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Ljubiša Kočinac

A TOPOLOGICAL ORDINAL INVARIANT

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Abstract. We define a new topological ordinal invariant for a given almost radial space.

Let τ be an infinite cardinal number. A τ-sequence $(x_\alpha : \alpha \in \tau)$ in a topological space X is a function from τ into X. If X is a topological space and $A \subset X$ we write

(i) LimA for the set of all $x \in X$ such that there is a τ-sequence in A converging to x [3], and

(ii) t-LimA for the set of all $x \in X$ such that there are a regular cardinal number τ and a τ-sequence $(x_\alpha : \alpha \in \tau)$ in A with the following properties:

(a) $x_\alpha \neq x_\beta$ for $\alpha \neq \beta$;

(b) $(x_\alpha : \alpha \in \tau)$ converges to x;

(c) $x \notin \overline{\{x_\alpha : \alpha < \beta\}}$ for every $\beta < \tau$ [2].

A space X is called pseudo-radial [3] (almost radial [1]) if every subset A of X for which $\mathrm{Lim}A \subset A$ ($\text{t-Lim}A \subset A$) holds is closed in X.

In [4], the notion of radial order ir(X) of a pseudo-radial space X was introduced and proved that $ir(X) \leq rt(X)^+$, where rt (often denoted by σ_c) denotes the radial tightness [1]. Here in a similar way we define the almost radial order of a given almost radial space. We suppose that all spaces are Hausdorff.

DEFINITION. Let X be an almost radial space and let A be a subset of X. We put

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$A_0 = A$;
$A_\alpha = \bigcup\{A_\beta : \beta<\alpha\}$, if α is a limit ordinal number;
$A_{\alpha+1} = \text{t-Lim}A_\alpha$
and define
$\text{iar}(A) = \min\{\alpha : A_\alpha = \overline{A}\}$,
$\text{iar}(X) = \sup\{\text{iar}(A) : A\subset X\}$.
We call iar(X) the <u>almost radial order</u> of X.

THEOREM. A space X is almost radial if and only if iar(X) exists; then $\text{iar}(X)\leq t(X)^+$, where $t(X)$ is the tightness of X and $t(X)^+$ is the successor of $t(X)$.

PROOF. We prove only the last (non-trivial) part of the theorem. Let X be an almost radial space and let $t(X)^+ = \lambda$. Suppose, to the contrary, that $\text{iar}(X)>\lambda$. This means that there exists a subset A of X such that the set $A_{\lambda+1}\setminus A_\lambda$ is non-empty. Choose a point $x\in A_{\lambda+1}\setminus A_\lambda$. There exists a subset B of A_λ such that x belongs to t-LimB and $|B|\leq t(X)$, since for any almost radial space the tightness is equal to the radial tightness [1;Th.2.9]. Every $y\in B$ belongs to some A_μ with $\mu<\lambda$. Since $\lambda= t(X)^+$ is a regular cardinal number and $|B|<\lambda$, there is $\eta<\lambda$ such that $B\subset A_\eta$. This implies $x\in \text{t-Lim}B\subset \text{t-Lim}A_\eta = A_{\eta+1}\subset A_\lambda$. But this contradicts the fact $x\notin A_\lambda$ and the theorem is proved.

If X is an almost radial space then clearly $\text{iar}(X)\geq \text{ir}(X)$.

QUESTION. Is there an almost radial space X with $\text{iar}(X)>\text{ir}(X)$?

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Ljubiša Kočinac

JEDNA TOPOLOŠKA ORDINALNA INVARIJANTA

Definisana je nova ordinalna invarijanta - indeks skoro-radijalnosti - u klasi skoro radijalnih topoloških prostora. Data je njegova ocena pomoću tesnoće prostora.

Filozofski fakultet
18000 Niš
Jugoslavija