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Multi-Stage Stochastic Model in Portfolio Selection Problem

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Abstract. This paper is a novel work of portfolio-selection problem solving using multi objective model considering four parameters, Expected return, downside beta coefficient, semivariance and conditional value at risk at a specified confidence level. Multi-period models can be defined as stochastic models. Early studies on portfolio selection developed using variance as a risk measure; although, theories and practices revealed that variance, considering its downsides, is not a desirable risk measure. To increase accuracy and overcoming negative aspects of variance, downside risk measures like semivarinace, downside beta covariance, value at risk and conditional value at risk was other risk measures that replaced in models. These risk measures all have advantages over variance and previous works using these parameters have shown improvements in the best portfolio selection. Purposed models are solved using genetic algorithm and for the topic completion, numerical example and plots to measure the performance of model in four dimensions are provided.

1. Introduction

Portfolio selection problem introduced by Markowitz [13] established a fundamental base for single-period portfolio selection by maximizing expected return for a certain level of risk or minimizing risk for a certain level of return. In the real world portfolio strategies are usually multi-period as the investors are able to rebalance their portfolio in each time period. Li and Ng [12] proposed an analytical optimal solution to the multi-period mean-variance model. Wei and Ye [24] developed a multi-period mean-variance portfolio selection model, taken bankruptcy constraint into consideration in stochastic markets. Gulpnar and Rustem [8] proposed a multi-period meanvariance optimization, containing the construction of scenario tree to present uncertainties and associated possibilities in future stages. Celikyurt and Ozekici [4] introduced several multi-period portfolio optimization models in stochastic markets using the meanvariance approach. As mentioned above, researchers mainly used variance as a risk measure. Later studies revealed measuring the actual investment risk and asymmetric return distributions are reasons of replacing downside risk measures, such as semi-variance, downside beta coefficient, Value-at-Risk (VaR) or Conditional Value-at-Risk (CVaR) with variance. Markowitz et al. [14] proposed the mean-semivariance model as an alternative to mean-variance model. Managing and controlling risk using Value at Risk (VaR), a risk measures, was proposed by Baumol (1963) and known as quantile in the statistical literatures. However, VaR is not an

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acceptable risk measure since does not have sub-additive characteristics (Artzner et al. [2]). Therefore, Rockafellar and Uryasev [19, 20], expressed another risk measure which was named Conditional Value at Risk (CVaR). Pflug [18] proved that CVaR is a coherent risk measure having properties such as monotonicity, sub-additivity, positive homogeneity, translation invariance, and convexity. CVaR is defined as average of larger losses than VaR. CVaR became so popular for its advantages like convexity (Pflug [18], Ogryczak and Ruszczynski [16]) and researchers use CVaR as a risk measure for portfolio and financial evaluations (John and Hafize [11], Huang et al. [10], Zhu and Fukushima [27], Yau et al. [25], Sawik [21], Claro and Pinho de Sousa [5]). Scaillet [22], considered a nonparametric estimation of CVaR by using kernel estimator. A group of fully non parametric estimators based on the empirical conditional quantile function are considered in Peracchi and Tanase [17]. Hong and Liu [9] used Monte Carlo simulation to calculate CVaR for portfolio optimization. Another nonparametric estimation of CVaR is proposed by Yu et al. [26] based on the kernel quantile estimation approach. Najafi and Mushakhian solved multi-stage stochastic mean-semi variance-CVaR model using evolution algorithms[15]. In their paper optimal weights determined using genetic algorithm and developed using downside beta coefficient, semi variance and CVaR as risk measures. Therefore Mean-semi variance-β-CVaR multi-stage stochastic model is intoduced for portfolio optimization. The purpose is to find the best weights for each cycle or stage. For this, genetic methods for weights improvements are used. The remainder of this paper is organized as follows. Risk measures definitions and formulations are explained in section 2. Proposed model described in section 3. In section 4, a numerical example is presented and genetic algorithm is used to solve proposed model. Section 5 sums up with a conclusion.

2. Genetic Algorithm

Genetic algorithm (introduced in 1960s and described by Goldburg [7]) is a well-known evolutionary algorithm, which is under Inspiration of natural evolution and looking for better future generations. Procedure of evolution contains recombination operations and elites selection. Genetic algorithm was firstly used by biologist; however, later it found its way to other branches of science where models had to be solved and finding the best solutions were complicated. In genetic algorithm selection, crossover and mutation methods are main methods used to evolve the under study population.

Evolution procedure starts with an initial population. Usually initial population is generated randomly. Next step is an arbitrary one where all of the members are evaluated according to the objective function. Next generation is going to be created by the application of elite ones (Figure 1). On the third step, next generation is created using mutation or crossover methods. In mutation, some randomly selected genes of a single selected chromosome are changed to form new generation (Figure 2). Crossover method is performed by two selected chromosomes (parents). Selected genes of parents are combined (e.g. linear combinations) and new chromosome is formed (Figure 3). This procedure continues till the desired criteria (e.g. numbers of iteration or level of precious) are satisfied otherwise the procedure backs to the step 1.

3. Basics and fundamentals

In this section, risk measures, definitions and other useful basics are explained.

3.1. Semi-variance

Semi-variance, a statistical measure, evaluates how far values which are less than mean are spread out from average. This measure makes investors be aware of risk of loss whenever return of their investment is less than its expected return. In fact they are warned about negative fluctuations more directly. The formula for semi-variance is as follows:

$$\text{Semi-variance} = \frac{1}{s}\sum_{n=1}^{s}(R_n - \bar{R})^2; \tag{1}$$

Where R_n are returns less than the expected return ($\bar{R}$) and s is the total number of returns below the expected return or numeber of senario.

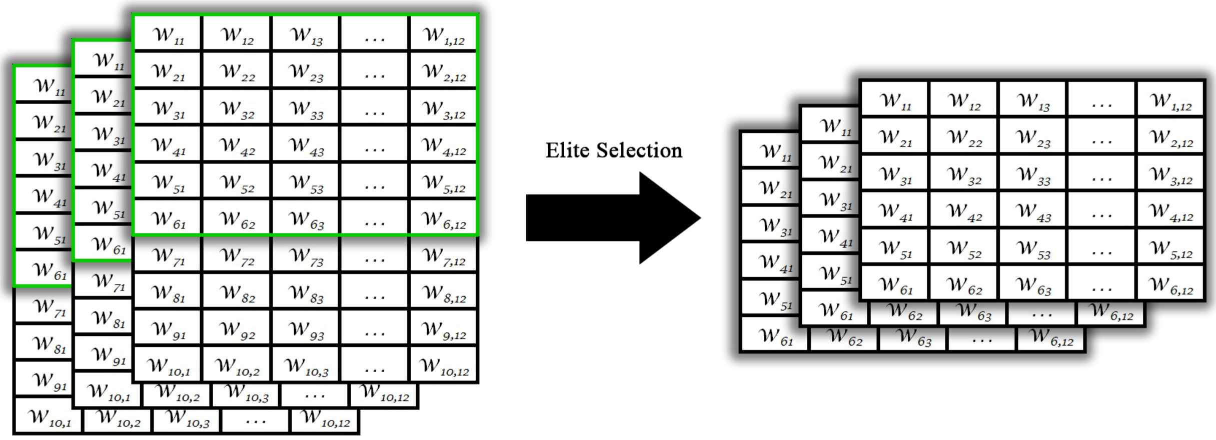

Figure 1: Selection of elite members

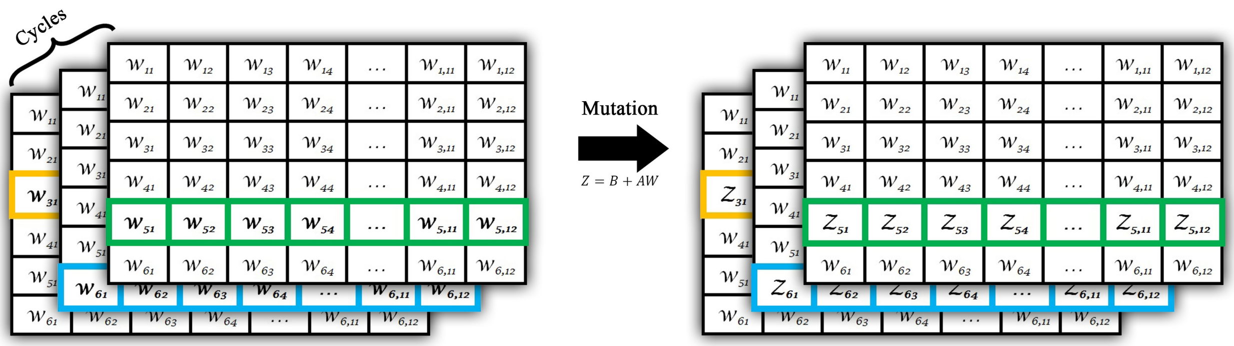

Figure 2: Mutation method. B and A are arbitrary values.

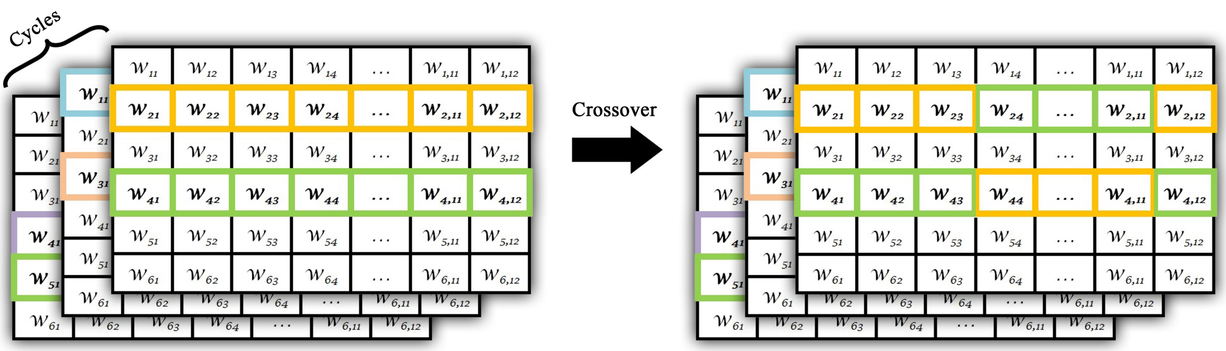

Figure 3: Crossover method.

3.2. *Value at Risk (VaR)*

VaR is defined as the maximum money that one may loss in a specified time interval. Mathematically VaR is defined as the quantile of a distribution. Suppose that P_t is the initial wealth and P_{t+k} is the secondary wealth after k periods; probability of loss is defined as:

$$p(-\Delta P_k < VaR) = \alpha \tag{2}$$

where ΔP_k is defined as $P_{k+t} - P_k$ and α is the confidence level. Based on to this definition probability of losing more than VaR is equal to 1-α. There are different methods for computing VaR, such as Variance-Covariance method, Historical and Monte Carlo simulations. Variance-covariance method only used for normally distributed data. As in most cases prices are not normally distributed, variance-covariance should be used with considerations. In Historical and Monte Carlo simulation methods there is no hypothesis for normally distributed data, thus these methods can be used with less restrictions.

3.3. *Conditional Value at Risk (CVaR)*

Let $x \in \phi \subset \mathbb{R}^n$ be a decision vector, $r \in \mathbb{R}^n$ be the random vector representing the value of under lying risk factors, and $f(x,r)$ be the corresponding loss. For simplicity, it is assumed that $r \in \mathbb{R}^n$ is a continuous random vector. For a given portfolio x, the probability of the loss not exceeding a threshold η, is given by the probability function $\mathbb{P}(.)$.

$$\psi(x,\eta) := \mathbb{P}(f(x,r) \leq \eta) \tag{3}$$

The VaR associated with a portfolio x and a specified confidence level α is the minimal η satisfying $(x,\eta) \geq \alpha$, that is:

$$VaR_\alpha(x) := inf\{\eta \in \mathbb{R}, \psi(x,\eta) \geq \alpha\}. \tag{4}$$

Since $\psi(x,\eta)$ is continuous by assumption, we have:

$$\mathbb{P}(f(x,r) \leq VaR_\alpha(x)) = \psi(x, VaR_\alpha(x)) = \alpha \tag{5}$$

CVaR is defined as the conditional expectation of the portfolio loss exceeding or equal to VaR

$$CVaR_\alpha(x) := E[f(x,r)|f(x,r) \geq VaR_\alpha(x) = \frac{1}{1-\alpha}\int_{VaR_\alpha(x)}^{+\infty} rd(r)dx] \tag{6}$$

where E is the expectation operator and $d(r)$ is the probability density function of the loss $f(x,r)$. Rockafellar and Uryasev [19] prove that CVaR has an equivalent definition as follows:

$$CVaR_\alpha(x) = min_\eta F_\alpha(x,\eta) \tag{7}$$

where $F_\alpha(x,\eta)$ is defined as:

$$F_\alpha(x,\eta) := \eta + \frac{1}{1-\alpha}E[(f(x,r) - \eta)^+] \tag{8}$$

with $(x)^+ = max\{x,0\}$. They also show that minimizing CVaR over $x \in \phi \subset \mathbb{R}^n$ is equivalent to minimizing $F_\alpha(x,\eta)$ over $(x,\eta) \in \phi \times \mathbb{R}$; i.e., $min_{x\in\phi} CVaR_\alpha(x) = min_{(x,\eta)\in\phi\times\mathbb{R}} F_\alpha(x,\eta)$. Furthermore, when ϕ is a convex set and $f(x,r)$ with respect to x is convex, the problem is a convex programming problem.

3.4. *Downside β coefficient*

Beta coefficient is a measure which talks over shares (portfolios) and market relation. Larger than 1 beta indicates higher violation of shares (portfolios) in comparison with market. More clearly movements in price of an understudy share are much more under effect of movements of market price. Beta coefficient is defined as the ratio of Market and Share Returns covariance to variance of market return. When a share has fewer movements in price, in comparison with market price, beta will be less than 1. Note that market's beta equals to 1. To calculate Beta, covariance of market and share returns can be restricted to ones which are less than mean [6]. This formula known as downside Beta coefficient and defined as:

$$\beta_{is} = \frac{E[min(R_i - \bar{R}_i, 0)min(R_m - \bar{R}_m, 0)]}{E[min((R_m - \bar{R}_m, 0)^2]} \tag{9}$$

Where R_m and $\bar{R}_m$ are market return and average of market's return respectively. R_i is share's return and $\bar{R}_i$ stands for average of under evaluation share.

4. Main results and methodology

Model is a multi objective one that should be changed one objective. One way is weighted sum of the objective function. Also,ϵ constrained method is another way for these problems. Multi-period portfolio problem can be defined as follows: there are N risky assets consist of bonds and stocks, one risk free asset; a planning horizon consists of T periods ($t = 0, 1, \cdots, T-1$). Uncertainly is stated through scenarios and each scenario describes a probability realization of all uncertain parameters in model. Each Scenario has a probability p_s, where $p_s > 0$ and $\sum p_s = 1$. For simplicity, we supposed $p_s = \frac{1}{s}$. Parameters and decision variables can be defined as follows:

Parameters:

$$\begin{array}{ll} P_s & \text{probability that scenario s occurs.} \\ r^s_{n,t} & \text{Return of asset n, in time period t under scenario s} \\ r^s_{m,t} & \text{Return of market, in time period t under scenario s} \\ d & \text{expected of wealth in the last time period} \\ b & \text{expected of CVaR} \\ c & \text{expected of semivariance} \\ w_0 & \text{wealth in the beginning of time period 0} \\ \alpha & \text{confidence level} \end{array} \tag{10}$$

Decision variables:

$$\begin{array}{ll} x^s_{n,t} & \text{amount of asset's weight n, in the beginning of time t before rebalancing.} \\ y^s_{n,t} & \text{amount of asset's weight n, in the beginning of time t after rebalancing.} \\ v^s_{n,t} & \text{amount of money bought of asset n, in the beginning of time t} \\ u^s_{n,t} & \text{amount of money sold of asset n, in the beginning of time t} \\ w^s_t & \text{wealth at the beginning of time t, under scenario s.} \end{array} \tag{11}$$

Our proposed model is as follows:

$$\begin{array}{ll} \min & \text{downside } \beta \text{ coefficient} \\ \min & \text{CVaR} \\ \max & \text{expected return} \\ \min & \text{semivariance} \\ \text{s.t.} & x \in S \end{array} \tag{12}$$

The above model is changed by using ϵ constrained method as follows:

$$\begin{array}{ll} \min & \text{downside } \beta \text{ coefficient(x)} \\ \text{s.t.} & \text{CVaR}(x,\alpha) \leq b \\ & E(x) \geq d \\ & \frac{1}{S}\sum_{n=1}^{S}(R_n - \overline{R})^2 \leq c \\ & x \in S \end{array} \tag{13}$$

In the following model objective function defined as the average of all under evalution parameters.

$$\begin{array}{ll} \min & 0.25 * \left(-E(x) + \frac{E[min(R_i - \bar{R}_i, 0) min(R_m + \bar{R}_m, 0)]}{E[min((R_m - \bar{R}_m, 0)^2]} + \frac{1}{S}\sum_{n=1}^{S}(R_n - \bar{R})^2 + \text{CVaR}(x,\alpha)\right) \\ & x \in S \end{array} \tag{14}$$

Finally, the model is defined as follows:

$$
\begin{array}{lll}
\min & \frac{E[\min(r^s_{n,t}-E(r^s_{n,t}),0)\min(r_{m,t}-E(r_{m,t}),0)]}{E[(r_{m,t}-E(r_{m,t}))]^2} & \\
s.t & \sum_{n=1}^{N} y^s_{n,t}E(r^s_{n,t}) = \sum_{n=1}^{N} x^s_{n,t}E(r^s_{n,t}) + v^s_{n,t} - u^s_{n,t} & \forall s \in S, \forall n \in N, \forall t \in T \\
& \sum_{n=1}^{N} w^s_{n,0} = w_0 & \forall s \in S \\
& \sum_{n=1}^{N} x^s_{n,t}E(r^s_{n,t}) = w^s_t & \forall s \in S, \forall n \in N, \forall t \in T \\
& \frac{1}{S}\sum_{n=1}^{N} (r^s_{n,t} - E(r^s_{n,t}))^2 \le c & \forall s \in S, \forall n \in N, \forall t \in T \\
& \frac{1}{s}\sum w^s_t \ge d & \forall s \in S, \forall t \in T \\
& \alpha_s \ge \eta - w^s_t & \forall s \in S \\
& \alpha_s \ge 0 & \forall s \in S \\
& \eta - \frac{1}{(1-\alpha)s}\sum_{s=1}^{S} \alpha_s \le b & \\
& \sum_{n=1}^{N} x^s_{n,t} = 1 & \\
& \sum_{n=1}^{N} y^s_{n,t} = 1 &
\end{array}
\tag{15}
$$

5. Empirical discussion

5.1. Data collection

This section goes through a numerical sample for previously discussed models. This paper is a new attempt to find the best matrix of weights using Evolutionary Algorithms". The idea of using evolutionary algorithms is based on the fact that optimization of portfolios on a time horizon and considering different investment cycles and scenarios and having this fact in mind that assets' combination can be changed during investment, needs large amounts of calculations. Therefore, evolutionary algorithms, although may not find the best solution, they can find good ones in much considerable time.

In this sample last daily price of 12 assets over a year (23/08/2015 to 22/08/2016) from Tehran Stock Exchange selected[1)]. National holidays omitted and missing data estimated through interpolation methods. Using interpolated data, daily returns over 246 days calculated. Four under evaluation parameters including expected return, semi variance, CVaR and downside β coefficient are calculated (Table 1).

Table 1: Calculated parameters of under study companies

Stock's Name	Semi Variance	CVaR	β	Expected Return
AMI	0.000787	0.000092	0.031000	0.421609
BG	0.000967	0.000501	0.046488	3.014368
BA	0.000854	0.000385	0.043585	2.009351
DC	-0.000532	0.000183	0.038174	0.863699
ENB	0.000800	0.000067	0.025103	0.765834
IT	0.002557	0.000345	0.043346	2.373823
IRIK	-0.000308	0.000132	0.029721	1.248270
KD	0.000825	0.000204	0.040225	1.166596
MB	0.001216	0.000573	0.046174	2.760882
PS	-0.002193	0.000278	0.038202	0.985883
PB	0.002726	0.000482	0.042222	2.324887
S&V	0.001850	0.000206	0.039535	1.603708

Investment period consists of 3 dependent cycles, where in each cycle combinations of assets can change to form optimal portfolios based on the market situation. The aim is to find the best matrix of weights for each cycle. Just as a matter of providing a series of investment opportunities for investors, in each cycle 20 different arrays of weights (portfolios) are provided. In this paper genetic methods for

[1)] http://www.tse.ir/

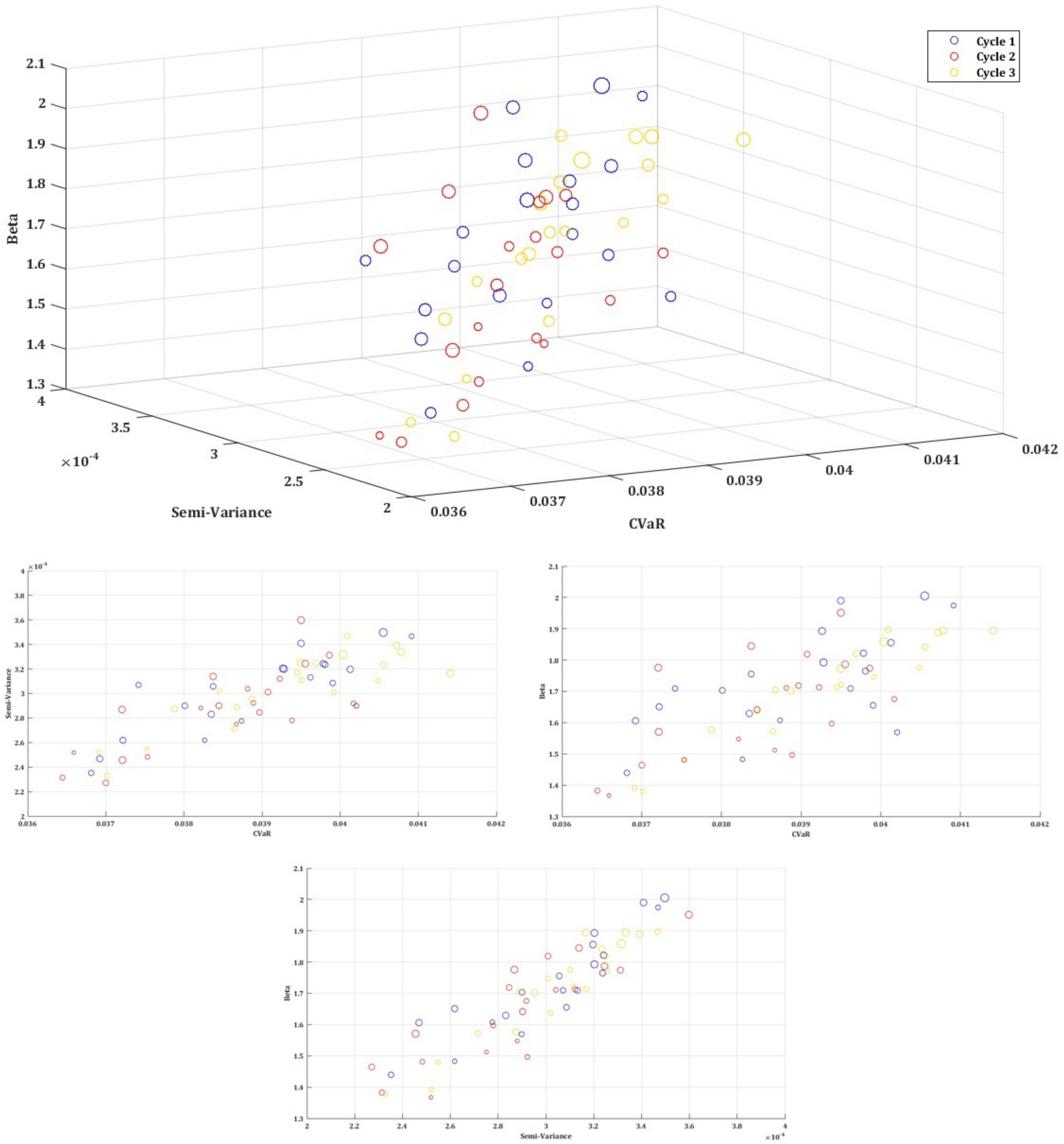

Figure 4: Portfolios constructed based on optimized weights. It is obvious that semi-varinace, beta and CVaR have direct and positive relations.

weights improvements are used. Procedure generally consists of a series of parents' chromosomes that new generations are going to be developed by a group of elite ones. Parents in 3 cycles by three random matrixes which have 6010 rows (parents) and 12 columns (assets) are generated. Mutation and crossover methods are used for improvements. In each iteration [2)] of optimization elite members based on weighted objective function (model 14) are selected and 20 members with the worst objective values are deleted. Procedure is as follows:

- A random matrix of weights with 3 dimensions generated. This matrix had 6010 rows (repetitions) and 12 columns (assets) and 3 layers (cycles). Note for each iteration, 20 of the worst weights combinations are omitted. In the last iteration only 20 rows with the least objective functions (elite ones) remain.
- In each cycle, 4 parameters for each portfolio calculated and objective function for all of them calculated. 20 of portfolios having the worst values (since model in minimization, 20 portfolios with the most maximum values) are omitted. Remained portfolios are elites going to the next step. In the first iteration after omitting 5990 rows remain.
- In the first step mutation method for improvement of weights used. 5 random numbers without repetition selected. Random numbers stand for selected rows. Selected rows called parent chromosomes. New generations built through linear calculations (Figure 2).
  Since weights must be positive values in range of [0,1], after calculations, negative weights and greater than 1s, replaced with 0 and 1 respectively.
- Crossover is another method used to improve future generations. In this method 2 rows (parent chromosomes) selected and randomly selected sections of chromosomes are exchanged (Figure 3).
- And finally in the last step, under study parameters and the objectove function for each portfolio with new weights calculated. New weights replaced with the previous ones, if the objective function value was smaller than previous one. This procedure starts over 300 times till the best weights are generated.

In figure 4 portfolios made according to optimized weights, are shown with circles. It can be seen that risk measures have positive and direct relation. Also, Circle diameters show expected return of each optimized portfolio. Circles with larger diameters have higher returns. However as expected there is not any relationship between expected return and risk measures.

In figure 5, 6010 portfolios constructed based on primary weights in all cycles are shown. In figure 6 optimized and primary weighted portfolios are shown together. An interesting conclusion is optimized weighted ones are less spread than not optimized ones. In fact most of optimized weighteds are in the middle of cloud of primary weighteds.

6. Conclusion

The paper introduced a new multistage model for portfolio selection considering four risk parameters. As mentioned through the paper, multi stage models by considering changes in the structure of portfolio over time are closer to conditions that investors experience. Proposed model in this paper developed using four risk parameters, semivarinace, downside beta covariance, value at risk and conditional value at risk. Four periods of 3 months was considered for investment. Solving such models needs using evolutionary algorithms. By using genetic algorithm model solved and optimal weights calculated. Outcomes revealed risk parameters are positively and directly related however, expected return and risk measures are independent. As the last point, visualization weighted portfolios are shown in four dimension diagrams where diameters of circles were indicators of expected returns. Also plots revealed that optimized portfolios are less speared.

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[2)]Number of iterations is 300.

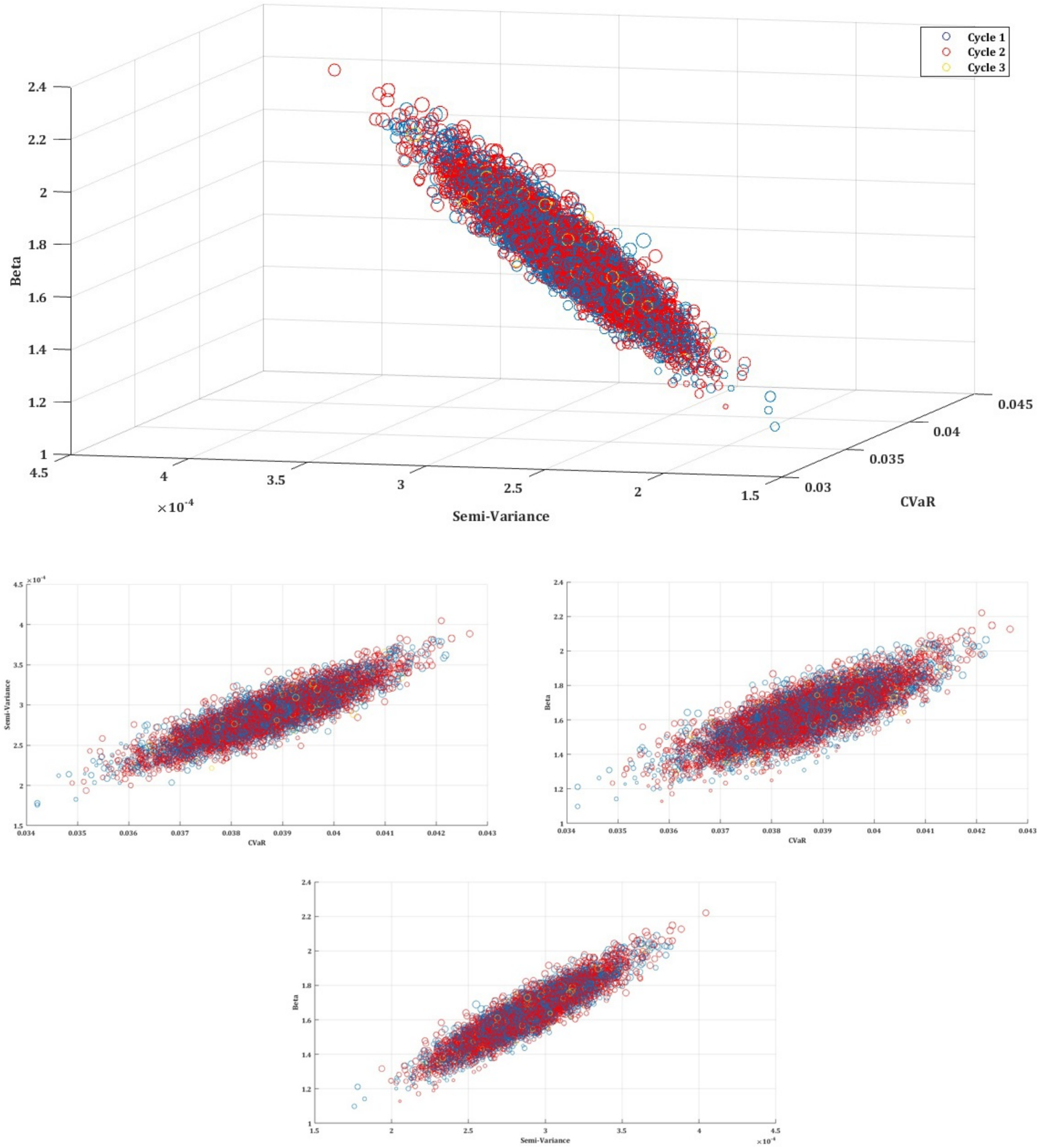

Figure 5: Cloud of primary wieghted portfolios in all cycles.

References

[1] P. Artzner, F. Delbaen, J.M. Eber, D. Heath, Coherent measures of risk, Math. Finance 9 (1999) 203–228.
[2] P. Artzner, F. Eber, J.M. Eber, D. Heath, Thinking coherently, Risk 10 (1997) 68–71.
[3] W.J. Baumol, An expected gain confidence limit criterion for portfolio selection, J. Management Sci. 10 (1963) 174–182.

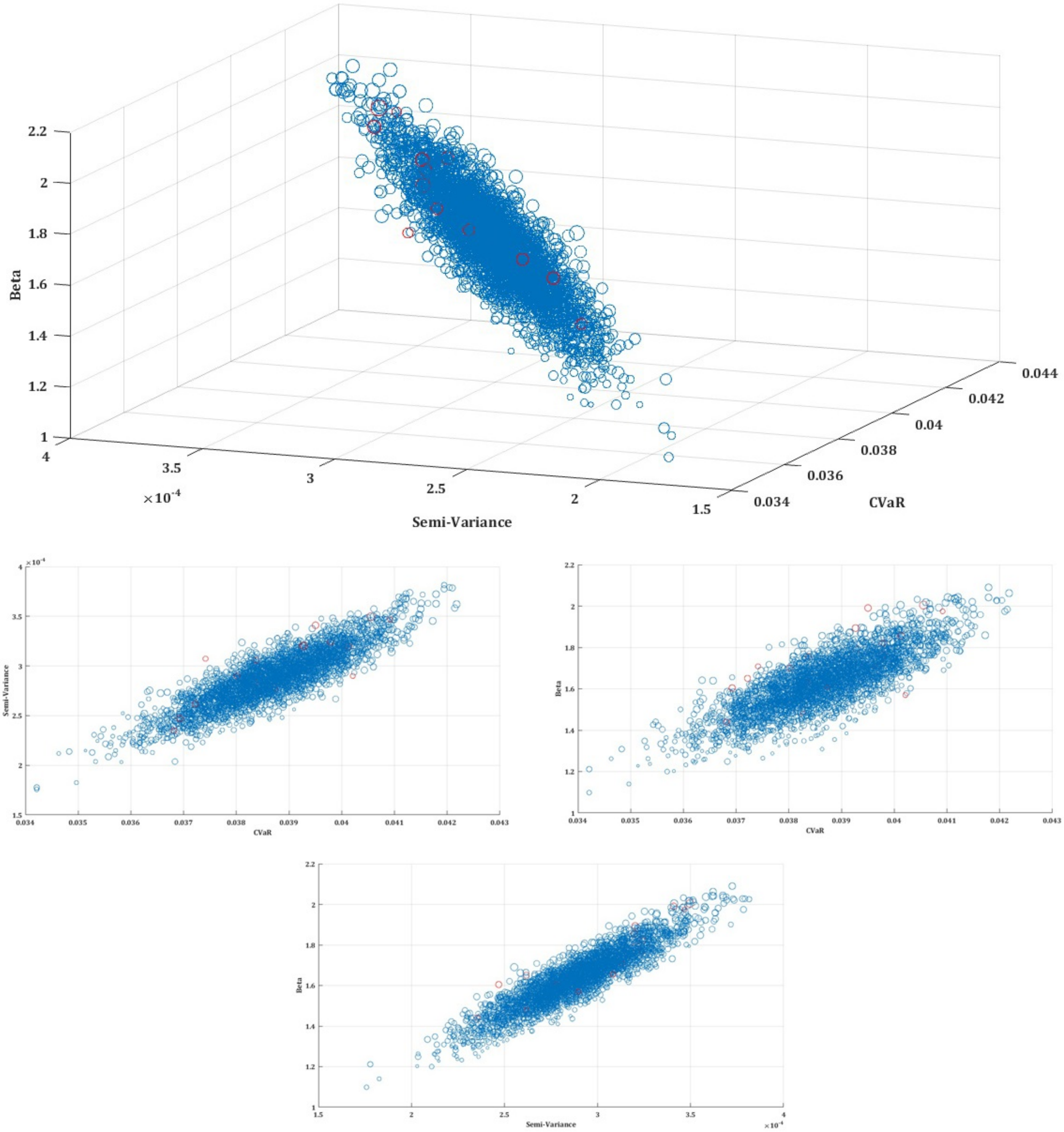

Figure 6: Primary and optimized weighted profiles. Optimized weighted profiles are gathered in the center of primary portfolio cloud.

[4] U. Celikyurt, S. Ozekici, Multi-period portfolio optimization models in stochastic markets using the mean-variance approach, Eur. J. Oper. Res. 1 (2007) 186–202.
[5] J. Claro, J. Pinho de Sousa, A multi objective meta heuristic for a meanrisk multistage capacity investment problem with process flexibility, Comput. Oper. Res. 39 (2012) 838–849.
[6] J. Estrada, Systematic risk in emerging markets: the D-CAPM, Emerging Markets (2002).
[7] D.E. Goldberg, Genetic Algorithms in Search, Optimization, and Machine learning, Addison-Wesley, New York, 1998.
[8] N. Gulpnar, B. Rustem, Worst-case robust decisions for multi-periodmean-variance portfolio optimization, Eur. J. Oper. Res. 183

(2007) 981–1000.
[9] L.J. Hong, G.W. Liu, Simulating sensitivities of conditional value at risk, Management Sci. 55 (2009) 281–293.
[10] D.S. Huang, S.S. Zhu, F.J. Fabozzi, M. Fukushima, Portfolio selection with uncertain exittime: a robust CVaR approach, J. Econ. Dynamics Control 32 (2008) 594–623.
[11] M.M. John, G.E. Hafize, Applying CVaR for decentralized risk management of financial companies, J. Banking Finance 30 (2006) 627–644.
[12] D. Li, W.L. Ng, Optimal dynamic portfolio selection: multi period mean-variance formulation, Math. Finance 10 (2000) 387–406.
[13] H. Markowitz, portfolio selection, J. Finance 91 (1952) 7–77.
[14] H. Markowitz, P. Todd, G. Xu, Y. Yamane, Computation of mean-semivariance efficient sets by the critical line algorithm, Ann. Oper. Res. 45 (1993) 307–317.
[15] A. Najafi, S. Mushkhian, Multi-stage stochastic mean-semivariance-CVaR portfolio optimization under transaction costs, Appl. Math. Comput. 256 (2015) 445–458.
[16] W. Ogryczak, A. Ruszczynski, Dual stochastic dominance and quantile risk measures, Internat. Trans. Oper. Res. 9 (2002) 661–680.
[17] F. Peracchi, A.V. Tanase, On estimating the conditional expected shortfall, Appl. Stoch. Models Business Indus. 24 (2008) 471–493.
[18] G.Ch. Pflug, Some remarks on the value-at-risk and the conditional value-at-risk, In: S. Uryasev (ed.), Probabilistic Constrained Optimization: Methodology and Applications, Dordrecht: Kluwer Academic Publishers (2000).
[19] T. Rockfeller, S. Uryasev, Optimization of conditional value-at-risk, J. Risk 2:3 (2000) 21–24.
[20] T. Rockfeller, S. Uryasev, Conditional value-at-risk for general loss distribution, J. Banking Finance 26 (2002) 1443–1471.
[21] T. Sawik, Selection of a dynamic supply portfolio in make-to-order environment with risks, Comput. Oper. Res. 38 (2011) 782–796.
[22] O. Scaillet, Nonparametric estimation and sensitivity analysis of expected shortfall, Math. Finance 14 (2004) 115–129.
[23] O. Scaillet, Nonparametric estimation of conditional expected shortfall, Insur. Risk Management J. 74 (2005) 639–660.
[24] S.Z. Wei, Z.X. Ye, Multi period optimization portfolio with bankrupty control in stochastic market, Appl. Math. Comput. 186 (2007) 414–425.
[25] S. Yau, R.H. Kwon, J.S. Rogers, D. Wu, Financial and operational decisions in the electricity sector: contract portfolio optimization with the conditional value-at-risk criterion, Internat. J. Prod. Econ. 134 (2011) 67–77.
[26] K. Yu, A. Allay, S. Yang, D.J. Hand, Kernel quantile based estimation of expected shortfall, J. Risk 12:4 (2010) 15–32.
[27] S.S. Zhu, M. Fukushima, Worst-case conditional value-at-risk with application to robust portfolio management, Oper. Res. 57 (2009) 1155–1168.