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# PAWLEY MULTIPLE ANTISYMMETRY THREE-DIMENSIONAL SPACE GROUPS $G_3^{l,p'}$ II. HEMISYMMORPHIC AND ASYMMORPHIC GROUPS

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**Abstract.** By use of antisymmetric characteristic method, hemisymmorphic and asymmorphic Pawley multiple antisymmetry three-dimensional space groups $G_3^{l,p'}$ $(p=3,4,6)$, are derived.

Crystallographic $(p')$-symmetry three-dimensional space groups (or Pawley colored symmetry groups) $G_3^{p'}$ $(p=3,4,6)$ are derived by A.F.Palistrant [1,2,3,4]. From 54 hemisymmorphic space groups $G_3$ are derived 562 junior $G_3^{p'}$ (75 $G_3^{3'}$ + 252 $G_3^{4'}$ + 235 $G_3^{6'}$). By the use of the generalized antisymmetric characteristic method ($AC$-method) [5,6,7], we will derive all hemisymmorphic crystallographic $(p',2^l)$-symmetry three-dimensional space groups $G_3^{l,p'}$ $(p=3,4,6)$.

## 1. Partial Catalogue of Hemisymmorphic $(p',2^l)$-symmetry Three-dimensional Space Groups $G_3^{l,p'}$ $(p=3,4,6)$

Using the theorethical bacground given in the Part I, we will continue with the derivation of all hemisymmorphic $(p',2^l)$- symmetry three-dimensional space groups. As an example, we are giving the derivation of such groups from the families 1h-5h. The remaining tables of this partial catalogue can be ordered from the author.

$1h$ $(Pb)$ $\{a,b,c\}(b/2m)$, $AC:\{b/2m,b/2mc\}\{b/2ma,b/2mac\}$, $IV$

| | | | |
|---|---|---|---|
| 1) | $\{a,b,c^{(3}\}(b/2m^{')})$, | $(3)(3)^1$, | $N_1=3$ $N_2=6$ |
| 2) | $\{a,b,c^{(4}\}(b/2m^{')})$, | $(4)(4)^2$, | $N_1=3$ |
| 3) | $\{a^{(2},b,c^{(4}\}(b/2m^{')})$, | $(4)(4)^2$, | $N_1=3$ |
| 4) | $\{a,b,c^{(6}\}(b/2m^{')})$, | $(4)(4)^2$, | $N_1=3$ |
| 5) | $\{a^{(2},b,c^{(3}\}(b/2m^{')})$, | $(3)(3)^2$, | $N_1=1$ |

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6) $\{a^{(2}, b, c^{(6}\}(b/2m^{')})$, $(4)(4)^2$, $N_1 = 3$

$2h$ $(Bb)$ $\{a, b, (a+c)/2\}(b/2m)$, $AC : \{b/2m, (a+c)/2b/2m\}$, $XXI$

1) $\{a, b, (a+c)/2^{(3}\}(b/2m)$, $(3)^1$, $N_1 = 1$

$3h$ $(P2/b)$ $\{a, b, c\}(2 : b/2m)$, $AC : \{2, 2b\}\{2c, 2ac\}\{\{b/2m, cb/2m\}, \{ab/2m, acb/2m\}\}$, $XXVI$

1) $\{a, b, c^{(3}\}(2 : b/2m^{')})$, $(3)(3)((3,3))^1$, $N_1 = 6$ $N_2 = 36$ $N_3 = 168$
2) $\{a^{(3}, b, c\}(2^{')} : b/2m)$, $(3)(3)((3,3))^1$, $N_1 = 6$ $N_2 = 36$ $N_3 = 168$
3) $\{a, b, c^{(4}\}(2 : b/2m^{')})$, $(3)(3)((4,4))^2$, $N_1 = 5$ $N_2 = 15$
4) $\{a, b, c^{(4}\}(2^{(2} : b/2m^{')})$, $(3)(3)((4,4))^2$, $N_1 = 5$ $N_2 = 16$
5) $\{a^{(2}, b, c^{(4}\}(2 : b/2m^{')})$, $(3)(3)((4,4))^2$, $N_1 = 5$ $N_2 = 16$
6) $\{a^{(4}, b, c\}(2^{')} : b/2m)$, $(4)(4)(3,3)^2$, $N_1 = 6$ $N_2 = 30$
7) $\{a^{(4}, b, c^{(2}\}(2^{')} : b/2m)$, $(4)(4)(3,3)^2$, $N_1 = 6$ $N_2 = 30$
8) $\{a, b^{(2}, c\}(2^{')} : b/2m^{(4})$, $(3)(3)((3,3))^2$, $N_1 = 4$ $N_2 = 12$
9) $\{a, b^{(2}, c^{(2}\}(2^{')} : b/2m^{(4})$, $(3)(3)((3,3))^2$, $N_1 = 4$ $N_2 = 12$
10) $\{a, b, c^{(3}\}(2^{(2} : b/2m^{')})$, $(3)(3)((3,3))^2$, $N_1 = 4$ $N_2 = 12$
11) $\{a, b, c^{(6}\}(2 : b/2m^{')})$, $(3)(3)((4,4))^2$, $N_1 = 5$ $N_2 = 16$
12) $\{a, b, c^{(6}\}(2^{(2} : b/2m^{')})$, $(3)(3)((4,4))^2$, $N_1 = 5$ $N_2 = 16$
13) $\{a^{(2}, b, c^{(3}\}(2 : b/2m^{')})$, $(4)(4)(3,3)^2$, $N_1 = 7$ $N_2 = 36$
14) $\{a^{(2}, b, c^{(6}\}(2 : b/2m^{')})$, $(4)(4)((4,4))^2$, $N_1 = 7$ $N_2 = 36$
15) $\{a^{(6}, b, c\}(2^{')} : b/2m)$, $(4)(4)(3,3)^2$, $N_1 = 6$ $N_2 = 30$
16) $\{a^{(3}, b, c^{(2}\}(2^{')} : b/2m)$, $(3)(3)((4,4))^2$, $N_1 = 6$ $N_2 = 24$
17) $\{a^{(6}, b, c^{(2}\}(2^{')} : b/2m)$, $(4)(4)((4,4))^2$, $N_1 = 6$ $N_2 = 30$
18) $\{a^{(3}, b, c\}(2^{')} : b/2m^{(2})$, $(3)(3)((3,3))^2$, $N_1 = 4$ $N_2 = 12$

$4h$ $(B2/b)$ $\{a, b, (a+c)/2\}(2 : b/2m)$, $AC : \{2\}\{b/2m, (a+c)/2b/2m\}$, $VI$

1) $\{a, b, (a+c)/2^{(3}\}(2 : b/2m^{')})$, $(2)(3)^1$, $N_1 = 4$ $N_2 = 12$
2) $\{a, b^{(3}, (a+c)/2\}(2^{')} : b/2m^{(-3})$, $(2)(3)^1$, $N_1 = 4$ $N_2 = 12$
3) $\{a, b, (a+c)/2^{(4}\}(2 : b/2m^{')})$, $(2)(4)^2$, $N_1 = 3$
4) $\{a, b, (a+c)/2^{(4}\}(2^{(2} : b/2m^{')})$, $(2)(4)^2$, $N_1 = 3$
5) $\{a^{(2}, b, (a+c)/2^{(4}\}(2^{')} : b/2m)$, $(2)(4)^2$, $N_1 = 4$
6) $\{a, b^{(2}, (a+c)/2\}(2^{')} : b/2m^{(4})$, $(2)(3)^2$, $N_1 = 2$
7) $\{a^{(2}, b, (a+c)/2^{')}\}(2^{''}) : b/2m^{''}))$, $(2)(4)^2$, $N_1 = 4$
8) $\{a, b, (a+c)/2^{(3}\}(2^{(2} : b/2m^{')})$, $(2)(3)^2$, $N_1 = 2$
9) $\{a, b, (a+c)/2^{(6}\}(2 : b/2m^{')})$, $(2)(4)^2$, $N_1 = 3$
10) $\{a, b, (a+c)/2^{(6}\}(2^{(2} : b/2m^{')})$, $(2)(4)^2$, $N_1 = 3$
11) $\{a, b^{(3}, (a+c)/2\}(2^{')} : b/2m^{(6})$, $(2)(3)^2$, $N_1 = 2$
12) $\{a^{(3}, b, (a+c)/2^{(6}\}(2^{')} : b/2m)$, $(2)(4)^2$, $N_1 = 4$

$5h$ $(Pcc2)$ $\{a, b, c\}(2c/2m)$, $AC : \{\{c/2m, c/2ma\}, \{c/2m2, c/2m2b\}\}$, $XXVII$

1) $\{a^{(3}, b, c\}(2^{')}c/2m^{')})$, $(3,3)^1$, $N_1 = 7$ $N_2 = 54$ $N_3 = 336$
2) $\{a^{(4}, b, c\}(2^{')}c/2m^{')})$, $(3,4)^2$, $N_1 = 6$ $N_2 = 30$
3) $\{a^{(4}, b, c\}(2^{')}c/2m^{(2'})$, $(3,4)^2$, $N_1 = 6$ $N_2 = 30$
4) $\{a^{(4}, b^{(2}, c\}(2^{')}c/2m^{')})$, $(3,4)^2$, $N_1 = 6$ $N_2 = 30$

5) $\{a^{(3}, b, c\}(2^{'})c/2m^{(2'}),$ $(3,3)^2,$ $N_1 = 5$ $N_2 = 24$
6) $\{a^{(6}, b, c\}(2^{'})c/2m^{'}),$ $(3,4)^2,$ $N_1 = 6$ $N_2 = 30$
7) $\{a^{(3}, b^{(2}, c\}(2^{'})c/2m^{'}),$ $(3,4)^2,$ $N_1 = 8$ $N_2 = 48$
8) $\{a^{(6}, b^{(2}, c\}(2^{'})c/2m^{'}),$ $(4,4)^2,$ $N_1 = 9$ $N_2 = 60$
9) $\{a^{(6}, b, c\}(2^{'})c/2m^{(2'}),$ $(3,4)^2,$ $N_1 = 6$ $N_2 = 30$

The numerical results of the derivation are given in Table 1.

Table 1.

| | (3') | (4') | (6') |
|---|---|---|---|
| $1h$ | 1 | 2 | 3 |
| $2h$ | 1 | 2 | 1 |
| $3h$ | 2 | 7 | 9 |
| $4h$ | 2 | 5 | 5 |
| $5h$ | 1 | 3 | 5 |
| $6h$ | 2 | 7 | 12 |
| $7h$ | 2 | 7 | 5 |
| $8h$ | 1 | 2 | 2 |
| $9h$ | 1 | 2 | 3 |
| $10h$ | 1 | 4 | 3 |
| $11h$ | 2 | 9 | 11 |
| $12h$ | 2 | 9 | 6 |
| $13h$ | 2 | 7 | 5 |
| $14h$ | 2 | 3 | 6 |
| $15h$ | 1 | 3 | 3 |
| $16h$ | 1 | 1 | 1 |
| $17h$ | 2 | 10 | 16 |
| $18h$ | 2 | 8 | 10 |
| $19h$ | 1 | 3 | 3 |
| $20h$ | 2 | 11 | 12 |
| $21h$ | 2 | 14 | 22 |
| $22h$ | 2 | 12 | 12 |
| $23h$ | 2 | 9 | 12 |
| $24h$ | 1 | 2 | 2 |
| $25h$ | | 4 | |
| $26h$ | | 4 | |
| $27h$ | | 2 | |
| $28h$ | | 5 | |
| $29h$ | 1 | 7 | 3 |
| $30h$ | 1 | 6 | 3 |
| $31h$ | 1 | 6 | 2 |
| $32h$ | 1 | 6 | 3 |
| $33h$ | 1 | 4 | 2 |
| $34h$ | 1 | 6 | 3 |
| $35h$ | 1 | 11 | 5 |
| $36h$ | 1 | 16 | 7 |
| $37h$ | 1 | 10 | 3 |
| $38h$ | 1 | 16 | 7 |
| $39h$ | 2 | | |
| $40h$ | 1 | | |
| $41h$ | 1 | | |
| $42h$ | 2 | 1 | 2 |
| $43h$ | 3 | 1 | 3 |
| $44h$ | 2 | | 2 |
| $45h$ | 3 | 1 | 3 |
| $46h$ | 4 | 1 | 4 |
| $47h$ | 2 | 1 | 2 |
| $48h$ | 3 | 2 | 9 |
| $51h$ | 1 | | |
| $52h$ | 1 | | |
| $53h$ | 1 | | 2 |
| $54h$ | 1 | | 1 |

| | $(3',2)$ | $(4',2)$ | $(6',2)$ | $(3',2^2)$ | $(4',2^2)$ | $(6',2^2)$ |
|---|---|---|---|---|---|---|
| $1h$ | 3 | 6 | 7 | 6 | | |
| $2h$ | 1 | | | | | |
| $3h$ | 12 | 35 | 50 | 72 | 132 | 212 |
| $4h$ | 8 | 16 | 14 | 24 | | |
| $5h$ | 7 | 18 | 34 | 54 | 90 | 192 |
| $6h$ | 20 | 60 | 102 | 192 | 384 | 648 |
| $7h$ | 8 | 18 | 13 | 24 | | |
| $8h$ | 3 | 2 | 4 | 6 | | |
| $9h$ | 6 | 8 | 12 | 24 | | |
| $10h$ | 6 | 12 | 12 | 24 | | |
| $11h$ | 20 | 76 | 94 | 192 | 480 | 600 |
| $12h$ | 12 | 36 | 24 | 48 | | |
| $13h$ | 12 | 28 | 20 | 48 | | |
| $14h$ | 12 | 12 | 24 | 48 | | |
| $15h$ | 6 | 10 | 12 | 24 | | |
| $16h$ | 2 | | | | | |
| $17h$ | 26 | 122 | 242 | 456 | 1872 | 4074 |
| $18h$ | 17 | 53 | 78 | 150 | 288 | 498 |
| $19h$ | 5 | 9 | 15 | 24 | 18 | 58 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 20*h* | 24 | 104 | 126 | 264 | 720 | 936 |
| 21*h* | 36 | 256 | 444 | 768 | 5088 | 8784 |
| 22*h* | 22 | 120 | 124 | 252 | 960 | 940 |
| 23*h* | 24 | 88 | 126 | 264 | 624 | 948 |
| 24*h* | 4 | 4 | 5 | 12 | | |
| 25*h* | | 12 | | | | |
| 26*h* | | 16 | | | | |
| 28*h* | | 20 | | | | |
| 29*h* | 4 | 17 | 8 | 12 | | |
| 30*h* | 4 | 16 | 8 | 12 | | |
| 31*h* | 4 | 16 | 5 | 12 | | |
| 32*h* | 4 | 15 | 8 | 12 | | |
| 33*h* | 3 | 4 | 4 | 6 | | |
| 34*h* | 4 | 18 | 8 | 12 | | |
| 35*h* | 10 | 104 | 48 | 96 | 336 | 840 |
| 36*h* | 10 | 134 | 60 | 96 | 840 | 384 |
| 37*h* | 6 | 40 | 12 | 24 | | |
| 38*h* | 14 | 192 | 84 | 168 | 1536 | 672 |
| 42*h* | 4 | | | | | |
| 43*h* | 6 | | | | | |
| 44*h* | 4 | | | | | |
| 45*h* | 6 | | | | | |
| 46*h* | 8 | | | | | |
| 47*h* | 4 | | | | | |
| 48*h* | 18 | 8 | 36 | 72 | | |
| 53*h* | 2 | | | | | |
| 54*h* | 2 | | | | | |

| | $(3',2^3)$ | $(4',2^3)$ | $(6',2^3)$ | $(3',2^4)$ |
|---|---|---|---|---|
| 3*h* | 336 | | | |
| 5*h* | 336 | | | |
| 6*h* | 1344 | | | |
| 11*h* | 1344 | | | |
| 17*h* | 8568 | 23520 | 53760 | 120960 |
| 18*h* | 1008 | | | |
| 19*h* | 84 | | | |
| 20*h* | 2016 | | | |
| 21*h* | 16128 | 64512 | 126336 | 241920 |
| 22*h* | 2016 | | | |
| 23*h* | 2016 | | | |
| 35*h* | 672 | | | |
| 36*h* | 672 | | | |
| 38*h* | 1344 | | | |

For the complete $(p',2^l)$-symmetry junior hemisymmorphic three-dimensional space groups of the $M^m$-type the numbers $N_m^{p'}$ ($p = 3,4,6$) are the following:

$$N_0^{p'} = 75G_3^{3'} + 252G_3^{4'} + 235G_3^{6'} = 562$$
$$N_1^{p'} = 413G3^{1,3'} + 1705G_3^{1,4'} + 1863G_3^{1,6'} = 3981$$

$$N_2^{p'} = 3498G_3^{2,3'} + 13368G_3^{2,4'} + 19786G_3^{2,6'} = 36652$$
$$N_3^{p'} = 37884G_3^{3,3'} + 88032G_3^{3,4'} + 180096G_3^{3,6'} = 306012$$
$$N_4^{p'} = 362880G_3^{4,3'} = 362880.$$

## 2. Partial Catalogue of Asymmorphic $(p', 2^l)$-symmetry Three-dimensional Space Groups $G_3^{l,p'}$ $(p = 3, 4, 6)$

Crystallographic $(p')$-symmetry three-dimensional space groups (or Pawley colored symmetry groups) $G_3^{p'}$ $(p = 3, 4, 6)$ are derived by A.F.Palistrant [1,2,3,4]. From 103 asymmorphic space groups $G_3$ are derived 980 junior $G_3^{p'}$ (138 $G_3^{3'}$ + 432 $G_3^{4'}$ + 410 $G_3^{6'}$). By the use of the generalized antisymmetric characteristic method ($AC$-method) [5,6,7], we will derive all asymmorphic crystallographic $(p', 2^l)$-symmetry three-dimensional space groups $G_3^{l,p'}$ $(p = 3, 4, 6)$.

According to the theorethical bacground given in the Part I, we will continue with the derivation of all asymmorphic $(p', 2^l)$- symmetry three-dimensional space groups. We are giving the sample of their derivation in the families 1a-3a. The remaining catalogues can be obtained from the author.

$1a$ $(P21)$ $\{a, b, c\}(c/22)$, $AC : \{c/22, c/22a, c/22b, c/22ab\}$, $XXXI$

1) $\{a^{(3}, b, c\}(c/22^{')})$, $(5)^1$, $N_1 = 1$ $N_2 = 1$
2) $\{a^{(4}, b, c\}(c/22^{')})$, $(9)^2$, $N_1 = 1$
3) $\{a(6, b, c\}(c/22^{')})$, $(9)^2$, $N_1 = 1$

$2a$ $(P21/m)$ $\{a, b, c\}(c/22 : m)$, $\{m\}\{c/22, c/22a, c/22b, c/22ab\}$, $III$

1) $\{a, b, c^{(3}\}(c/22^{(-3} : m^{')})$, $(2)(5)^1$, $N_1 = 4$ $N_2 = 16$ $N_3 = 56$
2) $\{a^{(3}, b, c\}(c/22^{')} : m)$, $(2)(5)^1$, $N_1 = 4$ $N_2 = 16$ $N_3 = 56$
3) $\{a^{(4}, b, c\}(c/22^{')} : m)$, $(2)(9)^2$, $N_1 = 5$ $N_2 = 18$
4) $\{a^{(4}, b, c\}(c/22^{')} : m^{(2)})$, $(2)(9)^2$, $N_1 = 5$ $N_2 = 18$
5) $\{a, b, c^{(2}\}(c/22^{(4} : m^{')})$, $(2)(5)^2$, $N_1 = 2$ $N_2 = 4$
6) $\{a^{(2}, b, c^{(2}\}(c/22^{(4} : m^{')})$, $(2)(5)^2$, $N_1 = 2$ $N_2 = 4$
7) $\{a^{(3}, b, c\}(c/22^{')} : m^{(2)})$, $(2)(5)^2$, $N_1 = 2$ $N_2 = 4$
8) $\{a^{(6}, b, c\}(c/22^{')} : m)$, $(2)(9)^2$, $N_1 = 5$ $N_2 = 18$
9) $\{a^{(6}, b, c\}(c/22^{')} : m^{(2)})$, $(2)(9)^2$, $N_1 = 5$ $N_2 = 18$
10) $\{a, b, c^{(3}\}(c/22^{(6} : m^{')})$, $(2)(5)^2$, $N_1 = 2$ $N_2 = 4$
11) $\{a^{(2}, b, c^{(3}\}(c/22^{(6} : m^{')})$, $(2)(9)^2$, $N_1 = 6$ $N_2 = 24$

$3a$ $(P21/b)$ $\{a, b, c\}(c/22 : b/2m)$, $AC : \{c/22, c/22a\}\{c/22b/2m, c/22b/2ma\}$, $IV$

1) $\{a, b, c^{(3}\}(c/22^{(-3} : b/2m^{')})$, $(3)(3)^1$, $N_1 = 3$ $N_2 = 6$
2) $\{a^{(3}, b, c\}(c/22^{')} : b/2m)$, $(3)(3)^1$, $N_1 = 3$ $N_2 = 6$
3) $\{a^{(4}, b, c\}(c/22^{')} : b/2m)$, $(4)(4)^2$, $N_1 = 3$

4) $\{a, b^{(2}, c\}(c/22^{')} : b/2m^{(4)})$, $(3)(3)^2$, $N_1 = 1$
5) $\{a, b, c^{(2}\}(c/22^{(4} : b/2m^{'}))$, $(3)(3)^2$, $N_1 = 1$
6) $\{a^{(2}, b, c^{(2}\}(c/22^{(4} : b/2m^{'}))$, $(3)(3)^2$, $N_1 = 1$
7) $\{a^{(6}, b, c\}(c/22^{')} : b/2m)$, $(4)(4)^2$, $N_1 = 3$
8) $\{a^{(3}, b, c\}(c/22^{')} : b/2m^{(2)})$, $(3)(3)^2$, $N_1 = 1$
9) $\{a, b, c^{(3}\}(c/22^{(6} : b/2m^{'}))$, $(3)(3)^2$, $N_1 = 1$
10) $\{a^{(2}, b, c^{(3}\}(c/22^{(6} : b/2m^{'}))$, $(4)(4)^2$, $N_1 = 3$

The complete results are given in Table 2.

Table 2.

| | (3') | (4') | (6') | | (3') | (4') | (6') | | (3') | (4') | (6') |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1a | 1 | 1 | 1 | 34a | | 2 | | 67a | 1 | 10 | 3 |
| 2a | 2 | 4 | 5 | 35a | | 2 | | 70a | 1 | 1 | 1 |
| 3a | 2 | 4 | 4 | 36a | | 7 | | 71a | 1 | 1 | 1 |
| 4a | 2 | 6 | 8 | 37a | | 6 | | 72a | 2 | 1 | 2 |
| 5a | 2 | 5 | 6 | 38a | | 4 | | 73a | 2 | 1 | 2 |
| 6a | 1 | 2 | 3 | 39a | | 3 | | 74a | 1 | | |
| 7a | 2 | 4 | 5 | 40a | 1 | 2 | 3 | 75a | 1 | | |
| 8a | 1 | 1 | 1 | 41a | 1 | 7 | 2 | 76a | 1 | | |
| 9a | 2 | 6 | 10 | 42a | 1 | 4 | 3 | 77a | 1 | | |
| 10a | 2 | 6 | 5 | 43a | 1 | 8 | 3 | 78a | 1 | | |
| 11a | 2 | 4 | 5 | 44a | 1 | 4 | 2 | 79a | 2 | | 2 |
| 12a | 2 | 3 | 2 | 45a | 1 | 4 | 2 | 80a | 2 | | 2 |
| 13a | 2 | 5 | 6 | 46a | 1 | 5 | 3 | 81a | 3 | 1 | 3 |
| 14a | 3 | 18 | 30 | 47a | 1 | 8 | 5 | 82a | 2 | 1 | 2 |
| 15a | 3 | 13 | 17 | 48a | 1 | 2 | 1 | 83a | 2 | 1 | 2 |
| 16a | 3 | 10 | 16 | 49a | 1 | 2 | 1 | 84a | 2 | 2 | 4 |
| 17a | 3 | 9 | 9 | 50a | 1 | 4 | 3 | 85a | 2 | 2 | 4 |
| 18a | 3 | 14 | 21 | 51a | 1 | 2 | 1 | 86a | 5 | 1 | 5 |
| 19a | 3 | 11 | 21 | 52a | 1 | 6 | 3 | 87a | 3 | 2 | 9 |
| 20a | 2 | 7 | 14 | 53a | 1 | 3 | 1 | 88a | 3 | 2 | 9 |
| 21a | 1 | 2 | 7 | 54a | 1 | 10 | 7 | 93a | 1 | | |
| 22a | 2 | 6 | 10 | 55a | 1 | 14 | 7 | 94a | 1 | | |
| 23a | 3 | 11 | 17 | 56a | 1 | 7 | 3 | 95a | 1 | | |
| 24a | 2 | 9 | 10 | 57a | 1 | 8 | 3 | 96a | 1 | | 1 |
| 25a | 2 | 6 | 5 | 58a | 1 | 6 | 3 | 97a | 1 | | |
| 26a | 3 | 7 | 9 | 59a | 1 | 6 | 3 | 98a | 1 | | 1 |
| 27a | 2 | 5 | 5 | 60a | 1 | 15 | 5 | 99a | 1 | | 2 |
| 28a | 3 | 7 | 9 | 61a | 1 | 14 | 5 | 100a | 1 | | 1 |
| 29a | 1 | 2 | 3 | 62a | 1 | 10 | 5 | 101a | 1 | | 1 |
| 30a | | 1 | | 63a | 1 | 10 | 3 | 102a | 1 | | 1 |
| 31a | | 1 | | 64a | 1 | 10 | 3 | 103a | 1 | | 2 |
| 32a | | 2 | | 65a | 1 | 10 | 3 | | | | |
| 33a | | 2 | | 66a | 1 | 7 | 3 | | | | |

| | $(3',2)$ | $(4',2)$ | $(6',2)$ | $(3',2^2)$ | $(4',2^2)$ | $(6',2^2)$ |
|---|---|---|---|---|---|---|
| 1a | 1 | 1 | 1 | 1 | | |
| 2a | 8 | 14 | 20 | 32 | 44 | 68 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 3*a* | 6 | 6 | 8 | 12 | | |
| 4*a* | 11 | 39 | 49 | 81 | 216 | 246 |
| 5*a* | 10 | 16 | 20 | 36 | | |
| 6*a* | 4 | 5 | 8 | 12 | | |
| 7*a* | 9 | 10 | 15 | 30 | | |
| 8*a* | 1 | | | | | |
| 9*a* | 14 | 36 | 64 | 108 | 180 | 354 |
| 10*a* | 8 | 14 | 13 | 24 | | |
| 11*a* | 8 | 10 | 13 | 24 | | |
| 12*a* | 4 | | | | | |
| 13*a* | 12 | 20 | 24 | 48 | | |
| 14*a* | 48 | 324 | 507 | 900 | 5904 | 8964 |
| 15*a* | 30 | 108 | 150 | 288 | 672 | 984 |
| 16*a* | 30 | 84 | 141 | 288 | 528 | 924 |
| 17*a* | 18 | 36 | 36 | 72 | | |
| 18*a* | 42 | 168 | 252 | 504 | 1344 | 2016 |
| 19*a* | 42 | 132 | 252 | 504 | 1056 | 2016 |
| 20*a* | 22 | 66 | 128 | 252 | 456 | 912 |
| 21*a* | 10 | 24 | 84 | 96 | 192 | 672 |
| 22*a* | 17 | 42 | 78 | 150 | 234 | 480 |
| 23*a* | 30 | 92 | 146 | 288 | 576 | 936 |
| 24*a* | 17 | 60 | 82 | 150 | 330 | 528 |
| 25*a* | 10 | 18 | 17 | 36 | | |
| 26*a* | 18 | 28 | 36 | 72 | | |
| 27*a* | 10 | 14 | 17 | 36 | | |
| 28*a* | 18 | 28 | 36 | 72 | | |
| 29*a* | 6 | 8 | 12 | 24 | | |
| 33*a* | | 8 | | | | |
| 36*a* | | 24 | | | | |
| 37*a* | | 16 | | | | |
| 38*a* | | 8 | | | | |
| 40*a* | 6 | 8 | 12 | 24 | | |
| 41*a* | 4 | 20 | 6 | 12 | | |
| 42*a* | 3 | 4 | 7 | 6 | | |
| 43*a* | 4 | 20 | 8 | 12 | | |
| 44*a* | 4 | 12 | 6 | 12 | | |
| 45*a* | 4 | 12 | 6 | 12 | | |
| 46*a* | 6 | 20 | 12 | 24 | | |
| 47*a* | 7 | 52 | 32 | 54 | 288 | 174 |
| 48*a* | 2 | | | | | |
| 49*a* | 2 | | | | | |
| 50*a* | 4 | 8 | 8 | 12 | | |
| 51*a* | 2 | | | | | |
| 52*a* | 4 | 14 | 11 | 12 | | |
| 53*a* | 2 | | | | | |
| 54*a* | 10 | 84 | 60 | 96 | 828 | 384 |
| 55*a* | 10 | 116 | 60 | 96 | 720 | 384 |
| 56*a* | 6 | 28 | 12 | 24 | | |
| 57*a* | 6 | 32 | 12 | 24 | | |

| | | | | | | |
|---|---|---|---|---|---|---|
| 58$a$ | 6 | 24 | 12 | 24 | | |
| 59$a$ | 6 | 24 | 12 | 24 | | |
| 60$a$ | 10 | 144 | 48 | 96 | 1536 | 336 |
| 61$a$ | 10 | 128 | 48 | 96 | 864 | 336 |
| 62$a$ | 8 | 60 | 36 | 60 | 240 | 192 |
| 63$a$ | 6 | 40 | 12 | 24 | | |
| 64$a$ | 6 | 40 | 12 | 24 | | |
| 65$a$ | 6 | 40 | 12 | 24 | | |
| 66$a$ | 6 | 28 | 12 | 24 | | |
| 67$a$ | 6 | 40 | 12 | 24 | | |
| 70$a$ | 1 | | | | | |
| 71$a$ | 1 | | | | | |
| 72$a$ | 2 | | | | | |
| 73$a$ | 2 | | | | | |
| 79$a$ | 4 | | | | | |
| 80$a$ | 4 | | | | | |
| 81$a$ | 6 | | | | | |
| 82$a$ | 4 | | | | | |
| 83$a$ | 4 | | | | | |
| 84$a$ | 8 | 6 | 10 | 24 | | |
| 85$a$ | 8 | 6 | 10 | 24 | | |
| 86$a$ | 10 | | | | | |
| 87$a$ | 18 | 8 | 36 | 72 | | |
| 88$a$ | 18 | 8 | 36 | 72 | | |
| 96$a$ | 2 | | | | | |
| 98$a$ | 1 | | | | | |
| 99$a$ | 2 | | | | | |
| 100$a$ | 2 | | | | | |
| 101$a$ | 2 | | | | | |
| 102$a$ | 2 | | | | | |
| 103$a$ | 4 | | 12 | 12 | | |

| | $(3',2^3)$ | $(4',2^3)$ | $(6',2^3)$ | $(3',2^4)$ |
|---|---|---|---|---|
| 2$a$ | 112 | | | |
| 4$a$ | 504 | | | |
| 9$a$ | 672 | | | |
| 14$a$ | 17136 | 80640 | 120960 | 241920 |
| 15$a$ | 2016 | | | |
| 16$a$ | 2016 | | | |
| 18$a$ | 4032 | | | |
| 19$a$ | 4032 | | | |
| 20$a$ | 2016 | | | |
| 21$a$ | 672 | | | |
| 22$a$ | 1008 | | | |
| 23$a$ | 2016 | | | |
| 24$a$ | 1008 | | | |
| 47$a$ | 336 | | | |
| 54$a$ | 672 | | | |
| 55$a$ | 672 | | | |
| 60$a$ | 672 | | | |

| | |
|---|---|
| 61*a* | 672 |
| 62*a* | 336 |

For the complete $(p', 2^l)$-symmetry junior asymmorphic three-dimensional space groups of the $M^m$-type the numbers $N_m^{p'}$ $(p = 3, 4, 6)$ are the following:

$$\begin{aligned}
N_0^{p'} &= 138G_3^{3'} + 432G_3^{4'} + 410G_3^{6'} = 980 \\
N_1^{p'} &= 725G_3^{1,3'} + 2485G_3^{1,4'} + 2781G_3^{1,6'} = 5991 \\
N_2^{p'} &= 5184G_3^{2,3'} + 16208G_3^{2,4'} + 20906G_3^{2,6'} = 42298 \\
N_3^{p'} &= 40600G_3^{3,3'} + 80640G_3^{3,4'} + 120960G_3^{3,6'} = 242200 \\
N_4^{p'} &= 241920G_3^{4,3'} = 241920.
\end{aligned}$$

## 3. $(p', 2^l)$-symmetry Three-dimensional Space Groups $G_3^{l,p'}$ $(p = 3, 4, 6)$

As the final result, for the complete $(p', 2^l)$-symmetry junior three-dimensional space groups of the $M^m$-type the numbers $N_m^{p'}$ $(p = 3, 4, 6)$ are the following:

$$\begin{aligned}
N_0^{p'} &= 309G_3^{3'} + 950G_3^{4'} + 953G_3^{6'} = 2212 \\
N_1^{p'} &= 1634G_3^{1,3'} + 6361G_3^{1,4'} + 7288G_3^{1,6'} = 15283 \\
N_2^{p'} &= 13391G_3^{2,3'} + 53664G_3^{2,4'} + 78825G_3^{2,6'} = 145880 \\
N_3^{p'} &= 150197G_3^{3,3'} + 441924G_3^{3,4'} + 967568G_3^{3,6'} = 1559689 \\
N_4^{p'} &= 1888320G_3^{4,3'} + 2056320G_3^{4,4'} + 10321920G_3^{4,6'} = 14266560 \\
N_5^{p'} &= 19998720G_3^{5,3'} = 19998720.
\end{aligned}$$

## References

The list of references is given in the first part of this paper, in FILOMAT (Niš) 9:1 (1995), 9–20.

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