# Portfolio Optimization By Using MeanSharp-βVaR and Multi Objective MeanSharp-βVaR Models

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**Abstract.**

The purpose of this study is to develop portfolio optimization and assets allocation using our proposed models. For this, three steps are considered. In the first step, the stock companies screen by their financial data. For second step, we need some inputs and outputs for solving Data Envelopment Analysis (DEA) models. Conventional DEA models assume non-negative data for inputs and outputs. However, many of these data take the negative value, therefore we propose the MeanSharp-βVaR (MShβV) model and the Multi Objective MeanSharp-βVaR (MOMShβV) model based on Range Directional Measure (RDM) that can take positive and negative values. Also, we consider one of downside risk measures named Value at Risk (VaR) and try to decrease it. After using our proposed models, the efficient stock companies will be selected for making the portfolio. In the third step, Multi Objective Decision Making (MODM) model was used to specify the capital allocation to the stock companies that was selected for the portfolio. Finally, a numerical example of the purposed method in Iranian stock companies is presented.

## 1. Introduction and Preliminaries

Portfolio selection and portfolio management are the most important problems from the past that has attracted the attention of investors. This means that the investor wants to maximize expected return and minimize uncertainty (risk). Rate of return (or simply return) of the investors wealth from beginning to the end of period is calculated as follows:
Return=

$$\frac{(\text{end of period wealth}) - (\text{beginning of period wealth})}{\text{beginning of period wealth}}, \tag{1}$$

or

Return=

$$\log(\text{end of period wealth}) - \log(\text{beginning of period wealth}). \tag{2}$$

Since Portfolio is a collection of assets, its return ($r_P$) can be calculated in a similar manner. Thus according to Markowitz [10], the investor should consider rates of returns associated with any of these portfolios as, what is called in statistics, a random variable. These variables can be described by mean ($r_P$) and standard deviation ($\sigma_P$), which are calculated as follows:

$$r_P = \sum_{i=1}^{n} \lambda_i \mu_i, \tag{3}$$

$$\sigma_P = [\sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_i \lambda_j \omega_{ij}]^{\frac{1}{2}}, \tag{4}$$

where $n$ is the number of assets in the portfolio, $\mu_i$ is expected return of asset $i$ calculated from equation 1 or 2 and $r_P$ is expected return of portfolio. Also, $\lambda_i$ is proportion of portfolios initial value invested in asset $i$ and $\sigma_P$ is standard deviation of portfolio, and $\omega_{ij}$ shows matrix of covariance of returns between assets $i$ and $j$. So optimal portfolio from a set of portfolios either offering maximum expected return among a varying levels of risk or minimum risk for a varying levels of expected returns . The mean-variance or Markowitz model is described as follow:

$$\begin{aligned}
\text{Min } z = & \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_i \lambda_j \omega_{ij}, \\
\text{s.t. } & \sum_{i=1}^{n} \lambda_i \mu_i \geq r_f, \\
& \sum_{i=1}^{n} \lambda_i = 1, \\
& 0 \leq \lambda_i \leq 1, \qquad i = 1, 2, \ldots, n.
\end{aligned} \tag{5}$$

The objective is finding a portfolio with the minimum risk under the situation that the corresponding expected return must be greater than riskless return rate ($r_f$). Sharpe [14] believed that risk is only depended to the expected return of a company and the expected return of the market. So, Sharpe [15] proposed his model for solving the portfolio selection problem.

Data Envelopment Analysis (DEA) is a nonparametric linear programming technique used to evaluate DMUs efficiency (see [1, 3]). The traditional DEA models cannot be used for the situation that DMUs include both negative and positive data for inputs and/or outputs. Portela et al. [12] expressed Range Directional Measure (RDM) model. In this DEA model input and output data can take positive and negative values. The RDM model is described as follow:

$$\begin{aligned}
\text{Max } & \theta \\
\text{s.t. } & \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io} - \theta R_{io} \quad i = 1, \ldots, m, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} + \theta R_{ro} \quad r = 1, \ldots, s, \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& \lambda_j \geq 0, \qquad\qquad j = 1, \ldots, n,
\end{aligned} \tag{6}$$

Where, directions can be defined as follows:

$$R_{io} = x_{io} - \min_{j} \left\{ x_{ij} : j = 1, \ldots, n \right\}, \qquad i = 1, \ldots, m, \tag{7}$$

$$R_{ro} = \max_{j} \left\{ y_{rj} : j = 1, \ldots, n \right\} - y_{ro}, \qquad r = 1, \ldots, s. \tag{8}$$

Ideal point (I) within the attendance of negative data is:

$$I = \left( \max_j \left\{ y_{rj} \; : r = 1, \dots, s \right\}, \; \min_j \left\{ x_{ij} \; : i = 1, \dots, m \right\} \right).$$ (9)

and the purpose is to project each under evaluation asset's point to this ideal point. Other models that use negative data are modified slacks-based measure model (MSBM), Emrouznejad [6] and semi-oriented radial measure (SORM), Sharp et al. [16].

At first, risk was defined as uncertainty to gain the expected return. Variance is used by Markowitz (1952) in his MV model. Today the definition of risk is more accurate and it is better that a measure of risk be coherent risk measure. After defenition semivariance, Markowitz et al.[11] proposed the mean-semivariance model as alternative of mean-variance model. One of the other risk measures for manage and control risk is Value at Risk (VaR) that proposed by Baumol [2] and is known as quantile in the literature. This risk measure focuses on returns come with high risk. A portfolios VaR is the maximal loss one expects to endure at the confidence level by holding that portfolio over the time horizon. Duffie and Pan [5] used VaR to measure the risk of firms. Silvapulle and Granger [17] estimated VaR by using regular statistics and nonparametric kernel approximation of density function, . Glasserman et al. [9] used the Monte Carlo method along with quadratic estimation to measure the portfolios VaR. Chen and Tang[4] verified other nonparametric approximation of VaR for related financial returns. A nonparametric estimation of dynamic VaR is developed by Jeong and Kang [8] based on the adaptive fluctuations estimation and the nonparametric quantiles estimation. Schaumburg [13] used the nonparametric quantile regression, along with the extreme value theory to predict VaR. There are different methods for computing the VaR, such as Variance-Covariance method, Historical simulation and Monte Carlo simulation. We used Monte Carlo simulation for computing the VaR. In this paper, we use three steps for selecting portfolio. The first step, we screen assets by their financial index. Then, we find more efficient assets for selecting by using Msh$\beta$v model and MoMsh$\beta$v model based on RDM model for negative data that seeks simultaneously to improve mean of return and sharp ratio, and decrease $\beta$ coefficient and VaR. Also, in these models VaR is considered as a risk measure. In the third step, the weigte is allocated to assets that were selected to make portfplio.

The organization of the paper is as follows. Section 2 describes proposed models and three stages for making portfolio. Section 3 presents the empirical discussion of the proposed methodology. Finally, concluding remarks are provided in section 4.

## 2. Modeling and Analyzing Portfolio

The proposed method is including three stages:

1. We screen the stock companies by their financial index.
2. We estimate the relative efficiency of stock companies that passe the first stage with our proposed models, efficient companies will construct the portfolio.
3. We allocate capital to the stock companies in the portfolio by using the MODM model.

### 2.1. Stock screening

The stock price movement has highly associate with current and future expected financial performance. Stock screening by financial indexes is an effective method for determining which companies are most likely to perform effectively in the long time. In this paper we used Return On Equity (ROE), Return On Assets (ROA) and Gross Net Profit to screening the stock companies.

1. ROE measures the rate of return for ownership interest (shareholders' equity) of common stock owners. ROE can be computed as:

$$ROE = \frac{\text{net income}}{\text{average shareholder equity}}.$$ (10)

2. ROA shows the percentage of how profitable a company's assets are in generating revenue. ROA can be computed as:

$$ROA = \frac{\text{net income}}{\text{average total assets}}.$$ (11)

3. One of the company profitability measures is Gross Net Profit. Gross Net Profit can be computed as:

$$\text{Gross Net Profit} = \frac{\text{net income}}{\text{Total sales}}.$$ (12)

## 2.2. Constructing Portfolio

The stock companies screened from the first stage are selected as the candidate for the portfolio. These candidates are the DMUs of DEA model that should be ranked based on their relative efficiencies. Here we proposed our models to measure the efficiency of each DMU. First we need to describe some definitions:

**Definition 2.1.** Assume that a portfolio is going to be selected from $n$ financial assets, $\lambda_i$ is the proportion of invested money in asset $i$. The set of our acceptable portfolios is:

$$\phi = \left\{ \lambda_i \in \mathbb{R}^n; \sum_{i=1}^{n} \lambda_i = 1, \ \lambda_i \geq 0 \right\}.$$ (13)

Return of portfolio $r(\lambda)$ is:

$$r(\lambda) = \sum_{i=1}^{n} \lambda_i r_i.$$ (14)

Expected return of this portfolio is:

$$E(r(\lambda)) = \sum_{i=1}^{n} \lambda_i E(r_i).$$ (15)

**Definition 2.2.** $\beta$-coefficient of an investment indicates whether the investment is more or less volatile than the market. $\beta$-coefficient is:

$$\beta_i = \frac{\text{Cov}(r_i, r_m)}{\text{Var}(r_m)}.$$ (16)

**Definition 2.3.** The Sharpe ratio also known as Reward to Variability Ratio (RVAR) is a way to examine the performance of an investment by adjusting for its risk. Sharpe ratio is:

$$RVAR = \frac{\mu_P - r_f}{\sigma_p}.$$ (17)

**Definition 2.4.** Weakly efficient frontier is described as:

$$\Delta^w(\phi) = \{(\mu, RVAR, \beta, VaR) \in S; \ (-\mu', -RVAR', \beta', VaR') < (-\mu, -RVAR, \beta, VaR) \Rightarrow (\mu', RVAR', \beta', VaR') \notin S\}.$$ (18)

This frontier is a part of the boundary of the disposal region set ($S$). The weakly frontier can contain points that are not reachable by real portfolios.

**Definition 2.5.** Strongly efficient frontier is described as:

$$\Delta^s\left(\phi\right) \;\; = \;\; \{(\mu, RVAR, \beta, VaR) \in S; \; (-\mu', -RVAR', \beta', VaR') \le (-\mu, -RVAR, \beta, VaR)$$
$$\text{and } (-\mu', -RVAR', \beta', VaR') \ne (-\mu, -RVAR, \beta, VaR) \Rightarrow (\mu', RVAR', \beta', VaR') \notin S\}. \quad (19)$$

In Definition 2.4 and 2.5, $\mu$, RVAR, $\beta$ and VaR are expected return (mean), Sharpe ratio, $\beta$-coefficient and Value at Risk (VaR) of a point in disposal region respectively. Similarly, $\mu'$, $RVAR'$, $\beta'$ and $VaR'$ are expected return (mean), Sharpe ratio, $\beta$-coefficient and Value at Risk (VaR) of an optional point in MeanSharp-$\beta$VaR space. As we know, weakly efficient frontier is included in the strongly efficient frontier.

**Proposed Models**

Based on the RDM model provided by Portela et al. (2004), we propose the MeanSharp-$\beta$VaR model (MSh$\beta$V) and the Multi Objective MeanSharp-$\beta$VaR (MOMSh$\beta$V) model. In each direction $g$ is:

$$g = (R_{\mu_o}, R_{RVAR_o}, R_{\beta_o}, R_{VaR_o}) \in [0, +\infty) \times [0, +\infty) \times [0, +\infty) \times [0, +\infty).$$

$$g = \left( \begin{array}{cc} \left(\max_j\left(\mu_j : j = 1, \ldots, n\right) - \mu_o\right) = R_{\mu_o}, & \left(\max_j\left(RVAR_j : j = 1, \ldots, n\right) - RVAR_o\right) = R_{RVAR_o}, \\ \left(\left[\beta_o - \min\left(\beta_j : j = 1, \ldots, n\right)\right]\right) = R_{\beta_o}, & \left(\left[VaR_o - \min\left(VaR_j : j = 1, \ldots, n\right)\right]\right) = R_{VaR_o} \end{array} \right). \quad (20)$$

Consider a vector with specified direction $g = (R_{\mu_o}, R_{RVAR_o}, R_{\beta_o}, R_{VaR_o})$ and an under evaluation asset $DMU_o = (\mu_o, RVAR_o, beta_o, VaR_o)$, the MSh$\beta$V model is description as follow:

$$
\begin{aligned}
\text{Max} \quad & \theta \\
\text{s.t.} \quad & E\left(r\left(\lambda\right)\right) \ge \mu_o + \theta R_{\mu_o}, \\
& RVAR(r\left(\lambda\right)) \ge RVAR_o + \theta R_{RVAR_o}, \\
& \beta(r\left(\lambda\right)) \le \beta_o - \theta R_{\beta_o}, \\
& VaR(r\left(\lambda\right)) \le VaR_o - \theta R_{VaR_o}, \\
& \sum_{i=1}^{n} \lambda_i = 1, \\
& \theta \ge 0, \; 0 \le \lambda_i \le 1, \qquad\qquad i = 1, \ldots, n.
\end{aligned}
\quad (21)
$$

Mechanism of the MSh$\beta$V model is just like the RDM model. The MeanSharp-$\beta$Risk (MSh$\beta$V) model seeks simultaneously to improve mean of return and Sharp ratio and to reduce $\beta$ coefficient and VaR in the direction of the vector g. The use of this model guarantees that a projected MSh$\beta$V point is part of the weakly efficient subset. To ensure that the projection of a MSh$\beta$R point is part of the strongly efficient subset, one should change proportional in all dimension. Therefore, we should introduce another model that project point proportionally.

Consider a vector with specified direction $g = (R_{\mu_o}, R_{RVAR_o}, R_{\beta_o}, R_{VaR_o})$ and an under evaluation asset $DMU_o = (\mu_o, RVAR_o \, \beta_o, VaR_o)$, by using multi objective function for the MSh$\beta$V model, the Multi Objective MeanSharp-$\beta$VaR (MOMS$\beta$V)model is description as follow:

$$
\begin{aligned}
\text{Max} \quad & (\theta_1, \theta_2, \theta_3, \theta_4) \\
\text{s.t.} \quad & E\left(r\left(\lambda\right)\right) \ge \mu_o + \theta_1 R_{\mu_o}, \\
& RVAR(r\left(\lambda\right)) \ge RVAR_o + \theta_2 R_{RVAR_o}, \\
& \beta(r\left(\lambda\right)) \le \beta_o - \theta_3 R_{\beta_o}, \\
& VaR(r\left(\lambda\right)) \le VaR_o - \theta_4 R_{VaR_o}, \\
& \sum_{i=1}^{n} \lambda_i = 1, \\
& \theta_1, \theta_2, \theta_3, \theta_4 \ge 0, \; 0 \le \lambda_i \le 1, \qquad i = 1, \ldots, n.
\end{aligned}
\quad (22)
$$

Multi objective functions are more flexible than single objective functions in determination of optimal directions. Multi objective functions try to maximize the average of objects (because of having more than

one parameter to maximize). Multi objective functions in here try to maximize $\theta$ in directions of mean, Sharpe ratio, $\beta$-coefficient and VaR separately. Mechanism of the MOMSh$\beta$V model is just like the MSh$\beta$V model. When amount of $\theta$ for the DMU equal is to zero, means that the DMU is efficient. In the other words, $1 - \theta$ is amount of the efficiency. But there is a fundamental difference between these models; the DMUs which are efficient in the MSh$\beta$V model locate on weakly efficient frontier but the DMUs which are efficient in the MOMSh$\beta$V model locate on strongly efficient frontier.

### 2.3. Capital Allocation

Huang C.Y et al. [6] used MODM to determine the capital allocation in the portfolio optimization problem. Assume that: A portfolio with maximum return and minimum risk together, named Positive Ideal Solution (PIS). Vice versa, a portfolio with maximum risk and minimum return together, named Negative Ideal Solution (NIS). $W_1$: weight allocated to decision return, $W_2$: weight allocated to decision risk, and $W_1 + W_2 = 1$. The MODM model is described as follow:

$$
\begin{aligned}
\text{Min } z = \quad & W_1\left(\frac{f_1^* - f_1(x)}{f_1^* - f_1^-}\right) + W_2\left(\frac{f_2(x) - f_2^*}{f_2^- - f_2^*}\right) \\
\text{s.t.} \quad & f_1(x) \geq r_f, \\
& \sum_{i=1}^{n} \lambda_i = 1, \\
& 0 \leq \lambda_i \leq 1, \qquad\qquad i = 1, 2, \ldots, n,
\end{aligned}
\tag{23}
$$

where

$$
f_1(x) = \sum_{i=1}^{n} \lambda_i \mu_i
\begin{cases}
f_1^* = \text{Max} \sum_{i=1}^{n} \lambda_i \mu_i, \\
f_1^- = \text{Min} \sum_{i=1}^{n} \lambda_i \mu_i,
\end{cases}
\tag{24}
$$

$$
f_2(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_i \lambda_j r_{R_i R_j} \sigma_{R_i} \sigma_{R_j}
\begin{cases}
f_2^* = \text{Min} \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_i \lambda_j r_{R_i R_j} \sigma_{R_i} \sigma_{R_j}, \\
f_2^- = \text{Max} \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_i \lambda_j r_{R_i R_j} \sigma_{R_i} \sigma_{R_j}.
\end{cases}
\tag{25}
$$

($f^*$ is PIS and $f^-$ is NIS).

The objective function represents the distance of both objectives (return and risk) from PIS, which is searching for closest solution for the PIS. This solution is the best portfolio that investor can select.

## 3. Empirical Discussion

### 3.1. Data Collection

The dataset was randomly collected from the stock's price of the 40 Iranian stock companies, from 25/04/2015 till 25/04/2016. Also missing data over holidays is estimated through interpolation .The dataset was obtained from `http://www.irvex.ir/index`.

### 3.2. Stock Screening

Here we use ROE > 0, ROA > 0 and Gross Net Profit > 0 to screen the stock companies. After screening the 40 Iranian stock companies, just 15 stock companies were selected. All of the stock companies are shown by company symbols, and the selected stock companies symbol is shown in bold text in Table 1.

Table 1: Symbol of stock companies that were used

| company symbols | company symbols | company symbols | company symbols | company symbols | company symbols | company symbols | company symbols |
|---|---|---|---|---|---|---|---|
| INDM1 | SISH1 | BIME1 | TAIR1 | GESF1 | GSHI1 | FNAR1 | BHSM1 |
| **AZAB1** | SHKR1 | **IPAR1** | ZMYD1 | **NAFT1** | FIBR1 | **TRNS1** | MNMH1 |
| SNRO1 | **DJBR1** | COMB1 | **KRTI1** | CRBN1 | **RENA1** | LEAB1 | **PSIR1** |
| SDOR1 | **DSIN1** | GSBE1 | SPDZ1 | **PASH1** | PLAK1 | **TRIR1** | LENT1 |
| **CONT1** | KLBR1 | **KHAZ1** | SIPA1 | SSEP1 | **SHND1** | TKNO1 | **GHAT1** |

Table 2: Inputs and outputs

| Stock companies | Inputs | | | | Outputs | |
|---|---|---|---|---|---|---|
| | $\beta$-coefficient | VaR (Monte Carlo simulation) | | | expected return | Sharpe ratio |
| | | 90 % | 95 % | 99 % | | |
| AZAB1 | 1.3417 | 0.0285 | 0.0352 | 0.0374 | 0.0026 | 0.09020 |
| CONT1 | 0.1089 | 0.0195 | 0.0310 | 0.0348 | 0.0085 | 0.09902 |
| DJBR1 | 0.6384 | 0.0088 | 0.0222 | 0.0267 | 0.0013 | 0.05388 |
| DSIN1 | 0.4935 | 0.0066 | 0.0217 | 0.0267 | 0.0023 | 0.11599 |
| IPAR1 | 0.4902 | 0.0115 | 0.0233 | 0.0273 | 0.0019 | 0.08545 |
| KHAZ1 | 0.8071 | 0.0321 | 0.0418 | 0.0451 | 0.0017 | 0.04872 |
| KRTI1 | 1.4303 | 0.0299 | 0.0557 | 0.0643 | -0.0003 | -0.01750 |
| NAFT1 | 0.9958 | 0.0321 | 0.0402 | 0.0429 | -0.0006 | -0.03976 |
| PASH1 | 0.3434 | 0.0046 | 0.0135 | 0.0165 | 0.0009 | 0.04232 |
| RENA1 | 1.5404 | 0.0320 | 0.0392 | 0.0416 | 0.0030 | 0.05445 |
| SHND1 | -1.3693 | 0.0296 | 0.0723 | 0.0866 | -0.0029 | -0.05442 |
| TRIR1 | -0.1302 | 0.0260 | 0.0686 | 0.0828 | -0.0035 | -0.08533 |
| TRNS1 | 0.6126 | 0.0212 | 0.0302 | 0.0332 | 0.0027 | 0.10848 |
| PSIR1 | 0.7381 | 0.0300 | 0.0442 | 0.0489 | 0.0011 | 0.02411 |
| GHAT1 | 1.0476 | 0.0326 | 0.0683 | 0.0802 | -0.0023 | -0.00002 |

*3.3. Constructing Portfolio*

As mentioned before, since we have negative data such as expected return, Sharpe ratio and $\beta$-coefficient, we have used the MSh$\beta$V model and the MOMSh$\beta$V model to calculate the efficiency of stock companies. The software Matlab was used to calculate value at risk of companies. Also the software GAMS was used to measure the relative efficiency of selected stock companies.

By comparing the results of Table 3, we can derive that results of the MOMSh$\beta$V model is generally better than the results of the MSh$\beta$V model because the multi objective functions are more accurate. Also, in the Monte Carlo simulation the higher confidence levels are more accurate than lower levels.

*3.4. Capital Allocation*

The weights allocated to the objectives of return and risk (return, risk) = $(W_1, W_2)$, rely on investor privilege.

As you see in Figure 1, the obtained weights from Table 4 are on the mean-variance frontier. It means that the weights which obtained by described MODM model in section 2, are the best portfolios. As you see, the MODM model helps the investor to allocate his/her capital, as he/she likes.

## 4. Conclusion

In this paper, we have described a method for portfolio optimization. In the first step, we used ROE, ROA and Gross Net Profit to screen the stock companies. In the second step, because of negative data,

Table 3: Inefficiency of the stock companies

| stock companies | θ with MShβV model | | | θ with MOMShβV model | | |
|---|---|---|---|---|---|---|
| | 90 % | 95 % | 99 % | 90 % | 95 % | 99 % |
| AZAB1 | 0.43 | 0.40 | 0.38 | 0.54 | 0.50 | 0.48 |
| CONT1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| DJBR1 | 0.17 | 0.19 | 0.20 | 0.43 | 0.32 | 0.30 |
| DSIN1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| IPAR1 | 0.13 | 0.09 | 0.09 | 0.44 | 0.31 | 0.28 |
| KHAZ1 | 0.42 | 0.36 | 0.35 | 0.63 | 0.61 | 0.61 |
| KRTI1 | 0.53 | 0.54 | 0.54 | 0.69 | 0.73 | 0.74 |
| NAFT1 | 0.50 | 0.42 | 0.41 | 0.68 | 0.65 | 0.64 |
| PASH1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| RENA1 | 0.52 | 0.48 | 0.47 | 0.67 | 0.63 | 0.62 |
| SHND1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| TRIR1 | 0.32 | 0.37 | 0.38 | 0.51 | 0.60 | 0.61 |
| TRNS1 | 0.13 | 0.13 | 0.13 | 0.47 | 0.38 | 0.35 |
| PSIR1 | 0.43 | 0.38 | 0.37 | 0.63 | 0.64 | 0.64 |
| GHAT1 | 0.51 | 0.52 | 0.53 | 0.68 | 0.73 | 0.74 |

Table 4: Capital allocation for VaR 95%, 99%

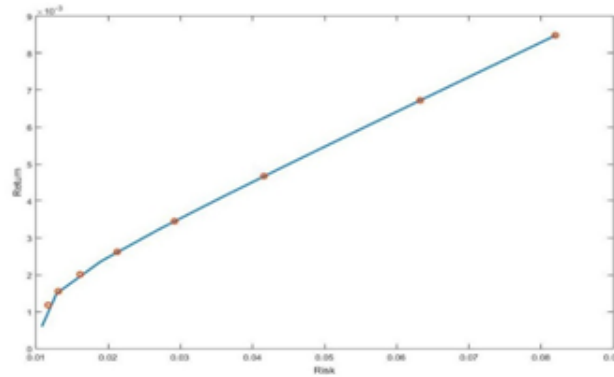| stock companies | (0.1,0.9) | (0.2,0.8) | (0.3,0.7) | (0.4,0.6) | (0.5,0.5) | (0.6,0.4) | (0.7,0.3) | (0.8,0.2) | (0.9,0.1) |
|---|---|---|---|---|---|---|---|---|---|
| CONT1 | 0.05 | 0.09 | 0.15 | 0.23 | 0.34 | 0.50 | 0.77 | 1 | 1 |
| DSIN1 | 0.93 | 0.91 | 0.85 | 0.77 | 0.66 | 0.50 | 0.23 | 0 | 0 |
| PASH1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| SHND1 | 0.02 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |



Figure 1: Obtained weights from Table 4 on the mean-variance frontier

we proposed the MShβV model and the MOMShβV model to calculate the relative efficiency of the stock companies. Multi objective functions are more accurate, so the general results of the MOMShβV model is generally better than the results of the MShβV model. Stock companies which are relatively efficient were selected for the portfolio. We used Monte Carlo simulation with different confidence levels to calculate the VaR. As you see, the results of higher confidence levels are much more accurate. In the third step, by using MODM model investors with different preferences of risk and return, can make their portfolio as they like.

Finally, the proposed method was applied to the 40 Iranian stock companies and the result was shown in the tables. For future studies, other financial parameters can be considered for the stock screening. Also, other risk measures can be considered to measure the risk.

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