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Some Invariants of Conformal Mappings of a Generalized Riemannian Space

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Abstract. Invariants of conformal mappings between non-symmetric affine connection spaces are obtained in this paper. Correlations between these invariants and the Weyl conformal curvature tensor are established. Before these invariants, it is obtained a necessary and sufficient condition for a mapping to be conformal. Some appurtenant invariants of conformal mappings are obtained.

1. Introduction and motivation

A lot of research papers, books and monographs are dedicated to development of the theory of Riemannian spaces and applications (see [1–23]).

Definition 1.1. [19, 22, 23] *An N-dimensional manifold $\mathcal{M}_N$ endowed with a metric tensor G_{ij}, $G_{ij} \neq G_{ji}$, is* ***the generalized Riemannian space*** $\mathbb{GR}_N$.

Because of the non-symmetry $G_{ij} \neq G_{ji}$, the symmetric and anti-symmetric part of the tensor G_{ij} are:

$$g_{ij} = \frac{1}{2}\Big(G_{ij} + G_{ji}\Big) \quad \text{and} \quad F_{ij} = \frac{1}{2}\Big(G_{ij} - G_{ji}\Big). \tag{1}$$

It evidently holds the equalities $G_{ij} = g_{ij} + F_{ij}$, $g_{ij} = g_{ji}$, $F_{ij} = -F_{ji}$. An N-dimensional manifold $\mathcal{M}_N$ endowed with the above defined symmetric metric tensor g_{ij} is the associated (Riemannian) space $\mathbb{R}_N$ of the space $\mathbb{GR}_N$.

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1.1. Affine connection of Riemannian spaces

Christoffel symbols of the first and second kind of the space $\mathbb{GR}_N$ are

$$\Gamma_{i.jk} = \frac{1}{2}(G_{ji,k} - G_{jk,i} + G_{ik,j}) \quad \text{and} \quad \Gamma^i_{jk} = g^{i\alpha}\Gamma_{\alpha.jk}, \tag{2}$$

for partial derivation denoted by comma and the symmetric contravariant metric tensor $\left(g^{ij}\right) = \left(g_{ij}\right)^{-1}$. The Christoffel symbols Γ^i_{jk} are affine connection coefficients of the space $\mathbb{GR}_N$.

Because of $\Gamma^i_{jk} \neq \Gamma^i_{kj}$, the symmetric and antisymmetric part of Γ^i_{jk} are respectively defined as:

$$\overset{0}{\Gamma}{}^i_{jk} = \frac{1}{2}\left(\Gamma^i_{jk} + \Gamma^i_{kj}\right) \quad \text{and} \quad T^i_{jk} = \frac{1}{2}\left(\Gamma^i_{jk} - \Gamma^i_{kj}\right). \tag{3}$$

The antisymmetric part T^i_{jk} is called *the torsion tensor* of the space $\mathbb{GR}_N$. It is easy to obtain that is

$$\overset{0}{\Gamma}{}^i_{jk} = \frac{1}{2}g^{i\alpha}\left(g_{j\alpha,k} - g_{jk,\alpha} + g_{\alpha k,j}\right) \quad \text{and} \quad T^i_{jk} = \frac{1}{2}g^{i\alpha}\left(F_{j\alpha,k} - F_{jk,\alpha} + F_{\alpha k,j}\right). \tag{4}$$

A space $\mathbb{R}_N$ endowed with affine connection $\overset{0}{\Gamma}{}^i_{jk}$ is *the associated space* of the space $\mathbb{GR}_N$.

Because of the symmetry $\overset{0}{\Gamma}{}^i_{jk} = \overset{0}{\Gamma}{}^i_{kj}$, it exists only one kind of covariant derivation with regard to the affine connection of the associated space $\mathbb{R}_N$ defined as

$$a^i_{j;k} := a^i_{j,k} + \overset{0}{\Gamma}{}^i_{\alpha k}a^\alpha_j - \overset{0}{\Gamma}{}^\alpha_{jk}a^i_\alpha, \tag{5}$$

for a tensor a of the type $(1,1)$. From this covariant derivative, it is derived one Ricci-type identity. The curvature tensor of associated space $\mathbb{R}_N$ obtained from this identity is

$$\overset{0}{R}{}^i_{jmn} = \overset{0}{\Gamma}{}^i_{jm;n} - \overset{0}{\Gamma}{}^i_{jn;m} = \overset{0}{\Gamma}{}^i_{jm,n} - \overset{0}{\Gamma}{}^i_{jn,m} + \overset{0}{\Gamma}{}^\alpha_{jm}\overset{0}{\Gamma}{}^i_{\alpha n} - \overset{0}{\Gamma}{}^\alpha_{jn}\overset{0}{\Gamma}{}^i_{\alpha m}. \tag{6}$$

Because of non-symmetry $\Gamma^i_{jk} \neq \Gamma^i_{kj}$, it was found four kinds of covariant differentiation (see [14, 15]) with regard to affine connection of the space $\mathbb{GR}_N$. For this reason, it exists twelve Ricci-type identities with regard to affine connection of this space. From these identities, it was obtained twelve curvature tensors of $\mathbb{GR}_N$:

$$R^i_{jmn} = \overset{0}{R}{}^i_{jmn} + uT^i_{jm;n} + u'T^i_{jn;m} + vT^\alpha_{jm}T^i_{\alpha n} + v'T^\alpha_{jn}T^i_{\alpha m} + wT^\alpha_{mn}T^i_{\alpha j}, \tag{7}$$

for the curvature tensor $\overset{0}{R}{}^i_{jmn}$ of the associated space and the corresponding real constants u, u', v, v', w. Five of these tensors are linearly independent:

$$\underset{1}{R}{}^i_{jmn} = \overset{0}{R}{}^i_{jmn} + T^i_{jm;n} - T^i_{jn;m} + T^\alpha_{jm}T^i_{\alpha n} - T^\alpha_{jn}T^i_{\alpha m}, \tag{8}$$

$$\underset{2}{R}{}^i_{jmn} = \overset{0}{R}{}^i_{jmn} - T^i_{jm;n} + T^i_{jn;m} + T^\alpha_{jm}T^i_{\alpha n} - T^\alpha_{jn}T^i_{\alpha m}, \tag{9}$$

$$\underset{3}{R}{}^i_{jmn} = \overset{0}{R}{}^i_{jmn} + T^i_{jm;n} + T^i_{jn;m} - T^\alpha_{jm}T^i_{\alpha n} + T^\alpha_{jn}T^i_{\alpha m} - 2T^\alpha_{mn}T^i_{\alpha j}, \tag{10}$$

$$\underset{4}{R}{}^i_{jmn} = \overset{0}{R}{}^i_{jmn} + T^i_{jm;n} + T^i_{jn;m} - T^\alpha_{jm}T^i_{\alpha n} + T^\alpha_{jn}T^i_{\alpha m} + 2T^\alpha_{mn}T^i_{\alpha j}, \tag{11}$$

$$\underset{5}{R}{}^i_{jmn} = \overset{0}{R}{}^i_{jmn} + T^\alpha_{jm}T^i_{\alpha n} + T^\alpha_{jn}T^i_{\alpha m}. \tag{12}$$

1.2. Conformal mappings and conformal curvature tensors

A conformal mapping of Riemannian space $\mathbb{R}_N$ [13] is a transformation that preserves local angles. Conformal mappings are very important in complex analysis. Moreover, these mappings are significant in different areas of physics and engineering.

Formally, conformal mapping $f : \mathbb{R}_N \to \overline{\mathbb{R}}_N$ is determined with the equation

$$\overline{g}_{ij} = e^{2\psi} g_{ij}, \tag{13}$$

for a scalar function ψ. The affine connection coefficients $\overset{0}{\Gamma}{}^i_{jk}$ and $\overset{0}{\overline{\Gamma}}{}^i_{jk}$ satisfy the equation

$$\overset{0}{\overline{\Gamma}}{}^i_{jk} = \overset{0}{\Gamma}{}^i_{jk} + \psi_j \delta^i_k + \psi_k \delta^i_j - \psi_\alpha g^{i\alpha} g_{jk}, \tag{14}$$

for $\psi_i = \psi_{,i}$. An invariant of the conformal mapping f is the Weyl conformal curvature tensor [2, 13]:

$$C^i_{jmn} = \overset{0}{R}{}^i_{jmn} + \frac{1}{N-2}\Big(\delta^i_m \overset{0}{R}_{jn} - \delta^i_n \overset{0}{R}_{jm} + \overset{0}{R}{}^i_m g_{jn} - \overset{0}{R}{}^i_n g_{jm}\Big) + \frac{\overset{0}{R}}{(N-1)(N-2)}\Big(\delta^i_m g_{jn} - \delta^i_n g_{jm}\Big), \tag{15}$$

for $\overset{0}{R}_{ij} = \overset{0}{R}{}^\alpha_{ij\alpha}, \overset{0}{R}{}^i_j = g^{i\alpha} \overset{0}{R}_{\alpha j}, \overset{0}{R} = \overset{0}{R}{}^\alpha_\alpha$.

A diffeomorphism $f : \mathbb{GR}_N \to \mathbb{G}\overline{\mathbb{R}}_N$ is the conformal mapping if the basic tensors G_{ij} and $\overline{G}_{ij}$ satisfy the equation [19, 22, 23]

$$\overline{G}_{ij} = e^{2\psi} G_{ij}, \tag{16}$$

for a scalar function ψ. The basic equation of mapping f is

$$\overline{\Gamma}{}^i_{jk} = \Gamma^i_{jk} + \delta^i_j \psi_k + \delta^i_k \psi_j - \psi_\alpha g^{i\alpha} g_{jk} + \xi^i_{jk}, \tag{17}$$

for the tensor ψ_i as above and the tensor ξ^i_{jk} anti-symmetric by indices j and k.

The Weyl conformal curvature tensor C^i_{jmn} is generalized for conformal mappings which preserve the torsion tensor T^i_{jk} (also called the equitorsion conformal mappings) in [22, 23]. Invariants of random conformal mappings of equidistant Riemannian spaces are obtained in [19]. The main aim of this paper is to generalize the Weyl conformal curvature tensor C^i_{jmn} for a random conformal mapping defined on a random generalized Riemannian space $\mathbb{GR}_N$. Furthermore, we will obtain some other invariants of conformal mappings and a necessary and sufficient condition for a mapping f to be conformal in here.

2. Generalizations of the Weyl conformal curvature tensor

Let $f : \mathbb{GR}_N \to \mathbb{G}\overline{\mathbb{R}}_N$ be a conformal mapping determined by the equation (17). The affine connection coefficients $\overset{0}{\Gamma}{}^i_{jk}$ and $\overset{0}{\overline{\Gamma}}{}^i_{jk}$ satisfy the equation

$$\overset{0}{\overline{\Gamma}}{}^i_{jk} = \overset{0}{\Gamma}{}^i_{jk} + \psi_j \delta^i_k + \psi_k \delta^i_j - \psi_\alpha g^{i\alpha} g_{jk}. \tag{18}$$

After contracting of this equation, we obtain that is

$$\psi_i = \frac{1}{N}\Big(\overset{0}{\overline{\Gamma}}{}^\alpha_{i\alpha} - \overset{0}{\Gamma}{}^\alpha_{i\alpha}\Big). \tag{19}$$

This result, involved in the equation (18), proves that it holds

$$\overline{\overset{0}{\Gamma}}{}^i_{jk} = \overset{0}{\Gamma}{}^i_{jk} + \frac{1}{N}\Big(\overline{\overset{0}{\Gamma}}{}^\alpha_{j\alpha}\delta^i_k + \overline{\overset{0}{\Gamma}}{}^\alpha_{k\alpha}\delta^i_j - \overline{\overset{0}{\Gamma}}{}^\beta_{\alpha\beta}\overline{g}^{i\alpha}\overline{g}_{jk}\Big) - \frac{1}{N}\Big(\overset{0}{\Gamma}{}^\alpha_{j\alpha}\delta^i_k + \overset{0}{\Gamma}{}^\alpha_{k\alpha}\delta^i_j - \overset{0}{\Gamma}{}^\beta_{\alpha\beta}g^{i\alpha}g_{jk}\Big). \tag{20}$$

From this equation, we obtain that it is satisfied

$$\overline{\widetilde{\Gamma}}{}^i_{jk} = \widetilde{\Gamma}{}^i_{jk},$$

for

$$\widetilde{\Gamma}{}^i_{jk} = \overset{0}{\Gamma}{}^i_{jk} - \frac{1}{N}\Big(\overset{0}{\Gamma}{}^\alpha_{j\alpha}\delta^i_k + \overset{0}{\Gamma}{}^\alpha_{k\alpha}\delta^i_j - \overset{0}{\Gamma}{}^\beta_{\alpha\beta}g^{i\alpha}g_{jk}\Big), \tag{21}$$

$$\overline{\widetilde{\Gamma}}{}^i_{jk} = \overline{\overset{0}{\Gamma}}{}^i_{jk} - \frac{1}{N}\Big(\overline{\overset{0}{\Gamma}}{}^\alpha_{j\alpha}\delta^i_k + \overline{\overset{0}{\Gamma}}{}^\alpha_{k\alpha}\delta^i_j - \overline{\overset{0}{\Gamma}}{}^\beta_{\alpha\beta}\overline{g}^{i\alpha}\overline{g}_{jk}\Big). \tag{22}$$

It holds the following proposition.

Proposition 2.1. *Let $f : \mathbb{GR}_N \to \mathbb{G}\overline{\mathbb{R}}_N$ be a conformal mapping of an affine connection space $\mathbb{GR}_N$. The geometrical object $\widetilde{\Gamma}{}^i_{jk}$ is an invariant of the mapping f.* □

Remark 2.2. *The invariant $\widetilde{\Gamma}{}^i_{jk}$ is analogy of generalized Thomas projective parameter* [17] *of associated space $\mathbb{R}_N$.*

The following equations are satisfied:

$$\overline{g}_{ij} = \frac{1}{2}(\overline{G}_{ij} + \overline{G}_{ji}) = \frac{1}{2}(e^{2\psi}G_{ij} + e^{2\psi}G_{ji}) = \frac{1}{2}e^{2\psi}(G_{ij} + G_{ji}) \overset{(1)}{=} e^{2\psi}g_{ij}, \tag{23}$$

$$\overline{F}_{ij} = \frac{1}{2}(\overline{G}_{ij} - \overline{G}_{ji}) = \frac{1}{2}(e^{2\psi}G_{ij} - e^{2\psi}G_{ji}) = \frac{1}{2}e^{2\psi}(G_{ij} - G_{ji}) \overset{(1)}{=} e^{2\psi}F_{ij}, \tag{24}$$

$$\delta^i_j = \overline{g}^{i\alpha}\overline{g}_{j\alpha} = \overline{g}^{i\alpha}e^{2\psi}g_{j\alpha} = g^{i\alpha}g_{j\alpha} \Longrightarrow \overline{g}^{i\alpha} = e^{-2\psi}g^{i\alpha}, \tag{25}$$

$$\overline{g}^{ij}\overline{g}_{mn} \overset{(23,25)}{=} \Big(e^{-2\psi}g^{ij}\Big)\Big(e^{2\psi}g_{mn}\Big) = g^{ij}g_{mn}, \tag{26}$$

$$\overline{g}^{ij}\overline{F}_{mn} \overset{(24,25)}{=} \Big(e^{-2\psi}g^{ij}\Big)\Big(e^{2\psi}F_{mn}\Big) = g^{ij}F_{mn}. \tag{27}$$

Based on the equations (25, 26, 27), we conclude that it is satisfied the following proposition:

Proposition 2.3. *Let $f : \mathbb{GR}_N \to \mathbb{G}\overline{\mathbb{R}}_N$ be a conformal mapping. The geometrical objects*

$$g^{ij}G_{mn}, \quad g^{ij}g_{mn}, \quad g^{ij}F_{mn}, \quad g^{ij}G_{mn,p} + g^{ij}_{,p}G_{mn}, \quad g^{ij}g_{mn,p} + g^{ij}_{,p}g_{mn}, \quad g^{ij}F_{mn,p} + g^{ij}_{,p}F_{mn} \tag{28}$$

are invariants of the mapping f. □

Let us now analyze the change of torsion tensor T^i_{jk} under the conformal mapping f.

Proposition 2.4. *The torsion tensors T^i_{jk} and $\overline{T}^i_{jk}$ of the spaces $\mathbb{GR}_N$ and $\mathbb{G}\overline{\mathbb{R}}_N$ are*

$$T^i_{jk} = \frac{1}{2}\Big(\Big(g^{i\alpha}F_{j\alpha}\Big)_{;k} - \Big(g^{i\alpha}F_{k\alpha}\Big)_{;j} - \Big(g^{i\alpha}F_{jk}\Big)_{;\alpha}\Big), \tag{29}$$

$$\overline{T}^i_{jk} = \frac{1}{2}\Big(\Big(\overline{g}^{i\alpha}\overline{F}_{j\alpha}\Big)_{\overline{;}k} - \Big(\overline{g}^{i\alpha}\overline{F}_{k\alpha}\Big)_{\overline{;}j} - \Big(\overline{g}^{i\alpha}\overline{F}_{jk}\Big)_{\overline{;}\alpha}\Big), \tag{30}$$

for covariant differentiations with regard to affine connections of $\mathbb{R}_N$ and $\overline{\mathbb{R}}_N$ denoted by ; and $\overline{;}$.

`Proof.` Let us prove the equation (29). The equation (30) may be proved in the same way.

It is satisfied the equalities

$$F_{ji;k} - F_{jk;i} + F_{ik;j} = F_{ji,k} - F_{jk,i} + F_{ik,j} \underbrace{-\overset{0}{\Gamma}{}^{\alpha}_{jk}F_{\alpha i} - \overset{0}{\Gamma}{}^{\alpha}_{ik}F_{j\alpha} + \overset{0}{\Gamma}{}^{\alpha}_{ji}F_{\alpha k} + \overset{0}{\Gamma}{}^{\alpha}_{ki}F_{j\alpha} - \overset{0}{\Gamma}{}^{\alpha}_{ij}F_{\alpha k} - \overset{0}{\Gamma}{}^{\alpha}_{kj}F_{i\alpha}}_{=0}$$

$$= F_{ji,k} - F_{jk,i} + F_{ik,j}.$$

From $2T^i_{jk} = \Gamma^i_{jk} - \Gamma^i_{kj} = g^{i\alpha}(\Gamma_{\alpha.jk} - \Gamma_{\alpha.kj})$, we obtain that it is satisfied

$$T^i_{jk} = \frac{1}{2}g^{i\alpha}(F_{j\alpha,k} - F_{jk,\alpha} + F_{\alpha k,j}) = \frac{1}{2}\Big(g^{i\alpha}F_{j\alpha;k} - g^{i\alpha}F_{jk;\alpha} + g^{i\alpha}F_{\alpha k;j}\Big).$$

Moreover, because $g^{ij}{}_{;k} = 0$ it holds $g^{ij}F_{mn;p} = \Big(g^{ij}F_{mn}\Big)_{;p}$, i.e.

$$T^i_{jk} = \frac{1}{2}\Big(\Big(g^{i\alpha}F_{j\alpha}\Big)_{;k} - \Big(g^{i\alpha}F_{jk}\Big)_{;\alpha} + \Big(g^{i\alpha}F_{\alpha k}\Big)_{;j}\Big),$$

which proves this proposition. □

From this proposition and the invariance (27), we obtain that is

$$\begin{aligned}
\Big(\overline{g}^{ij}\overline{F}_{mn}\Big)_{\bar{;}p} - \Big(g^{ij}F_{mn}\Big)_{;p} &= \overset{0}{\overline{\Gamma}}{}^{i}_{\alpha p}\overline{g}^{\alpha j}\overline{F}_{mn} + \overset{0}{\overline{\Gamma}}{}^{j}_{\alpha p}\overline{g}^{i\alpha}\overline{F}_{mn} - \overset{0}{\overline{\Gamma}}{}^{\alpha}_{mp}\overline{g}^{ij}\overline{F}_{\alpha n} - \overset{0}{\overline{\Gamma}}{}^{\alpha}_{np}\overline{g}^{ij}\overline{F}_{m\alpha}\\
&\quad - \overset{0}{\Gamma}{}^{i}_{\alpha p}g^{\alpha j}F_{mn} - \overset{0}{\Gamma}{}^{j}_{\alpha p}g^{i\alpha}F_{mn} + \overset{0}{\Gamma}{}^{\alpha}_{mp}g^{ij}F_{\alpha n} + \overset{0}{\Gamma}{}^{\alpha}_{np}g^{ij}F_{m\alpha},\\
\overline{T}^i_{jk} - T^i_{jk} &= \frac{1}{2}\Big(\Big(\overline{g}^{i\alpha}\overline{F}_{j\alpha}\Big)_{,k} + \overset{0}{\overline{\Gamma}}{}^{i}_{\alpha k}\overline{g}^{\alpha\beta}\overline{F}_{j\beta} - \overset{0}{\overline{\Gamma}}{}^{\alpha}_{jk}\overline{g}^{i\beta}\overline{F}_{\alpha\beta} - \Big(\overline{g}^{i\alpha}\overline{F}_{k\alpha}\Big)_{,j} - \overset{0}{\overline{\Gamma}}{}^{i}_{\alpha j}\overline{g}^{\alpha\beta}\overline{F}_{k\beta} + \overset{0}{\overline{\Gamma}}{}^{\alpha}_{kj}\overline{g}^{i\beta}F_{\alpha\beta} - \Big(\overline{g}^{i\alpha}\overline{F}_{jk}\Big)_{\bar{;}\alpha}\Big)\\
&\quad - \frac{1}{2}\Big(\Big(g^{i\alpha}F_{j\alpha}\Big)_{,k} + \overset{0}{\Gamma}{}^{i}_{\alpha k}g^{\alpha\beta}F_{j\beta} - \overset{0}{\Gamma}{}^{\alpha}_{jk}g^{i\beta}F_{\alpha\beta} - \Big(g^{i\alpha}F_{k\alpha}\Big)_{,j} - \overset{0}{\Gamma}{}^{i}_{\alpha j}g^{\alpha\beta}F_{k\beta} + \overset{0}{\Gamma}{}^{\alpha}_{kj}g^{i\beta}F_{\alpha\beta} - \Big(g^{i\alpha}F_{jk}\Big)_{;\alpha}\Big)\\
&\overset{(27)}{=} \frac{1}{2}\Big(\overline{g}^{\alpha\beta}\Big(\overset{0}{\overline{\Gamma}}{}^{i}_{\alpha k}\overline{F}_{j\beta} - \overset{0}{\overline{\Gamma}}{}^{i}_{\alpha j}\overline{F}_{k\beta}\Big) - \Big(\overset{0}{\overline{\Gamma}}{}^{i}_{\alpha\beta}\overline{g}^{\alpha\beta} + \overset{0}{\overline{\Gamma}}{}^{\beta}_{\alpha\beta}\overline{g}^{i\alpha}\Big)\overline{F}_{jk} + \overline{g}^{i\beta}\Big(\overset{0}{\overline{\Gamma}}{}^{\alpha}_{j\beta}\overline{F}_{\alpha k} - \overset{0}{\overline{\Gamma}}{}^{\alpha}_{k\beta}\overline{F}_{\alpha j}\Big)\Big)\\
&\quad - \frac{1}{2}\Big(g^{\alpha\beta}\Big(\overset{0}{\Gamma}{}^{i}_{\alpha k}F_{j\beta} - \overset{0}{\Gamma}{}^{i}_{\alpha j}F_{k\beta}\Big) - \Big(\overset{0}{\Gamma}{}^{i}_{\alpha\beta}g^{\alpha\beta} + \overset{0}{\Gamma}{}^{\beta}_{\alpha\beta}g^{i\alpha}\Big)F_{jk} + g^{i\beta}\Big(\overset{0}{\Gamma}{}^{\alpha}_{j\beta}F_{\alpha k} - \overset{0}{\Gamma}{}^{\alpha}_{k\beta}F_{\alpha j}\Big)\Big).
\end{aligned}$$

From these equalities, we get

$$\overline{T}^i_{jk} = T^i_{jk} + \tau^i_{jk} - \overline{\tau}^i_{jk}, \tag{31}$$

for

$$\tau^i_{jk} = -\frac{1}{2}\Big(g^{\alpha\beta}\Big(\overset{0}{\Gamma}{}^{i}_{\alpha k}F_{j\beta} - \overset{0}{\Gamma}{}^{i}_{\alpha j}F_{k\beta}\Big) - \Big(\overset{0}{\Gamma}{}^{i}_{\alpha\beta}g^{\alpha\beta} + \overset{0}{\Gamma}{}^{\beta}_{\alpha\beta}g^{i\alpha}\Big)F_{jk} + g^{i\beta}\Big(\overset{0}{\Gamma}{}^{\alpha}_{j\beta}F_{\alpha k} - \overset{0}{\Gamma}{}^{\alpha}_{k\beta}F_{\alpha j}\Big)\Big), \tag{32}$$

$$\overline{\tau}^i_{jk} = -\frac{1}{2}\Big(\overline{g}^{\alpha\beta}\Big(\overset{0}{\overline{\Gamma}}{}^{i}_{\alpha k}\overline{F}_{j\beta} - \overset{0}{\overline{\Gamma}}{}^{i}_{\alpha j}\overline{F}_{k\beta}\Big) - \Big(\overset{0}{\overline{\Gamma}}{}^{i}_{\alpha\beta}\overline{g}^{\alpha\beta} + \overset{0}{\overline{\Gamma}}{}^{\beta}_{\alpha\beta}\overline{g}^{i\alpha}\Big)\overline{F}_{jk} + \overline{g}^{i\beta}\Big(\overset{0}{\overline{\Gamma}}{}^{\alpha}_{j\beta}\overline{F}_{\alpha k} - \overset{0}{\overline{\Gamma}}{}^{\alpha}_{k\beta}\overline{F}_{\alpha j}\Big)\Big). \tag{33}$$

From the equation (31), we obtain

$$\overline{T}^i_{jk} + \overline{\tau}^i_{jk} = T^i_{jk} + \tau^i_{jk}. \tag{34}$$

From the invariants (21, 22), the equation (31) and the basic equation (17), we conclude that the following lemma holds:

Lemma 2.5. *Let* $f : \mathbb{GR}_N \to \mathbb{G}\overline{\mathbb{R}}_N$ *be a mapping between generalized Riemannian spaces* $\mathbb{GR}_N$ *and* $\mathbb{G}\overline{\mathbb{R}}_N$*.*

a) *If f is a conformal mapping, the geometrical object*

$$\hat{\Gamma}^i_{jk} = T^i_{jk} + \tau^i_{jk} \tag{35}$$

is an invariant of this mapping.

b) *The mapping f is a conformal mapping if and only if the geometrical object*

$$\begin{aligned}\overset{*}{\Gamma}{}^i_{jk} = \Gamma^i_{jk} &- \frac{1}{N}\Big(\overset{0}{\Gamma}{}^\alpha_{j\alpha}\delta^i_k + \overset{0}{\Gamma}{}^\alpha_{k\alpha}\delta^i_j - \overset{0}{\Gamma}{}^\beta_{\alpha\beta}g^{i\alpha}g_{jk}\Big)\\ &- \frac{1}{2}\bigg(g^{\alpha\beta}\Big(\overset{0}{\Gamma}{}^i_{\alpha k}F_{j\beta} - \overset{0}{\Gamma}{}^i_{\alpha j}F_{k\beta}\Big) - \Big(\overset{0}{\Gamma}{}^i_{\alpha\beta}g^{\alpha\beta} + \overset{0}{\Gamma}{}^\beta_{\alpha\beta}g^{i\alpha}\Big)F_{jk} + g^{i\beta}\Big(\overset{0}{\Gamma}{}^\alpha_{j\beta}F_{\alpha k} - \overset{0}{\Gamma}{}^\alpha_{k\beta}F_{\alpha j}\Big)\bigg)\end{aligned} \tag{36}$$

is an invariant of this mapping. □

The invariance $\hat{\overline{\Gamma}}{}^\alpha_{jm}\hat{\overline{\Gamma}}{}^i_{\alpha n} = \hat{\Gamma}^\alpha_{jm}\hat{\Gamma}^i_{\alpha n}$ proves that it is satisfied

$$\overline{T}{}^\alpha_{jm}\overline{T}{}^i_{\alpha n} = T^i_{jm}T^i_{\alpha n} + \tau^\alpha_{jm}T^i_{\alpha n} + \tau^i_{\alpha n}T^\alpha_{jm} + \tau^\alpha_{jm}\tau^i_{\alpha n} - \overline{\tau}{}^\alpha_{jm}\overline{T}{}^i_{\alpha n} - \overline{\tau}{}^i_{\alpha n}\overline{T}{}^\alpha_{jm} - \overline{\tau}{}^\alpha_{jm}\overline{\tau}{}^i_{\alpha n}. \tag{37}$$

From the invariance $\hat{\overline{\Gamma}}{}^i_{jm,n} = \hat{\Gamma}^i_{jm,n}$, we get

$$\hat{\overline{\Gamma}}{}^i_{jm\bar{;}n} - \hat{\Gamma}^i_{jm;n} = \overset{0}{\overline{\Gamma}}{}^i_{\alpha n}\hat{\overline{\Gamma}}{}^\alpha_{jm} - \overset{0}{\overline{\Gamma}}{}^\alpha_{jn}\hat{\overline{\Gamma}}{}^i_{\alpha m} - \overset{0}{\overline{\Gamma}}{}^\alpha_{mn}\hat{\overline{\Gamma}}{}^i_{j\alpha} - \overset{0}{\Gamma}{}^i_{\alpha n}\hat{\Gamma}^\alpha_{jm} + \overset{0}{\Gamma}{}^\alpha_{jn}\hat{\Gamma}^i_{\alpha m} + \overset{0}{\Gamma}{}^\alpha_{mn}\hat{\Gamma}^i_{j\alpha}, \tag{38}$$

i.e.

$$\overline{T}{}^i_{jm\bar{;}n} = T^i_{jm;n} + \overline{\sigma}{}^i_{jmn} - \sigma^i_{jmn} \tag{39}$$

for

$$\sigma^i_{jmn} = \overset{0}{\Gamma}{}^i_{\alpha n}\hat{\Gamma}^\alpha_{jm} - \overset{0}{\Gamma}{}^\alpha_{jn}\hat{\Gamma}^i_{\alpha m} - \overset{0}{\Gamma}{}^\alpha_{mn}\hat{\Gamma}^i_{j\alpha} - \tau^i_{jm;n}, \tag{40}$$

$$\overline{\sigma}{}^i_{jmn} = \overset{0}{\overline{\Gamma}}{}^i_{\alpha n}\hat{\overline{\Gamma}}{}^\alpha_{jm} - \overset{0}{\overline{\Gamma}}{}^\alpha_{jn}\hat{\overline{\Gamma}}{}^i_{\alpha m} - \overset{0}{\overline{\Gamma}}{}^\alpha_{mn}\hat{\overline{\Gamma}}{}^i_{j\alpha} - \overline{\tau}{}^i_{jm\bar{;}n}. \tag{41}$$

Let

$$R^i_{jmn} = \overset{0}{R}{}^i_{jmn} + u_0T^i_{jm;n} + u'_0T^i_{jn;m} + v_0T^\alpha_{jm}T^i_{\alpha n} + v'_0T^\alpha_{jn}T^i_{\alpha m} + w_0T^\alpha_{mn}T^i_{\alpha j} \tag{42}$$

be a curvature tensor of generalized Riemannian space $\mathbb{GR}_N$ expressed as linear function of the curvature tensor $\overset{0}{R}{}^i_{jmn}$ of the associated space $\mathbb{R}_N$. From the equation (42), after the contraction $i = n$ and the corresponding compositions with g^{ij}, we obtain that is

$$\overset{0}{R}{}^i_{jmn} = R^i_{jmn} - u_0T^i_{jm;n} - u'_0T^i_{jn;m} - v_0T^\alpha_{jm}T^i_{\alpha n} - v'_0T^\alpha_{jn}T^i_{\alpha m} - w_0T^\alpha_{mn}T^i_{\alpha j}, \tag{43}$$

$$\overset{0}{R}_{ij} = R_{ij} - u_0T^\alpha_{ij;\alpha} - (v'_0 + w_0)T^\alpha_{i\beta}T^\beta_{\alpha j}, \tag{44}$$

$$\overset{0}{R}{}^i_j = R^i_j - u_0g^{i\alpha}T^\beta_{\alpha j;\beta} - (v'_0 + w_0)g^{i\alpha}T^\beta_{\alpha\gamma}T^\gamma_{\beta j}, \tag{45}$$

$$\overset{0}{R} = R - (v'_0 + w_0)T^\alpha_{\gamma\beta}T^\beta_{\alpha\delta}g^{\gamma\delta}. \tag{46}$$

Because Weyl conformal curvature tensor C^i_{jmn} is an invariant of the mapping f, i.e. $\overline{C}^i_{jmn} = C^i_{jmn}$, and from the equations (43, 44, 45, 46), we obtain that it is satisfied

$$
\begin{aligned}
\overline{R}^i_{jmn} &= R^i_{jmn} - u_0 T^i_{jm;n} - u_0' T^i_{jn;m} - v_0 T^\alpha_{jm} T^i_{\alpha n} - v_0' T^\alpha_{jn} T^i_{\alpha m} - w_0 T^\alpha_{mn} T^i_{\alpha j} \\
&+ u_0 \overline{T}^i_{jm\bar{;}n} + u_0' \overline{T}^i_{jn\bar{;}m} + v_0 \overline{T}^\alpha_{jm} \overline{T}^i_{\alpha n} + v_0' \overline{T}^\alpha_{jn} \overline{T}^i_{\alpha m} + w_0 \overline{T}^\alpha_{mn} \overline{T}^i_{\alpha j} \\
&+ \frac{1}{N-2}\Big(\delta^i_m R_{jn} - \delta^i_n R_{jm} + R^i_m g_{jn} - R^i_n g_{jm} - \delta^i_m \overline{R}_{jn} + \delta^i_n \overline{R}_{jm} - \overline{R}^i_m \overline{g}_{jn} + \overline{R}^i_n \overline{g}_{jm}\Big) \\
&- \frac{u_0}{N-2}\Big(\delta^i_m T^\alpha_{jn;\alpha} - \delta^i_n T^\alpha_{jm;\alpha} - \delta^i_m \overline{T}^\alpha_{jn\bar{;}\alpha} + \delta^i_n \overline{T}^\alpha_{jm\bar{;}\alpha}\Big) \\
&- \frac{u_0}{N-2}\Big(g^{i\alpha} T^\beta_{\alpha m;\beta} g_{jn} - g^{i\alpha} T^\beta_{\alpha n;\beta} g_{jm} - \overline{g}^{i\alpha} \overline{T}^\beta_{\alpha m\bar{;}\beta} \overline{g}_{jn} + \overline{g}^{i\alpha} \overline{T}^\beta_{\alpha n\bar{;}\beta} \overline{g}_{jm}\Big) \\
&- \frac{v_0' + w_0}{N-2}\Big(\delta^i_m T^\alpha_{j\beta} T^\beta_{\alpha n} - \delta^i_n T^\alpha_{j\beta} T^\beta_{\alpha m} - \delta^i_m \overline{T}^\alpha_{j\beta} \overline{T}^\beta_{\alpha n} + \delta^i_n \overline{T}^\alpha_{j\beta} \overline{T}^\beta_{\alpha m}\Big) \\
&- \frac{v_0' + w_0}{N-2}\Big(g^{i\alpha} T^\beta_{\alpha\gamma} T^\gamma_{\beta m} g_{jn} - g^{i\alpha} T^\beta_{\alpha\gamma} T^\gamma_{\beta n} g_{jm} - \overline{g}^{i\alpha} \overline{T}^\beta_{\alpha\gamma} \overline{T}^\gamma_{\beta m} \overline{g}_{jn} + \overline{g}^{i\alpha} \overline{T}^\beta_{\alpha\gamma} \overline{T}^\gamma_{\beta n} \overline{g}_{jm}\Big) \\
&+ \frac{1}{(N-1)(N-2)}\Big(R(\delta^i_m g_{jn} - \delta^i_n g_{jm}) - \overline{R}(\delta^i_m \overline{g}_{jn} - \delta^i_n \overline{g}_{jm})\Big) \\
&- \frac{v_0' + w_0}{(N-1)(N-2)}\Big(T^\alpha_{\gamma\beta} T^\beta_{\alpha\delta} g^{\gamma\delta}\big(\delta^i_m g_{jn} - \delta^i_n g_{jm}\big) - \overline{T}^\alpha_{\gamma\beta} \overline{T}^\beta_{\alpha\delta} \overline{g}^{\gamma\delta}\big(\delta^i_m \overline{g}_{jn} - \delta^i_n \overline{g}_{jm}\big)\Big).
\end{aligned} \tag{47}
$$

From the equations (37, 38), we get:

$$\overline{T}^i_{jm\bar{;}n} = T^i_{jm;n} + \overline{\sigma}^i_{jmn} - \sigma^i_{jmn}, \tag{48}$$

$$\overline{T}^i_{jn\bar{;}m} = T^i_{jn;m} + \overline{\sigma}^i_{jnm} - \sigma^i_{jnm}, \tag{49}$$

$$\overline{T}^\alpha_{jm}\overline{T}^i_{\alpha n} = T^\alpha_{jm}T^i_{\alpha n} + \tau^\alpha_{jm}T^i_{\alpha n} + \tau^i_{\alpha n}T^\alpha_{jm} + \tau^\alpha_{jm}\tau^i_{\alpha n} - \overline{\tau}^\alpha_{jm}\overline{T}^i_{\alpha n} - \overline{\tau}^i_{\alpha n}\overline{T}^\alpha_{jm} - \overline{\tau}^\alpha_{jm}\overline{\tau}^i_{\alpha n}, \tag{50}$$

$$\overline{T}^\alpha_{jn}\overline{T}^i_{\alpha m} = T^\alpha_{jn}T^i_{\alpha m} + \tau^\alpha_{jn}T^i_{\alpha m} + \tau^i_{\alpha m}T^\alpha_{jn} + \tau^\alpha_{jn}\tau^i_{\alpha m} - \overline{\tau}^\alpha_{jn}\overline{T}^i_{\alpha m} - \overline{\tau}^i_{\alpha m}\overline{T}^\alpha_{jn} - \overline{\tau}^\alpha_{jn}\overline{\tau}^i_{\alpha m}, \tag{51}$$

$$\overline{T}^\alpha_{mn}\overline{T}^i_{\alpha j} = T^\alpha_{mn}T^i_{\alpha j} + \tau^\alpha_{mn}T^i_{\alpha j} + \tau^i_{\alpha j}T^\alpha_{mn} + \tau^\alpha_{mn}\tau^i_{\alpha j} - \overline{\tau}^\alpha_{mn}\overline{T}^i_{\alpha j} - \tau^i_{\alpha j}\overline{T}^\alpha_{mn} - \overline{\tau}^\alpha_{mn}\overline{\tau}^i_{\alpha j}. \tag{52}$$

By the equation (47), it is presented the transformation of curvature tensor $\overset{0}{R}{}^i_{jmn}$ to $\overset{0}{\overline{R}}{}^i_{jmn}$ as function of the curvature tensors R^i_{jmn} and $\overline{R}^i_{jmn}$ $\big($in the form (42)$\big)$ of the spaces $\mathbb{GR}_N$ and $\mathbb{G}\overline{\mathbb{R}}_N$. For this reason, from the corresponding linear combination of the equations (47 – 52):

$$(47) + u_0 \cdot (48) + u_0' \cdot (49) + v_0 \cdot (50) + v_0' \cdot (51) + w_0 \cdot (52)$$

we obtain that it holds the equality

$$\overline{\hat{C}}{}^i_{jmn} = \hat{C}^i_{jmn}$$

for

$$\begin{aligned}
\hat{C}^i_{jmn} &= R^i_{jmn} + \frac{1}{N-2}\Big(\delta^i_m R_{jn} - \delta^i_n R_{jm} + R^i_m g_{jn} - R^i_n g_{jm}\Big) + \frac{R}{(N-1)(N-2)}\Big(\delta^i_m g_{jn} - \delta^i_n g_{jm}\Big)\\
&- \frac{u_0}{N-2}\Big(\delta^i_m T^\alpha_{jn;\alpha} - \delta^i_n T^\alpha_{jm;\alpha} + g^{i\alpha}T^\beta_{\alpha m;\beta} g_{jn} - g^{i\alpha}T^\beta_{\alpha n;\beta} g_{jm}\Big)\\
&- \frac{v'_0 + w_0}{N-2}\Big(\delta^i_m T^\alpha_{j\beta}T^\beta_{\alpha n} - \delta^i_n T^\alpha_{j\beta}T^\beta_{\alpha m} + g^{i\alpha}T^\beta_{\alpha\gamma}T^\gamma_{\beta m} g_{jn} - g^{i\alpha}T^\beta_{\alpha\gamma}T^\gamma_{\beta n} g_{jm}\Big)\\
&- \frac{v'_0 + w_0}{(N-1)(N-2)} T^\alpha_{\gamma\beta}T^\beta_{\alpha\delta} g^{\gamma\delta}(\delta^i_m g_{jn} - \delta^i_n g_{jm})\\
&- u_0\sigma^i_{jmn} - u'_0\sigma^i_{jnm} + v_0(\tau^\alpha_{jm}T^i_{\alpha n} + \tau^i_{\alpha n}T^\alpha_{jm} + \tau^\alpha_{jm}\tau^i_{\alpha n})\\
&+ v'_0(\tau^\alpha_{jn}T^i_{\alpha m} + \tau^i_{\alpha m}T^\alpha_{jn} + \tau^\alpha_{jn}\tau^i_{\alpha m}) + w_0(\tau^\alpha_{mn}T^i_{\alpha j} + \tau^i_{\alpha j}T^\alpha_{mn} + \tau^\alpha_{mn}\tau^i_{\alpha j})
\end{aligned} \tag{53}$$

and the corresponding $\hat{\overline{C}}^i_{jmn}$. In this way, it is proved that the following theorem holds:

Theorem 2.6. *Let $f : \mathbb{GR}_N \to \mathbb{G}\overline{\mathbb{R}}_N$ be a conformal mapping of a generalized Riemannian space $\mathbb{GR}_N$. The geometrical objects* (53), *for the corresponding constants $u_0, u'_0, v_0, v'_0, w_0$ are invariants of the mapping f.* □

Corollary 2.7. *The invariant $\hat{C}^i_{jmn}$ of conformal mapping $f : \mathbb{GR}_N \to \mathbb{G}\overline{\mathbb{R}}_N$ and the Weyl conformal curvature tensor C^i_{jmn} of the associated space $\mathbb{R}_N$ satisfy the equation*

$$\begin{aligned}
\hat{C}^i_{jmn} &= C^i_{jmn} - u_0\sigma^i_{jmn} - u'_0\sigma^i_{jnm} + v_0(\tau^\alpha_{jm}T^i_{\alpha n} + \tau^i_{\alpha n}T^\alpha_{jm} + \tau^\alpha_{jm}\tau^i_{\alpha n})\\
&+ v'_0(\tau^\alpha_{jn}T^i_{\alpha m} + \tau^i_{\alpha m}T^\alpha_{jn} + \tau^\alpha_{jn}\tau^i_{\alpha m}) + w_0(\tau^\alpha_{mn}T^i_{\alpha j} + \tau^i_{\alpha j}T^\alpha_{mn} + \tau^\alpha_{mn}\tau^i_{\alpha j})
\end{aligned} \tag{54}$$

for the above defined σ^i_{jmn} and τ^i_{jk}. □

Corollary 2.8. *The invariants $\underset{k}{\hat{C}}{}^i_{jmn}$ of conformal mapping $f : \mathbb{GR}_N \to \mathbb{G}\overline{\mathbb{R}}_N$ derived from the linearly independent curvature tensors $\underset{k}{R}{}^i_{jmn}, k = 1, \ldots, 5$, given in the equations* (8–12) *are:*

$$\begin{aligned}
\underset{1}{\hat{C}}{}^i_{jmn} &= \underset{1}{R}{}^i_{jmn} + \frac{1}{N-2}\Big(\delta^i_m \underset{1}{R}_{jn} - \delta^i_n \underset{1}{R}_{jm} + \underset{1}{R}{}^i_m g_{jn} - \underset{1}{R}{}^i_n g_{jm}\Big) + \frac{1}{(N-1)(N-2)}\underset{1}{R}\Big(\delta^i_m g_{jn} - \delta^i_n g_{jm}\Big)\\
&- \frac{1}{N-2}\Big(\delta^i_m T^\alpha_{jn;\alpha} - \delta^i_n T^\alpha_{jm;\alpha} + g^{i\alpha}T^\beta_{\alpha m;\beta} g_{jn} - g^{i\alpha}T^\beta_{\alpha n;\beta} g_{jm}\Big)\\
&+ \frac{1}{N-2}\Big(\delta^i_m T^\alpha_{j\beta}T^\beta_{\alpha n} - \delta^i_n T^\alpha_{j\beta}T^\beta_{\alpha m} + g^{i\alpha}T^\beta_{\alpha\gamma}T^\gamma_{\beta m} g_{jn} - g^{i\alpha}T^\beta_{\alpha\gamma}T^\gamma_{\beta n} g_{jm}\Big)\\
&+ \frac{1}{(N-1)(N-2)} T^\alpha_{\gamma\beta}T^\beta_{\alpha\delta} g^{\gamma\delta}(\delta^i_m g_{jn} - \delta^i_n g_{jm})\\
&- \sigma^i_{jmn} + \sigma^i_{jnm} + \tau^\alpha_{jm}T^i_{\alpha n} + \tau^i_{\alpha n}T^\alpha_{jm} + \tau^\alpha_{jm}\tau^i_{\alpha n} - \tau^\alpha_{jn}T^i_{\alpha m} - \tau^i_{\alpha m}T^\alpha_{jn} - \tau^\alpha_{jn}\tau^i_{\alpha m},
\end{aligned} \tag{55}$$

$$\begin{aligned}
\underset{2}{\hat{C}}{}^i_{jmn} &= \underset{2}{R}{}^i_{jmn} + \frac{1}{N-2}\Big(\delta^i_m \underset{2}{R}_{jn} - \delta^i_n \underset{2}{R}_{jm} + \underset{2}{R}{}^i_m g_{jn} - \underset{2}{R}{}^i_n g_{jm}\Big) + \frac{1}{(N-1)(N-2)}\underset{2}{R}\Big(\delta^i_m g_{jn} - \delta^i_n g_{jm}\Big)\\
&+ \frac{1}{N-2}\Big(\delta^i_m T^\alpha_{jn;\alpha} - \delta^i_n T^\alpha_{jm;\alpha} + g^{i\alpha}T^\beta_{\alpha m;\beta} g_{jn} - g^{i\alpha}T^\beta_{\alpha n;\beta} g_{jm}\Big)\\
&+ \frac{1}{N-2}\Big(\delta^i_m T^\alpha_{j\beta}T^\beta_{\alpha n} - \delta^i_n T^\alpha_{j\beta}T^\beta_{\alpha m} + g^{i\alpha}T^\beta_{\alpha\gamma}T^\gamma_{\beta m} g_{jn} - g^{i\alpha}T^\beta_{\alpha\gamma}T^\gamma_{\beta n} g_{jm}\Big)\\
&+ \frac{1}{(N-1)(N-2)} T^\alpha_{\gamma\beta}T^\beta_{\alpha\delta} g^{\gamma\delta}(\delta^i_m g_{jn} - \delta^i_n g_{jm})\\
&+ \sigma^i_{jmn} - \sigma^i_{jnm} + \tau^\alpha_{jm}T^i_{\alpha n} + \tau^i_{\alpha n}T^\alpha_{jm} + \tau^\alpha_{jm}\tau^i_{\alpha n} - \tau^\alpha_{jn}T^i_{\alpha m} - \tau^i_{\alpha m}T^\alpha_{jn} - \tau^\alpha_{jn}\tau^i_{\alpha m},
\end{aligned} \tag{56}$$

$$\begin{aligned}
\hat{\underset{3}{C}}{}^i_{jmn} &= \underset{3}{R}{}^i_{jmn} + \frac{1}{N-2}\Big(\delta^i_m \underset{3}{R}_{jn} - \delta^i_n \underset{3}{R}_{jm} + \underset{3}{R}{}^i_m g_{jn} - \underset{3}{R}{}^i_n g_{jm}\Big) + \frac{1}{(N-1)(N-2)}\underset{3}{R}\Big(\delta^i_m g_{jn} - \delta^i_n g_{jm}\Big)\\
&- \frac{1}{N-2}\Big(\delta^i_m T^\alpha_{jn;\alpha} - \delta^i_n T^\alpha_{jm;\alpha} + g^{i\alpha}T^\beta_{\alpha m;\beta} g_{jn} - g^{i\alpha}T^\beta_{\alpha n;\beta} g_{jm}\Big)\\
&+ \frac{1}{N-2}\Big(\delta^i_m T^\alpha_{j\beta}T^\beta_{\alpha n} - \delta^i_n T^\alpha_{j\beta}T^\beta_{\alpha m} + g^{i\alpha}T^\beta_{\alpha\gamma}T^\gamma_{\beta m} g_{jn} - g^{i\alpha}T^\beta_{\alpha\gamma}T^\gamma_{\beta n} g_{jm}\Big)\\
&+ \frac{1}{(N-1)(N-2)}T^\alpha_{\gamma\beta}T^\beta_{\alpha\delta}g^{\gamma\delta}(\delta^i_m g_{jn} - \delta^i_n g_{jm})\\
&- \sigma^i_{jmn} - \sigma^i_{jnm} - \tau^\alpha_{jm}T^i_{\alpha n} - \tau^i_{\alpha n}T^\alpha_{jm} - \tau^\alpha_{jm}\tau^i_{\alpha n} + \tau^\alpha_{jn}T^i_{\alpha m} + \tau^i_{\alpha m}T^\alpha_{jn} + \tau^\alpha_{jn}\tau^i_{\alpha m}\\
&- 2(\tau^\alpha_{mn}T^i_{\alpha j} + \tau^i_{\alpha j}T^\alpha_{mn} + \tau^\alpha_{mn}\tau^i_{\alpha j}),
\end{aligned} \tag{57}$$

$$\begin{aligned}
\hat{\underset{4}{C}}{}^i_{jmn} &= \underset{4}{R}{}^i_{jmn} + \frac{1}{N-2}\Big(\delta^i_m \underset{4}{R}_{jn} - \delta^i_n \underset{4}{R}_{jm} + \underset{4}{R}{}^i_m g_{jn} - \underset{4}{R}{}^i_n g_{jm}\Big) + \frac{1}{(N-1)(N-2)}\underset{4}{R}\Big(\delta^i_m g_{jn} - \delta^i_n g_{jm}\Big)\\
&- \frac{1}{N-2}\Big(\delta^i_m T^\alpha_{jn;\alpha} - \delta^i_n T^\alpha_{jm;\alpha} + g^{i\alpha}T^\beta_{\alpha m;\beta} g_{jn} - g^{i\alpha}T^\beta_{\alpha n;\beta} g_{jm}\Big)\\
&- \frac{3}{N-2}\Big(\delta^i_m T^\alpha_{j\beta}T^\beta_{\alpha n} - \delta^i_n T^\alpha_{j\beta}T^\beta_{\alpha m} + g^{i\alpha}T^\beta_{\alpha\gamma}T^\gamma_{\beta m} g_{jn} - g^{i\alpha}T^\beta_{\alpha\gamma}T^\gamma_{\beta n} g_{jm}\Big)\\
&- \frac{3}{(N-1)(N-2)}T^\alpha_{\gamma\beta}T^\beta_{\alpha\delta}g^{\gamma\delta}(\delta^i_m g_{jn} - \delta^i_n g_{jm})\\
&- \sigma^i_{jmn} - \sigma^i_{jnm} - \tau^\alpha_{jm}T^i_{\alpha n} - \tau^i_{\alpha n}T^\alpha_{jm} - \tau^\alpha_{jm}\tau^i_{\alpha n} + \tau^\alpha_{jn}T^i_{\alpha m} + \tau^i_{\alpha m}T^\alpha_{jn} + \tau^\alpha_{jn}\tau^i_{\alpha m}\\
&+ 2(\tau^\alpha_{mn}T^i_{\alpha j} + \tau^i_{\alpha j}T^\alpha_{mn} + \tau^\alpha_{mn}\tau^i_{\alpha j}),
\end{aligned} \tag{58}$$

$$\begin{aligned}
\hat{\underset{5}{C}}{}^i_{jmn} &= \underset{5}{R}{}^i_{jmn} + \frac{1}{N-2}\Big(\delta^i_m \underset{5}{R}_{jn} - \delta^i_n \underset{5}{R}_{jm} + \underset{5}{R}{}^i_m g_{jn} - \underset{5}{R}{}^i_n g_{jm}\Big) + \frac{1}{(N-1)(N-2)}\underset{5}{R}\Big(\delta^i_m g_{jn} - \delta^i_n g_{jm}\Big)\\
&- \frac{1}{N-2}\Big(\delta^i_m T^\alpha_{j\beta}T^\beta_{\alpha n} - \delta^i_n T^\alpha_{j\beta}T^\beta_{\alpha m} + g^{i\alpha}T^\beta_{\alpha\gamma}T^\gamma_{\beta m} g_{jn} - g^{i\alpha}T^\beta_{\alpha\gamma}T^\gamma_{\beta n} g_{jm}\Big)\\
&- \frac{1}{(N-1)(N-2)}T^\alpha_{\gamma\beta}T^\beta_{\alpha\delta}g^{\gamma\delta}(\delta^i_m g_{jn} - \delta^i_n g_{jm})\\
&+ \tau^\alpha_{jm}T^i_{\alpha n} + \tau^i_{\alpha n}T^\alpha_{jm} + \tau^\alpha_{jm}\tau^i_{\alpha n} + \tau^\alpha_{jn}T^i_{\alpha m} + \tau^i_{\alpha m}T^\alpha_{jn} + \tau^\alpha_{jn}\tau^i_{\alpha m},
\end{aligned} \tag{59}$$

for $\underset{k}{R}_{ij} = \underset{k}{R}{}^\alpha_{ij\alpha}, \underset{k}{R}{}^i_j = g^{i\alpha}\underset{k}{R}_{\alpha j}, \underset{k}{R} = \underset{k}{R}{}^\alpha_\alpha, k = 1,\dots,5$, *and the above defined objects* τ^i_{jk} *and* σ^i_{jmn}. □

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