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THE NATURAL PARTIAL ORDER ON THE ABEL-GRASSMANN'S GROUPOIDS

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Abstract. A left almost semigroup, or LA-semigroup, is a groupoid S satisfying the left invertive law

$$(ab)c = (cb)a, \tag{1}$$

for every $a, b, c \in S$, [8].

Condition (1) is in fact a left Abel-Grassmann's law, [4], and notion LA-semi- group reminds of associativity. Since concerned structure is not associative in order to avoid confusion we shall use notion Abel-Grassmann's groupoids or AG-groupoids.

In this paper we define relation which is a natural partial order relation on AG-band, AG^* and AG^{**}-groupoids. Also we introduce the notion of r-cancelative π-inverse AG^*-groupoid and on this structure we consider the natural partial order and maximal elements.

1. Introduction

On an AG-groupoid holds *medial* law

$$(ab)(cd) = (ac)(bd) \tag{2}$$

for every $a, b, c, d \in S$. It has been shown in [8] that

$$(ab)c = b(ca), \tag{3}$$

$$(ab)c = b(ac) \tag{4}$$

are equivalent on an AG-groupoid S for every $a, b, c \in S$. The AG-groupoid S on which holds statement (3) or (4) we denote by AG^*-*groupoid.* The set $E(S)$ of all idempotents of an AG^*-groupoid S is a commutative semigroup, i.e. $E(S)$ is a semillatice, [9].

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Let on AG-groupoid S is true

$$a(bc) = b(ac) \tag{5}$$

for every $a, b, c \in S$. Then this class of AG-groupoids we shall call an AG^{**}-groupoids. If S has left identity then S is an AG^{**}-groupoid, [10].

By N we denote the set of all positive integers.

Definition 1.1. [3] Let S be an AG-groupoid, then $a \in S$ is an m-associative element ($m \in N$) if all words length $p \leq m$ ($p \in N$) which consists of element a has the same value and those words we denote with a^p. □

Now, if $a \in S$ is an m-associative element, then for every $n, p, q \in N$,- $n \leq m$ and $p + q = n$ we have $a^n = a^p a^q$.

An AG-groupoid S is called an *inverse AG-groupoid* if for each $a \in S$ there exists $a' \in S$ such that $(aa')a = a$, $(a'a)a' = a'$ and a' is an *inverse* for a, [9]. As usually we shall denote by $V(a)$ the set of all inverses of $a \in S$. If $a, b \in S$, $a' \in V(a)$, $b' \in V(b)$, then $a'b' \in V(ab)$, [9].

Definition 1.2. [3] An AG-groupoid S is *π-inverse AG-groupoid* if for every $a \in S$ there exists $m \in N$ such that a is an m-associative element and there exists $p \in N$ ($p \leq m$) and $a' \in S$ such that $(a^p a')a^p = a^p, (a'a^p)a' = a'$. □

Then a is a π-regular element of S, a^p is regular element of S and the set of all regular elements of S we denote with $RegS$.

Let S be a π-inverse AG-groupoid, then we define a mapping $r : S \longrightarrow RegS$ with $r(a) = a^n$ where n is the smallest positive integer such that $a^n \in RegS$, [3].

If S is an π-inverse AG^*-groupoid then $RegS$ is an inverse AG^*-groupoid and by Theorem 2.1., [3], it follows that S is a commutative inverse semigroup. For $a \in RegS$ by a^{-1} we denote an inverse element of a (which is unique).

For some results about π-inverse and k-inverse semigroups see [1], [2], [5] and [6].

For undefined notions and notations we refer to [1],[2] and [7].

2. The natural partial order

In this paragraph we shall give the natural partial order on the set of idempotents of the AG-groupoid. We shall also modify this relation to be a natural partial order on AG^* and AG^{**}-groupoids.

Definition 2.1. Let S be an AG-groupoid and $a^2 = a$ for every $a \in S$, then S is an AG-band.

If S is an AG-groupoid and $E(S) \neq \varnothing$, then from $e, f \in E(S)$ by medial law we have $(ef)^2 = (ef)(ef) = (ee)(ff) = ef$ and so $E(S)$ is an AG-band.

Lemma 2.1. *Let S be an AG-groupoid, $E(S) \neq \varnothing$ and $e, f \in E(S)$. Then*

$$e = ef \iff e = fe \ .$$

Proof. According to (1) we have

(i) $$fe = (ff)e = (ef)f$$

and dually

(ii) $$ef = (ff)e \ .$$

Now, if $ef = e$ then (i) implies $fe = ef$ i.e. $fe = e$. If $fe = e$, then (ii) implies $ef = e$. □

A groupoid S is a left (right) distributive if for every $a, b, c \in S$ holds $a(bc) = (ab)(ad)$ ($(ab)c = (ac)(bc)$).

Let S be a groupoid in which holds medial law and $a^2 = a$ for each $a \in S$. Then for $a, b, c \in S$ it follows that

$$a(bc) = (aa)(bc) = (ab)(ac), \quad (ab)c = (ab)(cc) = (ac)(bc) \ ,$$

and so S is a left and right distributive groupoid. Hence, AG-band is a left and right distributive groupoid.

Theorem 2.1. *Let S be a left and right distributive groupoid in whish $a^2 = a$ for each $a \in S$. Then the relation $\leq$ defined on S by*

$$e \leq f \iff e = ef = fe$$

is a (natural) partial order relation and $\leq$ is compatible.

Proof. Clearly, $e \leq e$ and relation $\leq$ is reflexive. Also, from $e \leq f$ and $f \leq e$ it follows that $e = f$ and relation $\leq$ is antisymmetric. Let $e \leq f \iff e = ef = fe$, $f \leq g \iff f = fg = gf$. Then

$$eg = (ef)g = (eg)(fg) = (eg)f = (ef)(gf) = ef = e \ ,$$
$$ge = g(fe) = (gf)(ge) = f(ge) = (fg)(fe) = fe = e \ .$$

Hence, $e \leq g$ and relation $\leq$ is transitive and so $\leq$ is a partial order on S.

From $e \leq f$ and $g \in S$ we have

$$eg = (ef)g = (eg)(fg) \ , \qquad eg = (fe)g = (fg)(eg)$$

and so $eg \leq fg$. Also,

$$ge = g(ef) = (ge)(gf) \ , \qquad ge = g(fe) = (gf)(ge)$$

and so $ge \leq gf$. Hence, $\leq$ is compatible with operation on S. □

Corollary 2.1. *Let S be an AG-band then the relation $\leq$ defined with*

$$e \leq f \iff e = ef$$

is a (natural) partial order relation and $\leq$ is compatible.

Proof. By Lemma 2.1. and Theorem 2.1. □

Lemma 2.2. *AG^*-groupoid S satisfies all permutation identities*

$$(6) \qquad (x_1x_2)(x_3x_4) = (x_{p(1)}x_{p(2)})(x_{p(3)}x_{p(4)})$$

where $\{p(1), p(2), p(3), p(4)\}$ means any permutation of set $\{1, 2, 3, 4\}$.

Proof. Let x_1, x_2, x_3, x_4 be an arbitrary elements from S. Then we have

$$\begin{aligned}(x_1x_2)(x_3x_4) &= (x_4(x_1x_2))x_3 = ((x_2x_4)x_1)x_3 = (x_3x_1)(x_2x_4)\\ &= (x_3x_1)(x_4x_2) = (x_3x_4)(x_1x_2).\end{aligned}$$

From this it holds that S^2 is a commutative AG^*-groupoid, so it is a commutative semigroup. Now by (3) and (4) we conclude that (6) holds. □

Lemma 2.3. *Let S be an AG^*-groupoid, $E(S) \neq \varnothing$, then for every $a \in S$ and every $e \in E(S)$ it holds $ea = ae$.*

Proof. Let $a \in S$, $e \in E(S)$, then by (6) we have

$$ea = (ee)a = (ae)e = (ae)(ee) = (ea)(ee) = (ea)e = a(ee) = ae. \quad \square$$

If S is an AG-groupoid, then by $S^1 = S \cup \{1\}$ we denote the groupoid in which 1 is identity (analogously as in semigroup theory). Clearly, S^1 is not an AG-groupoid.

Let S be the AG^*-groupoid, we can define relation $\leq$ as follows:

$$(7) \qquad (\forall a, b \in S) \quad a \leq b \quad \iff \quad (\exists e \in E(S^1)) \quad a = eb.$$

Theorem 2.2. *Relation $\leq$ defined on the AG^*-groupoid S with* (7) *is a natural partial order relation and it is compatible.*

Proof. Reflexivity is obvious since for any element $a \in S$ it holds $a = 1a$.

For antisymmetry let us suppose that $a \leq b$ and $b \leq a$, then there exist elements $e, f \in E(S^1)$ such that $a = eb$ and $b = fa$. If $e = 1$ or $f = 1$ antisymmetry follows directly. If $e, f \in E(S)$ then from (3) and (4) it holds:

$$\begin{aligned}a &= eb = e(fa) = (fe)a = (fe)(eb) = (fe)(be)\\ &= (fb)(ee) = (fb)e = (eb)f = af.\end{aligned}$$

Now by Lemma 2.3. it follows that $a = af = fa = b$ so we have proved antisymmetry.

Now let us suppose that $a \leq b$ and $b \leq c$, then exist elements $e, f \in E(S^1)$ such that $a = eb$ and $b = fc$. If $e = 1$ or $f = 1$ transitivity follows directly. Let $e, f \in E(S)$ then $a = e(fc) = (fe)c$. Since fe belongs to $E(S)$ it follows $a \leq c$ and transitivity holds.

Let $a \leq b$ and $c \in S$ then $a = eb$ for some $e \in E(S^1)$. If $e = 1$ then compatibility is true. If $e \in E(S)$ then

$$ac = (eb)c = (cb)e = (cb)(ee) = (ee)(bc) = e(bc),$$
$$ca = c(eb) = (ec)b = (bc)e = (bc)(ee) = (ee)(cb) = e(cb).$$

Hence, $ac \leq bc$ and $ca \leq cb$. □

Let S be an AG^{**}-groupoid and $a, b, c, d \in S$, then

$$(ab)(cd) = c((ab)d) = c((db)a) = (db)(ca). \tag{8}$$

If we consider relation $\leq$ on an AG^{**}-groupoid then we obtain the next theorem.

Theorem 2.3. *Relation $\leq$ defined as above is a natural partial order relation on an AG^{**}-groupoid S and it is compatible.*

Proof. Reflexivity is obvious since for any element $a \in S$ it holds $a = 1a$.

For antisymmetry let us suppose that $a \leq b$ and $b \leq a$, then exist elements $e, f \in E(S^1)$ such that $a = eb$ and $b = fa$. If $e = 1$ or $f = 1$ antisymmetry follows directly. Let $e, f \in E(S)$ then by (5) it holds:

$$a = eb = e(fa) \overset{(5)}{=} f(ea) = (ff)(ea) \overset{(1)}{=} ((ea)f)f$$
$$\overset{(1)}{=} ((fa)e)f = (be)f$$

and

$$b = fa = f((be)f) \overset{(5)}{=} (be)(ff) = (be)f = a\ .$$

So we have proved antisymmetry.

Now let us suppose that $a \leq b$ and $b \leq c$, then exist elements $e, f \in E(S^1)$ such that $a = eb$ and $b = fc$. If $e = 1$ or $f = 1$ transitivity follows directly. If $e, f \in E(S)$ then

$$a = eb = e(fc) = (ee)(fc) \overset{(8)}{=} (ce)(fe) \overset{(1)}{=} ((fe)e)c \overset{(1)}{=} ((ee)f)c = (ef)c\ .$$

Since ef belongs to $E(S^1)$ it follows $a \leq c$ and transitivity holds.

Let $a \leq b$ and $c \in S$, then there exists $e \in E(S^1)$ such that $a = eb$. If $e = 1$ then compatibility is true. If $e \in E(S)$ then by (8) we have

$$ac = (eb)c = (cb)e = (cb)(ee) = (eb)(ec) = (ee)(bc) = e(bc),$$
$$ca = c(eb) = e(cb)$$

and so $ac \leq bc$ and $ca \leq cb$. □

Since AG-groupoid with left identity is an AG^{**}-groupoid we obtain next corollary.

Corollary 2.3. *Let S be an AG-groupoid with left identity. The relation $\leq$ defined on S by*

$$a \leq b \iff (\exists e \in E(S))\ a = eb$$

is a natural partial order relation on S and it is compatible. □

3. The natural partial order on the π-inverse AG^*-groupoids

Let S be an AG^*-groupoid, then for each $a \in S$ the set $L(a) = a \cup Sa$ is a minimal left ideal of S containing a, [12].

Now, on AG^*-groupoid S for $a, b \in S$ we define the relation $\mathcal{L}$ by

$$a\mathcal{L}b \iff a \cup Sa = b \cup Sb.$$

Then $\mathcal{L}$ is an equivalence relation and by L_a we denote an equivalence class for $a \in S$. A relation $\preccurlyeq$ defined on $\mathcal{L}$-classes by

$$L_a \preccurlyeq L_b \iff a \cup Sa \subseteq b \cup Sb$$

is, clearly, a partial order on $S/\mathcal{L}$.

Lemma 3.1. *Let S be a π-inverse AG^*-groupoid, $a, b \in S$ and $a\mathcal{L}b$, then from $a \in RegS$ it follows that $b \in RegS$.*

Proof. From $a\mathcal{L}b$ and $a \neq b$ it follows that there exist $u, v \in S$ such that $b = ua$, $a = vb$. Since $a \in RegS$, then there exists $x \in RegS$ such that $(ax)a = a$ and $(xa)x = x$. Since $RegS$ is a commutative inverse semigroup we have $ax = xa$. Now

$$\begin{aligned} b = ua = u((ax)a) &\overset{(3)}{=} u(x(aa)) \overset{(4)}{=} (xu)(aa) \overset{(2)}{=} (xa)(ua) \\ &= (ax)(ua) = (ax)b \overset{(4)}{=} x(ab) \overset{(3)}{=} x(ba) \overset{(4)}{=} (bx)a \\ &= (bx)(vb) \overset{(6)}{=} (vx)(bb) \overset{(4)}{=} (b(vx))b, \end{aligned}$$

$$\begin{aligned} ((vx)b)(vx) &\overset{(4)}{=} (x(vb))(vx) = (xa)(vx) \overset{(2)}{=} (xv)(ax) = (xv)(xa) \\ &\overset{(3)}{=} v((xa)x) = vx. \end{aligned}$$

Hence, b and vx are mutually inverse and so $b \in RegS$. □

Let S be a π-inverse AG-groupoid, then we can define the relation $\tilde{\mathcal{L}}$ with

$$a\tilde{\mathcal{L}}b \iff Sr(a) = Sr(b)$$

where $a, b \in S$. Clearly, $r(a) \in Sr(a)$ and $\tilde{\mathcal{L}}$ is an equivalence relation. Since r(a)=r(r(a)) we have that $a\tilde{\mathcal{L}}r(a)$. On $\tilde{\mathcal{L}}$-classes we define the relation $\preccurlyeq$ with

$$\tilde{L}_a \preccurlyeq \tilde{L}_b \iff Sr(a) \subseteq Sr(b)$$

for $a, b \in S$. This relation is obviously partial order relation on $S/_{\tilde{\mathcal{L}}}$.

If S is a π-inverse AG^*-groupoid, then $\mathcal{L}\mid_{RegS} \equiv \tilde{\mathcal{L}}\mid_{RegS}$ and it is well known Green's relation for commutative inverse semigroup $RegS$.

Definition 3.1. An AG-groupoid S is an r-cancelative π-inverse AG^*-groupoid if for every $a, b \in S - RegS$ it holds

$$r(a) = r(b) \Longrightarrow a = b \quad \square$$

Hence, on r-cancelative π-inverse AG^*-groupoid S we have $|S - RegS| \leq |RegS|$.

Example 3.1. Let S be a AG-groupoid defined by the following Cayley table:

	1	2	3	4
1	2	2	2	2
2	2	2	2	2
3	2	2	3	3
4	1	1	3	3

Then $RegS = E(S) = \{2,3\}$, $r(1) = 1^2 = 2$, $r(4) = 4^2 = 3$ and S is an r-cancelative AG-groupoid.

Theorem 3.2. *Let S be a r-cancelative π -inverse AG^*-groupoid. For $a, b \in S$ we define*

$$(10) \qquad a \leq b \iff L_a \preccurlyeq L_b \wedge r(a) = r(a)r(b)^{-1}r(a) = r(a)r(b)^{-1}r(b).$$

Then the relation $\leq$ is a (natural) partial order relation on S.

Proof. Since $r(a) = r(a)r(a)^{-1}r(a)$ we have $a \leq a$. Let us suppose that $a \leq b$ and $b \leq a$, then $L_a = L_b$, $r(a) = r(a)r(b)^{-1}r(a) = r(a)r(b)^{-1}r(b)$ and

$r(b) = r(b)r(a)^{-1}r(b) = r(b)r(a)^{-1}r(a)$. Now, using the fact that $RegS$ is a commutative semigroup, it holds

$$\begin{aligned} r(a) &= r(a)r(b)^{-1}r(a) = r(a)r(b)^{-1}r(b)r(a)^{-1}r(a) \\ &= r(a)r(b)^{-1}r(a)r(a)^{-1}r(b) = r(a)r(a)^{-1}r(b) = r(b). \end{aligned} \tag{11}$$

Now, if $a, b \in RegS$, then $r(a) = a$, $r(b) = b$ and by (10) holds $a = b$. If $a, b \in S - RegS$, then since S is r-cancelative from $r(a) = r(b)$ it follows that $a = b$. By Lemma 2.2. the case that $a \in RegS$ and $b \in S - RegS$ ($a \in RegS$, $b \in S - RegS$) is impossible. Hence, a relation $\leq$ is antisymmetric.

Let $a \leq b$, $b \leq c$, then $L_a \preccurlyeq L_c$, $r(a) = r(a)r(b)^{-1}r(a) = r(a)r(b)^{-1}r(b)$ and $r(b) = r(b)r(c)^{-1}r(b) = r(b)r(c)^{-1}r(c)$. Now we have

$$\begin{aligned} r(a) &= r(a)r(b)^{-1}r(a) = r(a)r(b)^{-1}r(a)r(b)^{-1}r(b) \\ &= r(a)r(b)^{-1}r(b)r(a)r(b)^{-1} = r(a)r(b)^{-1}r(b)r(c)^{-1}r(b)r(a)r(b)^{-1} \\ &= r(a)r(c)^{-1}r(a)r(b)^{-1}r(b) = r(a)r(c)^{-1}r(a), \\ r(a) &= r(a)r(b)^{-1}r(b) = r(a)r(b)^{-1}r(b)r(c)^{-1}r(c) \\ &= r(a)r(c)^{-1}r(c) \end{aligned}$$

and so by (9) we have $a \leq c$. Hence, $\leq$ is transitive and so $\leq$ is a partial order relation on S. □

By the following theorem we introduce some equivalent definitions for the natural partial order on the r-cancelative π-inverse AG^*-groupoid.

Theorem 3.3. *The following statements are equivalent on the r-cancelative π-inverse AG^*-groupoid S:*

$$\begin{aligned} &(i) && a \leq b, \\ &(ii) && L_a \preccurlyeq L_b \wedge \tilde{L}_a \preccurlyeq \tilde{L}_b \wedge (\exists e \in \tilde{L}_a \cap E(S))\ r(a) = er(b), \\ &(iii) && L_a \preccurlyeq L_b \wedge r(a) = r(a)r(a)^{-1}r(b), \\ &(iv) && L_a \preccurlyeq L_b \wedge (\exists e \in E(S))\ r(a) = er(b). \end{aligned}$$

Proof. (i)$\Longrightarrow$ (ii) Let $a, b \in S$, $a \leq b$ defined with (10) and $e = r(a)r(b)^{-1}$, then $e^2 = r(a)r(b)^{-1}r(a)r(b)^{-1} = r(a)r(b)^{-1} = e$, i.e. $e \in E(S)$. Also, $r(a) = er(a)$. Since $RegS$ is a commutative semigroup we have

$$Sr(a) = Ser(a) \subseteq Se = Sr(a)r(b)^{-1} \subseteq Sr(a).$$

Hence $\tilde{L}_e = \tilde{L}_a$ and $e \in E(L_a) = \tilde{L}_a \cap E(S)$. From $r(a) = r(a)r(b)r(b)^{-1}$ it follows that $Sr(a) \subseteq Sr(b)$ so $\tilde{L}_a \preccurlyeq \tilde{L}_b$.

(ii)$\Longrightarrow$ (iii) Since restriction of $\tilde{\mathcal{L}}$ on $RegS$ is a known Green's relation on inverse semigroup $RegS$, then by Proposition 3.6.[6] from $e \in \tilde{L}_a \cap E(S)$ we have $e = r(a)r(a)^{-1}$. Hence, $r(a) = r(a)r(a)^{-1}r(b)$

(iii)$\Longrightarrow$ (iv) Is clear since $e = r(a)r(a)^{-1} \in E(S)$.

(iv)$\Longrightarrow$ (i) Let (iv) holds, then by commutativity of $RegS$ we have

$$\begin{aligned} r(a)r(b)^{-1}r(a) &= er(b)r(b)^{-1}er(b) = er(b)r(b)^{-1}r(b)e \\ &= er(b)e = er(a) = r(a), \\ r(a)r(b)^{-1}r(b) &= er(b)r(b)^{-1}r(b) = er(b) = r(a) \end{aligned}$$

so (i) holds. □

We shall now describe the maximal elements on a π-inverse AG^*-groupoid.

Definition 3.2. The element a of a r-cancelative π-inverse AG^*-groupoid S is *maximal* if it is maximal with respect to the natural partial order $\leq$ on S. □

Let S be a π-inverse AG-groupoid and let

$$A = \{x \in S \mid (\exists y \in S - RegS)\ x = r(y)\},\ B = RegS - A,$$

then the following lemma is true.

Lemma 3.2. *Let S be the r-cancelative π-inverse AG^*-groupoid, then for all $a \in S$ it holds that $r(a) \leq a$.*

Proof. If $r(a) = a^n$ then

$$\begin{aligned} Sr(a) &= Sa^n = S(a^{n-1}a) = \bigcup_{x \in S} x(a^{n-1}a) = \bigcup_{x \in S} (a^{n-1}x)a \subseteq Sa, \\ r(a) &= (r(a)r(a)^{-1})r(a) = (r(a)r(a)^{-1})(a^{n-1}a) = \\ &\quad (a^{n-1}(r(a)r(a)^{-1})a) \in Sa \end{aligned}$$

and so $r(a) \cup Sr(a) \subseteq Sa \subseteq a \cup Sa$, whence $L_{r(a)} \preccurlyeq L_a$. Also, $r(a) = er(a)$ where $e = r(a)r(a)^{-1} \in E(S)$. By Theorem 3.1. we have $r(a) \leq a$. □

Definition 3.3. The element a of a π-inverse AG-groupoid S is *strongly π-inverse* if

$$(\forall x \in S)((r(a) = (r(a)x)r(a) \Longleftrightarrow x = (xr(a))x). \quad \square$$

In this case $x \in RegS$.

Theorem 3.3. *Let S be an r-cancelative π-inverse AG^*-groupoid, then every strongly π-inverse element from $S - A$ is maximal.*

Proof. Suppose that $a \in S$ is a strongly π-inverse element and $b \in S$ such that $a \leq b$. Then by Theorem 3.1. we have $r(a) = r(a)r(b)^{-1}r(a) = r(a)r(b)^{-1}r(b)$ and since a is strongly π-inverse we have
$r(b)^{-1} = r(b)^{-1}r(a)r(b)^{-1}$. Now

$$r(b) = r(b)r(b)^{-1}r(b) = r(b)r(b)^{-1}r(a)r(b)^{-1}r(b) = r(b)r(b)^{-1}r(a) = r(a).$$

Let $a \in S - RegS$. If $b \in S - RegS$ then since S is r-cancelative from $r(a) = r(b)$ it follows that $a = b$. If $b \in RegS$ then $r(b) = b = r(a)$ and by Lemma 3.2 we have $b \leq a$. Now, $a \leq b$ and $b \leq a$ gives $a = b$ what is impossible.

Suppose that $a \in RegS$. If $b \in RegS$ then $r(a) = a = b = r(b)$ and a is maximal element. If $b \in S - RegS$ then $r(a) = a = r(b)$, which is impossible, since $a \in B$.

It is obvious that $a \in A$ is not maximal element because there exists $x \in S - RegS$ such that $a = r(x) \leq x$. □

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