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ΣC-ULTRAFILTERS, P-SETS AND HCC-PROPERTY

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Abstract. In this paper we further investigate the results given in [10], [11], [12]. In Section 2 we consider ΣC-filters (ultrafilters). Let X be a σ-compact, dense subspace of a locally compact space Y. The space Y is compact if and only if every ΣC-ultrafilter on X converges to some point in Y. In Section 3 we consider P-sets, ΣC-filters (ultrafilters) and HCC property. A locally compact space X is HCC if and only if every ΣC ultrafilter on X converges. In section 3 we also consider HCC extensions of locally compact spaces.

1. Introduction

The closure of a subset A of a space X is denoted by $cl_X(A)$. In this paper we assume that all spaces are Hausdorff. For notions and definitions not given here see [5], [6], [12].

Let X be a topological space. Then:

$exp(X)$ denotes the space of all nonempty closed subsets of X with finite topology. The finite topology on $exp(X)$ is generated by open collection of the form

$$\langle U_1, \dots, U_n\rangle = \{F \in exp(X) : F \subset \bigcup_{i=1}^{n} U_i \wedge F \cap U_i \neq \varnothing, i \in \{1, \dots, n\}\},$$

where $U_1, \dots, U_n$ are open subsets of X;

$\mathfrak{K}(X)$ denotes the family of all nonempty compact subsets of X;

$\Sigma(X)$ denotes the family of all σ-compact subsets of X;

$\Sigma(x)$ denotes the set of all σ-compact neighbourhoods of $x \in X$;

$\mathfrak{P}_X$ denotes the set of all P-points of X;

$W\mathfrak{P}_X$ denotes the set of all weak P-points of X.

We use the standard definitions for filter base, filter and ultrafilter. An open filter base, filter, ultrafilter is a filter base, filter, ultrafilter consisting exclusively of open sets.

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Definition 1.1. A Hausdorff space X is called *absolutely closed* (or *H-closed*) if X is closed in every Hausdorff space in which is embedded ([6]).

Theorem 1.2. *Let X be a Hausdorff space, then the following are equivalent:*

(a) X is absolutely closed,

(b) Every open filter base on X has a cluster point,

(c) Every open cover of X has a finite dense subsystem (whose union is dense in X),

(d) Every open ultrafilter on X converges ([6]). $\square$

2. ΣC-filters

Definition 2.1. A *ΣC-filter* is a nonempty subfamily $\mathfrak{F} \subset \Sigma(X)$ satisfying the following conditions:

(a) $\varnothing \notin \mathfrak{F}$.

(b) If $A_1,\ A_2 \in \mathfrak{F}$, then $A_1 \cap A_2 \in \mathfrak{F}$.

(c) If $A \in \mathfrak{F}$ and G is σ-compact, $A \subset G$, then $G \in \mathfrak{F}$.

A filter $\mathfrak{U}$ in $\Sigma(X)$ is a maximal filter or a *ΣC-ultrafilter* in $\Sigma(X)$, if for every filter $\mathfrak{F}$ in $\Sigma(X)$ that contains $\mathfrak{U}$ we have $\mathfrak{F} = \mathfrak{U}$.

A filter base in $\Sigma(X)$ is a nonempty family $\mathfrak{G} \subset \Sigma(X)$ such that $\varnothing \notin \mathfrak{G}$ and if $A_1,\ A_2 \in \mathfrak{G}$, then there exists an $A_3 \in \mathfrak{G}$ such that $A_3 \subset A_1 \cap A_2$.

One readily sees that for any filter base $\mathfrak{G}$ in $\Sigma(X)$, the family

$$\mathfrak{F}_{\mathfrak{G}} = \{A \in \Sigma(X) : \textit{there exists a } B \in \mathfrak{G} \textit{ such that } B \subset A\},$$

is a ΣC-filter in $\Sigma(X)$.

Definition 2.2. Let X be a locally compact space.

(a) A point $x \in X$ is called a *limit* of a ΣC-filter $\mathfrak{F}$ if $\Sigma(x) \subset \mathfrak{F}$; we then say that the ΣC-filter $\mathfrak{F}$ converges to x and write $x \in \lim \mathfrak{F}$.

(b) A point x is called a *limit* of a filter base $\mathfrak{G} \subset \Sigma(X)$ if $x \in \lim \mathfrak{F}_{\mathfrak{G}}$; we then say that the filter base $\mathfrak{G}$ converges to x and write $x \in \lim \mathfrak{G}$.

Remark. Clearly, $x \in \lim \mathfrak{G}$ if and only if every compact neighbourhood of x contains a member of $\mathfrak{G}$.

Definition 2.3. Let X be a locally compact space. A point x is called a *cluster point* of a ΣC-filter $\mathfrak{F}$ (of a filter base $\mathfrak{G}$) if x belongs to the closure of every member of $\mathfrak{F}$ (of $\mathfrak{G}$).

Remark. Clearly, x is a cluster point of a ΣC-filter $\mathfrak{F}$ (of a filter base $\mathfrak{G}$) if and only if every compact neighborhood of x intersects all members of $\mathfrak{F}$ (of $\mathfrak{G}$). This implies, in particular, that every cluster point of a ΣC-ultrafilter is a limit of this ultrafilter.

Lemma 2.4. *If $\mathfrak{U}$ is a ΣC-ultrafilter in $\Sigma(X)$, the following holds:*

(a) If $A \in \Sigma(X)$, then $A \cap U \neq \varnothing$ for all $U \in \mathfrak{U}$ iff $A \in \mathfrak{U}$.

(b) If A_1, A_2 are σ-compact subsets of X and $A_1 \cup A_2 \in \mathfrak{U}$, then $A_1 \in \mathfrak{U}$ or $A_2 \in \mathfrak{U}$.

Proof. (a) $\Leftarrow$: If $A \in \mathfrak{U}$, then $A \cap U \neq \varnothing$ for all $U \in \mathfrak{U}$.

$\Rightarrow$: If $A \cap U \neq \varnothing$ for all $U \in \mathfrak{U}$ and $A \notin \mathfrak{U}$, then $\mathfrak{U} \cup \{A\}$ is a filter base in $\Sigma(X)$, that contains $\mathfrak{U}$. Since $\mathfrak{U}$ is a ΣC-ultrafilter in $\Sigma(X)$, it follows that $A \in \mathfrak{U}$.

(b) : Suppose that $A_1 \notin \mathfrak{U}$, $A_2 \notin \mathfrak{U}$ and $A_1 \cup A_2 \in \mathfrak{U}$. Let $\mathfrak{G}$ be a subfamily of $\Sigma(X)$. The set $A \in \Sigma(X)$ is a member of $\mathfrak{G}$ iff $A \cup A_1 \in \mathfrak{U}$. Clearly, $\mathfrak{G}$ is a ΣC-filter that contains $\mathfrak{U}$. Since $\mathfrak{U}$ is a ΣC-ultrafilter in $\Sigma(X)$, it follows that $A_1 \in \mathfrak{U}$ or $A_2 \in \mathfrak{U}$. This completes the proof. $\square$

Lemma 2.5. *Let X be a σ-compact (closed) subset of a topological space Y and let $\mathfrak{F}$ be a ΣC-filter in $\Sigma(Y)$. The family $\mathfrak{F}_X = \mathfrak{F} \cap X = \{F \cap X : F \in \mathfrak{F}\}$ is a ΣC-filter in $\Sigma(X)$ if and only if $F \cap X \neq \varnothing$, for every $F \in \mathfrak{F}$.*

Proof. (a) Empty set $\varnothing \notin \mathfrak{F}_X \iff F \cap X \neq \varnothing$ for all $F \in \mathfrak{F}$. Furthermore, every member of $\mathfrak{F}_X$ is a σ-compact subset of X.

(b) Let sets $A_1 \cap X$ and $A_2 \cap X$ be contained in $\mathfrak{F}_X$. Then $(A_1 \cap X) \cap (A_2 \cap X) = (A_1 \cap A_2) \cap X \in \mathfrak{F}_X$, $(A_1 \cap A_2 \in \mathfrak{F})$.

(c) Also, if $A \cap X \in \mathfrak{F}$ and B is a σ-compact subset in $\Sigma(X)$, $A \subset B$, then $A \cup B \in \Sigma(Y)$ and $A \cup B \in \mathfrak{F}$. We have $B = (A \cup B) \cap X \in \mathfrak{F}_X$.

So, we have shown that $\mathfrak{F}_X$ is a ΣC-filter on X. $\square$

The following is an immediate consequence of Lemmas 2.5. and 2.4.

Lemma 2.6. *Let X be a σ-compact subset of a topological space Y and let $\mathfrak{F}$ be a ΣC-ultrafilter on Y. The family $\mathfrak{F}_X = \mathfrak{F} \cap X = \{F \cap X : F \in \mathfrak{F}\}$ is a ΣC - ultrafilter in $\Sigma(X)$ if and only if $X \in \mathfrak{F}$.* $\square$

Lemma 2.7. *Let X be a σ-compact subspace of a locally compact space Y. If every ΣC-ultrafilter on Y converges, then every ΣC-ultrafilter on X converges to some point in $cl_Y(X)$.*

Proof. Let $\mathfrak{U}$ be a ΣC-ultrafilter on X. Since the subset $X \subset Y$ is σ-compact, it is easy to see that $X \in \mathfrak{U}$. It is clear that family $\mathfrak{U}$ is a ΣC-filter base on Y. Let $\mathfrak{U}'$ be the ΣC-ultrafilter on Y generated by $\mathfrak{U}$. Now suppose $\mathfrak{U}' \to p \in Y$. By Definition 2.3, $p \in \lim \mathfrak{U}' \iff p \in cl_Y(U')$ for each $U' \in \mathfrak{U}'$. Since the family $\mathfrak{U} \subset U'$, the point $p \in cl_Y(U)$ for each $U \in \mathfrak{U}$. Hence $p \in \lim \mathfrak{U}$. This completes the proof. $\square$

Proposition 2.8. *Let X be a σ-compact, dense subspace of a locally compact space Y. The space Y is compact if and only if every ΣC-ultrafilter on X converges to some point in Y.*

Proof. Let Y be a compact space. It is known that every ultrafilter on Y converges; in particular, every ΣC-ultrafilter on Y converges. From Lemma 2.7, it follows that every ΣC-ultrafilter on X converges to some point in Y.

Conversely, suppose that every ΣC-ultrafilter on X converges. We shall prove that every open ultrafilter on Y converges. Since Y is locally compact and Hausdorff it is Tychonoff. By Theorem 1.2, Y is a compact space. If $\mathfrak{U}'$ is an open ultrafilter on Y and $\mathfrak{U} = \mathfrak{U}' \cap X = \{U' \cap X : U' \in \mathfrak{U}'\}$, then, by Lemma 1.5, $\mathfrak{U}$ is an open ultrafilter on X. Clearly the family $\mathfrak{B} = \{cl_Y(U') \cap X : U' \in \mathfrak{U}'\}$ is a filter base in $\Sigma(X)$ (ΣC-filter base on X). Let $\mathfrak{F}$ be the ΣC-ultrafilter on X generated by $\mathfrak{B}$. Now suppose that $\mathfrak{F} \to p \in Y = cl_Y(X)$. From Definition 2.3, it follows that $p \in \lim \mathfrak{F} \Leftrightarrow p \in cl_Y(cl_y(U') \cap X)$ for each $U' \in \mathfrak{U}'$. Therefore, for each $U' \in \mathfrak{U}$, we have that $p \in cl_Y(U') \cap cl_Y(X) = cl_Y(U') \cap Y$. Since, $p \in cl_Y(U')$, for each $U' \in \mathfrak{U}'$, it is clear that $\mathfrak{U}'$ converges to p. $\square$

Corollary 2.9. *Let $\mathfrak{U}$ be a ΣC-ultrafilter on X. If some $U \in \mathfrak{U}$ is a compact subset of X, then $\mathfrak{U}$ is a convergent ultrafilter on X.* $\square$

3. P-sets, weak P-sets and HCC property

Definition 3.1. Let X be a topological space. A point $p \in X$ is said to be a *P-point* if the intersection of countably many neighborhoods of p is a neighborhood of p ([12]).

It can be shown that a point $p \in X$ is a P-point if and only if every F_σ-set that is contained in $X \setminus \{p\}$ has the closure contained in $X \setminus \{p\}$.

Definition 3.2. Let X be a topological space. A set $A \subset X$ is said to be a *P-set* if the intersection of countably many neighborhoods of A is a neighborhood of A.

It is easy to see that every open set of X is a P-set.

The reader can easily prove the following lemma.

Lemma 3.3. *Let X be a topological space. The set $A \subset X$ is a P-set if and only if every F_σ-set that is contained in $X \setminus A$ has the closure contained in $X \setminus A$. If X is a compact space, then the set $A \subset X$ is a P-set if and only if every σ-compact set that is contained in $X \setminus A$ has the compact closure contained in $X \setminus A$.* $\square$

Remark. The set $\mathfrak{P}_X$ ($\mathfrak{P}_X$ denotes the set of all P-points of X) is a P-set. This is a direct consequence of Definition 3.1. and Lemma 3.3. The converse is not necessarily true (see Example 3.4.)

Example 3.4. Let $\mathbb{R}$ be the set of real numbers with the ususal metric topology. Every open interval (a, b) is a P-set, since for every F_σ-set $A \subset (-\infty, a] \cup [b, +\infty)$ the closure $cl_{\mathbb{R}}(A) \subset (-\infty, a] \cup [b, +\infty)$. But $\mathfrak{P}_X = \emptyset$, since $\mathbb{R}$ is second countable.

Lemma 3.5. *Let X be a locally compact space. A compact set $A \subset X$ is a P-set if and only if every σ-compact set that is contained in $X \setminus A$ has the closure contained in $X \setminus A$.*

Proof. $\Rightarrow$: Obvious.

$\Leftarrow$: Let M be an F_σ-set that is contained in $X \setminus A$. Since X is locally compact, for every open set $V \subset X$ that contains A there exists an open set $U \subset X$ such that $A \subset U \subset cl_X(U) \subset V$ and $cl_X(U)$ is compact. The set $M \cap cl_X(U)$ is a σ-compact set that is contained in $X \setminus A$. By assumption the closure $cl_X(M \cap cl_X(U)) \subset X \setminus A$. Furthermore, the closure $cl_X(M) \subset X \setminus A$. According to Lemma 3.3, the set A is a P-set. $\square$

Definition 3.6. A space X is HCC (*hypercountably compact*) if every σ-compact set in X has the compact closure in X ([11]).

Theorem 3.7. *For every Hausdorff locally compact space X the following conditions are equivalent:*

(a) *The space X is HCC.*

(b) *For every compactification cX the remainder $cX \setminus c(X)$ is a P-set in cX.*

(c) *There exists a compactification cX of the space X such that the remainder $cX \setminus c(X)$ is a P-set in cX.*

(d) *Every ΣC-filter base on X has a cluster point.*

(e) *Every ΣC-ultrafilter on X converges.*

Proof. $(a) \Rightarrow (b)$ Since X is locally compact, for every compactification cX the remainder $cX \setminus c(X)$ is a closed (compact) set in cX. If X is an HCC space, then every σ-compact set in $c(X)$ has the compact closure in $c(X)$. By Lemma 3.5., $cX \setminus c(X)$ is a P-set.

$(b) \Rightarrow (c)$ Obvious.

$(c) \Rightarrow (a)$ Since X is locally compact and Hausdorff it is Tychonoff. Therefore, there exists Stone-Čech compactification βX. The remainder $X^* = \beta X \setminus X$ is a compact set in βX. Let $X^* = \beta X \setminus X$ be the P-set in βX. According to Lemma 3.5, every σ-compact set $M \subset \beta X \setminus X^*$ has the closure $cl_{\beta X}(M) \subset \beta X \setminus X^* = X$. Hence, X is HCC.

$(d) \Leftrightarrow (e)$ Obvious.

$(a) \Rightarrow (d)$ Let $\mathfrak{U}$ be a ΣC-ultrafilter on X and U_0 be a member of $\mathfrak{U}$. Since X is CC(countably compact), $cl_X(U_0) = Y$ is a compact subspace of X. Let $\mathfrak{U}'$ be the trace of $\mathfrak{U}$ on Y. By Lemma 2.5, $\mathfrak{U}'$ is a ΣC-filter on Y. Since Y is compact, there exists a point $p \in Y$ such that p is a cluster point of $\mathfrak{U}'$. Clearly, the point p is a cluster point of $\mathfrak{U}$. Hence every ΣC - ultrafilter on X converges.

$(d) \Rightarrow (a)$ Suppose that every ΣC-ultrafilter on X converges and let A be any σ-compact subset of X. The family $\mathfrak{F}_A = \{B \in \Sigma(X) : A \subset B\}$ is a ΣC-filter base on X. Let $\mathfrak{U}$ be the ΣC-ultrafilter on X generated by $\mathfrak{F}_A$ and let $\mathfrak{U}'$ be the trace of $\mathfrak{U}$ on A. By Lemmas 2.6 and 2.7, $\mathfrak{U}'$ is a ΣC-ultrafilter on A and $\mathfrak{U}' \rightarrow p \in cl_X(A)$. According to Proposition 2.8, the set $cl_X(A)$ is a compact subspace of X. Hence, X is an HCC space. $\square$

Definition 3.8. A pair (Y, c), is called an HCC (SCC) *extension* of a space X, if Y is an HCC (SCC) space and $c : X \longrightarrow Y$ is a homeomorphic embedding of X in Y such that $cl_Y(c(X)) = Y$.*

Theorem 3.9. *Let X be a Tychonoff space which is not an HCC space and let cX be a compactification of X with the following properties:*

(a) The set $\mathfrak{P}_{cX}$ is not empty set.

(b) The set $\mathfrak{P}_{cX} \subset cX \setminus X$. Then there exists an HCC extension χX of X such that χX is a subspace of cX.

Proof. Consider the subspace $H(X) = cX \setminus \mathfrak{P}_{cX}$ of cX. Since cX is compact, by Theorem 3.7, the remainder $cX \setminus \mathfrak{P}_{cX}$ is an HCC subspace of cX. Let χX be the closure of $c(X)$ in $H(X)$ $(c(X) \approx X)$. The subspace

$$\chi X = cl_{H(X)}(c(X)) = cl_{cX}(c(X)) \cap H(X) = cX \cap H(X).$$

Hence, $\chi X = H(X)$. Furthermore, the mapping $i : c(X) \longrightarrow H(X)$ defined by $i(y) = y,\ y \in c(X)$, is a homeomorphic embedding of $c(X)$ in $H(X)$. The mapping $i \circ c : X \longrightarrow H(X)$ is a homeomorphic embedding of X in $H(X)$. By Definition 3.8, pair $(H(X), i \circ c)$ is an HCC extension of space X. $\square$

Proposition 3.10. *Let X be a topological space and let A be any closed subset of X. The subset A is a P-set if and only if the point $A \in exp(X)$ is a P-point.*

Proof. Suppose that a closed subset $A \subset X$ is a P-set and let $\{\langle U_1^i, \ldots, U_n^i\rangle : i \in N\}$ be any countable family neighbourhoods of $A \in exp(X)$. For every $i \in N$, the subset $A \subset X$ is contained in $U_i = \cup\{U_k^i : k = 1, \ldots, n\}$. Since A is a P-set, there exists a neighborhood U of A such that for every

*A space X is SCC (*strongly countably compact*) if every countable subset in X has compact closure in X (see [7]).

$i \in N$, $A \subset U \subset U_i$. Furthermore, for every $i \in N$, the neighborhood $\rangle U \langle$ of $A \in exp(X)$ is contained in $\langle U_1^i, \dots, U_n^i \rangle$. Hence $A \in exp(X)$ is a P-point.

Conversely, suppose that $A \in exp(X)$ is a P-point and let $\{U_n : n \in N\}$ be any countable family of neighborhoods of $A \subset X$. The sets $\langle U_n \rangle$, $n \in N$, are neighborhoods of $A \in exp(X)$. Since $A \in exp(X)$ is a P-point, there exists a neighbourhood $\langle V_1, \dots, V_n \rangle$ of $A \in exp(X)$ such that $A \in \langle V_1, \dots, V_n \rangle \subset \langle U_n \rangle$, $n \in N$. The set $A \subset X$ is contained in $V = \cup\{V_k : k = 1, \dots, n\}$ and for every $n \in N$ we have $V \subset U_n$. Hence the set $A \subset X$ is a P-set. $\square$

Theorem 3.11. *Let X be a locally compact HCC space. If X is normal, then there exists an HCC extension of $exp(X)$ and the space $exp(X)$ has the SCC property.*

Proof. It is clear that every HCC spase is a SCC space. By result of J.Keesling (see [7], Theorem 5), we have that $exp(X)$ is SCC. Furthermore, the subspace $\mathfrak{K}(X) \subset exp(X)$ is a dense HCC subspace of $exp(X)$ (see [11], Theorem 2.2). By assumption the space $exp(X)$ has a compactification and the mapping $F : exp(X) \longrightarrow exp(\beta X)$ defined by $F(A) = cl_{\beta X}(A)$, $A \in exp(X)$, is a homeomorphic embedding (see [7]). By Theorem 3.7, the set $\beta X \setminus X$ is a closed P-set in βX and applying Proposition 3.10, the point $\beta X \setminus X \in exp(X)$ is a P-point. It is clear that space $exp(\beta X)$ is compact and by Theorem 3.2 in [11], the subspace $exp(\beta X) \setminus \{\beta X \setminus X\}$ is an HCC space. It is easy to see that the point $\{\beta X \setminus X\}$ is not contained in $F(exp(X))$. Hence $F(exp(X)) \subset exp(\beta X) \setminus \{\beta X \setminus X\}$. Since the property HCC is hereditary with respect to closed sets, the subspace $cl_{exp(\beta X) \setminus \{\beta X \setminus X\}}(F(exp(X)))$ is an HCC space and pair $(cl_{exp(\beta X) \setminus \{\beta X \setminus X\}}(F(exp(X))), F)$ is an HCC extension $exp(X)$. $\square$

Definition 3.12. Let X be a topological space. A set $A \subset X$ is a *weak P-set* if $A \cap cl(F) = \varnothing$ for each countable set F contained in $X \setminus A$.

It is easy to see that every P-set is a weak P-set. Furthermore, if $W\mathfrak{P}_X$ denotes the set of all weak P-points* of X, then the set $W\mathfrak{P}_X$ is a weak P-set. The converse is not necessarily true (see Example 3.4).

The proofs of the following two theorems are similiar to the proofs of Theorem 3.7 and Theorem 3.9 and will be omitted.

Theorem 3.13. *For every Hausdorff locally compact space X the following conditions are equivalent:*

(a) The space X is SCC.

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*A point $p \in X$ is a *weak P-point* if $p \notin cl_X(F)$ for each countable subset $F \subset X \setminus \{p\}$ (see [11]).

(b) *For every compactification cX the remainder $cX \setminus c(X)$ is a weak P-set in cX.*

(c) *There exists a compactification cX of the space X such that the remainder $cX \setminus c(X)$ is a weak P-set in cX.* $\square$

Theorem 3.14. *Let X be a Tychonoff space which is not an HCC space and let cX be a compactification of X with the following properties:*

(a) *The set $W\mathfrak{P}_{cX}$ is not empty set.*

(b) *The set $W\mathfrak{P}_{cX} \subset cX \setminus X$.*

Then there exists a SCC extension σX of X such that σX is a subspace of cX. $\square$

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