ N, i = 1,2,...,I, j = 1,2,...,J \\ \quad \delta_j \in \{0,1\}, i = 1,2,...,I, j = 1,2,...,J \end{cases} \Leftrightarrow \begin{cases} min\ \overline{R}_{inf}(\alpha,\beta) \\ subject\ to: \\ \quad min\ \overline{C}_{inf}(\chi,\gamma) \\ \quad \sum_{j=1}^{j} y_{ij} = x_i\ , i = 1,2,...,I \\ \quad x_i\ \le D_i\delta_i\ , i = 1,2,...,I \\ \quad \sum_{i=1}^{I} y_{ij} = t_j, j = 1,2,...,J \\ \quad x_i\ ,\ y_{ij} \in\ N, i = 1,2,...,I, j = 1,2,...,J \\ \quad \delta_j \in \{0,1\}, i = 1,2,...,I, j = 1,2,...,J \end{cases}$$

The left-hand figure is our model, the objective is to seek the minimal risk and cost under the condition that satisfies certain constraints. The lower-level objective is to minimize the cost, and the upper-level objective is to minimize the risk. The two inequalities are chance measure constraints with credibility measure $\alpha$ , $\chi$ and probability measure $\beta$ ,$\gamma$ . The first equality and the following inequality are used to ensure depot capacity and the second equality is the customer demand constraint. The other symbols are the domains of the related variables. Based on the definition of pessimistic value, the left-hand model is equivalent to the right-hand model. To solve the bi-level chance constraint, we need to design an intelligent algorithm in the following section.

## 3. Solution Algorithm

As the model we proposed is a nonlinear problem, the ordinary solution cannot solve it to obtain a satisfying result. Next, we design a hybrid algorithm which integrates two algorithms-GA and fuzzy random simulation to explore the optimal answer.

### 3.1. Fuzzy random simulation

To solve the mentioned models, we must first dispose of the chance measure function:

$$U_1(x_i, y_{ij}) \to Ch\{R(x_i, y_{ij}, \widetilde{\eta_i}, \widetilde{\tau_{ij}}) \le \overline{R}\}(\alpha) \ge \beta$$

$$U_2(x_i, y_{ij}) \to Ch\{C(x_i, y_{ij}, \widetilde{\zeta_{ij}}) \le \overline{C}\}(\chi) \ge \gamma$$

where the variables are defined before.

Both functions are similar in form and content. To compute the $\overline{R}$ value and $\overline{C}$ value, the procedures about the fuzzy random simulation algorithm [15] are described as follows:

Step 1: Produce the random variable $\theta_k$ from the sample space ($k = 1,2,...N$) according to the random distribution Pr , where $N$ is a sufficiently large integer.

Step 2. Generate the fuzzy random variable $\xi$ , which means, multiply or add the random variable to the corresponding fuzzy variable.

Steps 3. Calculate the joint membership for the new fuzzy random variable and then use the fuzzy simulation algorithm [16] to calculate $\alpha$-pessimistic of $R$ ,

$$\overline{R}_k = inf\{R_k|Cr\{R(\xi(w_n)) \le R_k\} \ge \alpha\}, k = 1,2,...N$$

or $\chi$-pessimistic value of

$$\overline{C}_k = inf\{C_k | Cr\{C(\xi(w_n)) \le C_k\} \ge \chi\}, k = 1, 2, ...N$$

.
Step 4. Repeat steps 1 to 3 for the given $N$ times.
Step 5. Set $N_1$ as the integer part of $\beta * N$ for the risk inequality and $N_2$ as the integer part of $\gamma * N$ for the cost inequality.
Step 6. Return the $N_1 - th$ largest element of the array $\{\overline{R}_1, \overline{R}_2, ..., \overline{R}_N\}$ as the $(\alpha, \beta)$ chance level risk value, and return the $N_2 - th$ largest element of the array $\{\overline{C}_1, \overline{C}_2, ..., \overline{C}_N\}$ , namely the $(\chi, \gamma)$ chance level cost value.

### 3.2. Genetic algorithm

#### 3.2.1. Initialization process

The length of the chromosome is decided by two numbers: the number of the depots and the customers. It consists of two parts: the form of the chromosome $[\delta_1, \delta_2, ...\delta_i, y_{11}, y_{12}, ...y_{1j}, y_{21}, y_{22}, ...y_{2j}, ...y_{i1}, y_{i1}, ...y_{ij}]$ and the first part, which is encoded as $0-1$ variable. Once the depot $i$ is chosen to transport hazardous materials to the customer, the variable $\delta_i$ is initialized as 1. The second part, $y_{ij}$ , is randomly generated such that $\sum_{j=1}^{j} y_{ij} = x_i, x_i \le D_i$ , $\sum_{i=1}^{I} y_{ij} = t_j$ and then the chromosomes feasibility is checked given chance level constraints. If it meets the constraints conditions, the chromosome is retained. The above process is repeated until a feasible chromosome $Z_1, Z_2, ...Z_{popsize}$ is completely produced.

#### 3.2.2. Select process

The proposed model has two objectives. Once the four parameters are predetermined, the feasibility chromosomes are finished in the initialization process. Therefore, we use the roulette wheel selection method to choose parents. Its operating principle is a fitness-proportional selection process to choose larger proportion chromosome to be the parent chromosome. Although the model has bi-level objective, the lower level objective is adopted as the fitness value. Therefore, the evaluation function is defined as $eval(C)$ .

Table 1: Crossover process

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\delta_1$ | $y_{11}$ | $y_{12}$ | ... | $y_{1k}$ | ... | $y_{1J}$ | $\delta'_1$ | $y'_{11}$ | $y'_{12}$ | ... | $y'_{1k}$ | ... | $y'_{1J}$ |
| $\delta_2$ | $y_{21}$ | $y_{22}$ | ... | $y_{2k}$ | ... | $y_{2J}$ | $\delta'_2$ | $y'_{21}$ | $y'_{22}$ | ... | $y'_{2k}$ | ... | $y'_{2J}$ |
| ... | ... | ... | ... | ... | ... | ... | ... ... | ... | ... | ... | ... | ... | |
| $\delta_I$ | $y_{I1}$ | $y_{I2}$ | ... | $y_{Ik}$ | ... | $y_{IJ}$ | $\delta'_I$ | $y'_{I1}$ | $y'_{I2}$ | ... | $y'_{Ik}$ | ... | $y'_{IJ}$ |
| | | | | | | ↓ | | | | | | | |
| $\delta_1$ | $y_{11}$ | $y_{12}$ | ... | $y'_{1k}$ | ... | $y'_{1J}$ | $\delta'_1$ | $y'_{11}$ | $y'_{12}$ | ... | $y_{1k}$ | ... | $y_{1J}$ |
| $\delta_2$ | $y_{21}$ | $y_{22}$ | ... | $y'_{2k}$ | ... | $y'_{2J}$ | $\delta'_2$ | $y'_{21}$ | $y'_{22}$ | ... | $y_{2k}$ | ... | $y_{2J}$ |
| ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| $\delta_I$ | $y_{I1}$ | $y_{I2}$ | ... | $y'_{Ik}$ | ... | $y'_{IJ}$ | $\delta'_I$ | $y'_{I1}$ | $y'_{I2}$ | ... | $y_{Ik}$ | ... | $y_{IJ}$ |

#### 3.2.3. Crossover process

The crossover probability $p\_c$ is predefined before the GA starts to work. Repeat the following process for the entire population: randomly generate a number $r$ , $r \in [0, 1]$, if $r < p\_c$ is established, the chromosome $Z_k$ is selected to be a parent to do crossover operation.

To better understanding the crossover operation, we turn the chosen parent chromosome into a matrix, then divide the matrix into pair $(Z_1, Z_2), (Z_3, Z_4)...$ ,take $(Z_1, Z_2)$ as an example, an integer between the interval $k$ ,$k \in [1, J]$ is randomly generated ,swap from $k$ column to the end of the row for all the rows in the corresponding matrix. The process can be seen in the Table1.

After this operation, the new chromosome is still needed to check whether it is satisfying the constraint, if the chromosome is not, it must be abandoned, otherwise calculate the new objective.

### 3.2.4. *Mutation process*

A parameter $p\_m$ is defined as the mutation probability and repeat the following process from $k = 1, 2, ...popsize$ : a number $r$ ,$r \in [0, 1]$ is randomly generated, if $r < p\_m$ is established, the chromosome $Z_k$ is selected to be a parent to do mutation operation.

The mutation operation is also taken on the matrix, differ from the traditional mutation operation, we need to randomly produce four numbers $m, n \in (1, I)$ , $u, v \in (1, J)$ and $m \neq n$ , $u \neq v$ ,then select gene $y_{mu}, y_{mv}, y_{nu}, y_{nv}$ to mutate, then continue to randomly generate $d \in [0, min(y_{mu}, y_{nv})]$, set $y_{mu} \leftarrow y_{mu} - d, y_{nv} \leftarrow y_{nv} - d, y_{mv} \leftarrow y_{mv} + d, y_{nv} \leftarrow y_{nv} + d$ , the whole process can be shown in Table 2..

Table 2: Crossover process

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\delta_1$ | $y_{11}$ | $y_{12}$ | ... | $y_{1u}$ | ... | $y_{1v}$ | ... | $y_{1J}$ |
| $\delta_m$ | $y_{m1}$ | $y_{m2}$ | ... | $y_{mu}$ | ... | $y_{mv}$ | ... | $y_{mJ}$ |
| ... | ... | ... | ... | ... | ... | ... | ... | ... |
| $\delta_n$ | $y_{n1}$ | $y_{n2}$ | ... | $y_{nu}$ | ... | $y_{nv}$ | ... | $y_{nJ}$ |
| $\delta_I$ | $y_{I1}$ | $y_{I2}$ | ... | $y_{Iu}$ | ... | $y_{Iv}$ | ... | $y_{IJ}$ |

↓

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\delta_1$ | $y_{11}$ | $y_{12}$ | ... | $y_{1u}$ | ... | $y_{1v}$ | ... | $y_{1J}$ |
| $\delta_m$ | $y_{m1}$ | $y_{m2}$ | ... | $y_{mu} - d$ | ... | $y_{mv} + d$ | ... | $y_{mJ}$ |
| ... | ... | ... | ... | ... | ... | ... | ... | ... |
| $\delta_n$ | $y_{n1}$ | $y_{n2}$ | ... | $y_{nu} + d$ | ... | $y_{nv} - d$ | ... | $y_{nJ}$ |
| $\delta_I$ | $y_{I1}$ | $y_{I2}$ | ... | $y_{Iu}$ | ... | $y_{Iv}$ | ... | $y_{IJ}$ |

### 3.3. *Hybrid algorithm*

A hybrid algorithm integrating the above two algorithms is presented next, the concept of credibility is introduced in the paper [17].

Step 1. Initialize GA parameters such as *popsize* , $p_m$ , $p_c$ ,$max\_gen$ , the number of fuzzy random simulation cycles $N$ , then obtain *popsize* feasibility chromosomes on the premise that meeting all the constraints.
Step 2. Do the crossover operation and mutation operation on the feasibility chromosomes, after checking the new chromosome meeting the constraints, update the chromosome population.
Step 3. Calculate the lower level objective value and fitness through fuzzy random simulation, meanwhile save the same chromosomes upper level value to set $V$.
Step 4. Do the select operation by using the roulette wheel method.
Step 5. Repeat from Step 2 to Step 4 for certain cycles.
Step 6. Rank all the elements in the set $V$, take the least element as the final result, report the corresponding chromosome as the best distribution solution.

## 4. Case study

In this section, a small scale experiment is presented to account for the feasibility of the proposed model and algorithm. This experiment includes three depots $D_1, D_2, D_3$ and six customers $C1, C2, ...C6$ ,respectively. The fuzzy transportation costs and the number of influenced people are presented in Table 3 and Table 5, the random factors, i.e. accident frequency are presented in Table 4 and Table 6. Part data is from paper [13].

Table 3: Transportation cost

| | C1 | C2 | C3 | C4 | C5 | C6 |
|---|---|---|---|---|---|---|
| $D_1$ | (250, 260, 270, 280) | (220, 255, 260, 300) | (360, 400, 500, 550) | (200, 260, 300, 350) | (180, 200, 210, 240) | (800, 850, 950, 1000) |
| $D_2$ | (250, 255, 260, 265) | (200, 240, 260, 310) | (400, 450, 500, 550) | (180, 200, 210, 230) | (500, 600, 650, 700) | (300, 400, 450, 500) |
| $D_3$ | (220, 240, 250, 270) | (800, 850, 880, 920) | (270, 300, 320, 350) | (250, 300, 400, 420) | (240, 300, 320, 350) | (150, 200, 230, 250) |

Table 4: Transportation cost uniform distribution

| $\rho$ | C1 | C2 | C3 | C4 | C5 | C6 |
|---|---|---|---|---|---|---|
| $D_1$ | $U(30, 40)$ | $U(20, 40)$ | $U(15, 40)$ | $U(20, 30)$ | $U(15, 45)$ | $(U(20, 35)$ |
| $D_2$ | $U(20, 30)$ | $U(40, 60)$ | $U(30, 50)$ | $U(10, 30)$ | $U(40, 80)$ | $(U(30, 70)$ |
| $D_3$ | $U(10, 40)$ | $U(60, 90)$ | $U(30, 60)$ | $U(20, 50)$ | $U(30, 50)$ | $(U(20, 50)$ |

Table 5: Number of people that may be influenced

| | C1 | C2 | C3 | C4 | C5 | C6 |
|---|---|---|---|---|---|---|
| $D_1$ | (300, 350, 380, 450) | (200, 250, 270, 320) | (100, 120, 140, 160) | (500, 600, 800, 900) | (1000, 1100, 1300, 1500) | (200, 240, 360, 420) |
| $D_2$ | (150, 180, 200, 230) | (220, 240, 250, 280) | (80, 100, 150, 180) | (300, 330, 400, 420) | (300, 400, 450, 550) | (500, 700, 850, 950) |
| $D_3$ | (200, 220, 240, 260) | (100, 110, 120, 140) | (150, 200, 210, 240) | (800, 840, 890, 930) | (200, 220, 240, 260) | (660, 680, 700, 750) |

The capacities of depots are set to be s = (1200, 1100, 1000)t , and the demand amounts of the customers are ct = (296, 250, 355, 400, 215, 300)t , the fixed cost a = (400000, 36000000, 30000000)t , the unit vehicle cost are b = (90, 100, 100)Yuan . The parameters about GA are set to be $popsize = 50$ , $max\_gen = 100$ , $N = 1000$.

To compare the final results about cost and risk, we need to choose the decision maker acceptable level of risk and cost. In theory, the chance level pair can be set to be any number within [0,1], but according to the realistic meaning, we choose ten different pair parameters, risk is set to be $\alpha = \beta = [0.75, 0.80, 0.90, 0.92, 0.94, 0.96, 0.98, 0.99]$ , and cost is set to be $\chi = \gamma = [0.75, 0.80, 0.90, 0.92, 0.94, 0.96, 0.98, 0.99]$, respectively, the corresponding result of corresponding cases are presented as follows in Table 7.

From the table, we can know, when the chance level $(\alpha, \beta, \chi, \gamma)$ is to be $(0.75, 0.75, 0.75, 0.75)$ , the minimum cost and minimum risk are only 66592612 and 13.6107, with the growing of chance level, the cost and risk is also increasing. This trend is in accordance with the VaR model and CVaR model proposed in the paper [18]. Although the risk and the cost is high, the credibility measure and probability is also growing, this means a lot to the decision maker.

Table 6: Number of people that may be influenced uniform distribution

| $r$ | C1 | C2 | C3 | C4 | C5 | C6 |
|---|---|---|---|---|---|---|
| $D_1$ | $U(0.8*10^{-5}, 1.2*10^{-5})$ | $U(1.8*10^{-5}, 2.2*10^{-5})$ | $U(2.5*10^{-5}, 3.2*10^{-5})$ | $U(1.2*10^{-5}, 2.0*10^{-5})$ | $U(1.2*10^{-5}, 2.0*10^{-5})$ | $U(1.5*10^{-5}, 2.5*10^{-5})$ |
| $D_2$ | $U(0.9*10^{-5}, 1.5*10^{-5})$ | $U(1.2*10^{-5}, 1.8*10^{-5})$ | $U(1.5*10^{-5}, 2.5*10^{-5})$ | $U(1.3*10^{-5}, 2.0*10^{-5})$ | $U(1.2*10^{-5}, 2.0*10^{-5})$ | $U(1.2*10^{-5}, 2.0*10^{-5})$ |
| $D_3$ | $U(2.0*10^{-5}, 2.5*10^{-5})$ | $U(2.0*10^{-5}, 2.8*10^{-5})$ | $U(2.0*10^{-5}, 2.5*10^{-5})$ | $U(1.0*10^{-5}, 1.8*10^{-5})$ | $U(1.5*10^{-5}, 2.5*10^{-5})$ | $U(2.5*10^{-5}, 3.2*10^{-5})$ |

Table 7: Cost and risk result for different cases

| $p\_c$ | $p\_m$ | $\alpha$ | $\beta$ | $\chi$ | $\gamma$ | $\overline{C}$ | $\overline{R}$ | $Error$ |
|---|---|---|---|---|---|---|---|---|
| 0.4 | 0.1 | 0.75 | 0.75 | 0.75 | 0.75 | 66592612 | 13.6107 | 49.59% |
| 0.4 | 0.1 | 0.75 | 0.75 | 0.80 | 0.80 | 66599429 | 13.6347 | 49.50% |
| 0.4 | 0.1 | 0.80 | 0.80 | 0.75 | 0.75 | 66592612 | 14.1164 | 47.77% |
| 0.4 | 0.1 | 0.80 | 0.80 | 0.80 | 0.80 | 66599429 | 14.1222 | 47.76% |
| 0.4 | 0.1 | 0.90 | 0.90 | 0.90 | 0.90 | 66619396 | 15.2140 | 43.66% |
| 0.4 | 0.1 | 0.92 | 0.92 | 0.92 | 0.92 | 66621963 | 15.0462 | 44.29% |
| 0.4 | 0.1 | 0.94 | 0.94 | 0.94 | 0.94 | 66622224 | 15.1567 | 43.88% |
| 0.4 | 0.1 | 0.96 | 0.96 | 0.96 | 0.96 | 66626294 | 15.4064 | 42.96% |
| 0.4 | 0.1 | 0.98 | 0.98 | 0.98 | 0.98 | 66630675 | 15.7425 | 41.70% |
| 0.4 | 0.1 | 0.99 | 0.99 | 0.99 | 0.99 | 66638045 | 16.5371 | 38.77% |

Comparing with the CCP model by Wei [13], we define a relative ratio to better understand the risk reduction, $Error = [Risk(CCP) - Risk(Thispaper)]/Risk(CCP)$ ,from the last column of Table 7,we can see that risk reduction has a significant effect, when the $(\alpha, \beta, \chi, \gamma)$ is $(0.75, 0.75, 0.75, 0.75)$ , the reduction can be as much as half, it plays an important role in helping decision makers to make a decision from a series of alternative plans. We choose the best distribution solution to be shown when $(\alpha, \beta, \chi, \gamma)$ is set to be $(0.9, 0.9, 0.9, 0.9)$ as Table 8.

Table 8: Distribution Result

| | $Custo.1$ | $Custo.2$ | $Custo.3$ | $Custo.4$ | $Custo.5$ | $Custo.6$ |
|---|---|---|---|---|---|---|
| $Depot1$ | 73 | 205 | 57 | 87 | 152 | 96 |
| $Depot2$ | 77 | 19 | 50 | 244 | 39 | 17 |
| $Depot3$ | 146 | 26 | 248 | 69 | 24 | 187 |

The experiment is coded in C++ language using the software Visual Studio 2012 performed on an experimental computer with an Intel core i5 and 12G RAM, the total runtime of experiment is 48 hours. Note that the main time is spent on the initialization of the chromosome, the higher of the confidence level, the large computation is. Although the proposed algorithm is time-consuming, it can remarkably cut down the risk factor, provide better solutions to decision makers.

## 5. Conclusion

This paper proposed a bi-level minimum objective programming with chance constraint model for hazardous materials distribution, it can be understood that the distribution solution could guarantee the risk reach to the minimum based on the cost minimum. The decision maker can flexibility make choice of the distribution solution according to their acceptable range of risk. However, there still exist two big problems, one of them is during the initialization process of the hybrid algorithm, it occupies a mass of time, it can be accounted for that there are so many fuzzy variable and random variable, so we need to explore more effective intelligent algorithm to settle this problem. The other is that we use a small scale case to verify our model and algorithm, the practical applicability of big scale case is feasible in theory, but it may occur some unexpected matter in practice, such as how to define the range of big scale case, the big scale case means there are much more fuzzy random variables, so it is more different to find the ideal solution and so on. However, the above-mentioned problems point out our research directions in the future.

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