

The mean-Value at Risk static portfolio optimization using genetic algorithm

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Abstract. In this paper we solve the problem of static portfolio allocation based on historical Value at Risk (VaR) by using genetic algorithm (GA). VaR is a predominantly used measure of risk of extreme quantiles in modern finance. For estimation of historical static portfolio VaR, calculation of time series of portfolio returns is required. To avoid daily recalculations of proportion of capital invested in portfolio assets, we introduce a novel set of weight parameters based on proportion of shares. Optimal portfolio allocation in the VaR context is computationally very complex since VaR is not a coherent risk metric while number of local optima increases exponentially with the number of securities. We presented two different single-objective and a multiobjective technique for generating mean-VaR efficient frontiers. Results document good risk/reward characteristics of solution portfolios while there is a trade-off between the ability to control diversity of solutions and computation time.

Keywords: Genetic algorithm, Static portfolio optimization, Value at Risk, Mean-VaR efficient frontier.

1. Introduction

Market risk, as defined by Alexander [2], is a risk resulting from adverse moves in prices of liquid financial instruments. Portfolio allocation is about choosing the best mix from the opportunity set of securities to achieve maximal level of expected return while minimizing risk. By investing in a portfolio of securities some of the risk of individual securities may be diversified away. The fundamental principle of diversification was formally introduced by Markowitz [25]. The Markowitz problem boils down to the determination of portfolio with minimal possible risk for a given level of return, where risk is measured by variance (standard deviation). A set of

solutions of Markowitz problem for different levels of return form the so-called efficient frontier which represents the optimal trade-off between the risk and return.

When using variance to estimate risk we implicitly assume that returns are normally distributed, i.e. that distribution is fully explained by first two moments, return and standard deviation. However, empirical distributions of returns typically are asymmetric distributions with more events in tails relative to normal distribution suggesting that part of the risk is hidden in the higher moments of distribution. The importance of third moment in portfolio optimization was first suggested by Samuelson [29] while Markowitz [26] suggested semivariance for the measure of downside risk.

Nowadays, investors and regulators are mostly concerned about the risk of extreme quantiles. The risk of extreme quantiles is typically measured by value at risk (VaR) and conditional value at risk (CVaR). Although it is not a coherent risk metric [30], VaR is a predominantly used risk measure of extreme quantiles, in particular upon the introduction of new banking regulations for market risk in 1996 [5].

By definition, VaR is the α -quantile of distribution. Unlike variance, VaR of a portfolio cannot be estimated analytically, as a function of underlying constituents' parameters. Portfolio VaR can be estimated analytically, but only if we assume that portfolio data distribution (in value or return terms) can be accurately approximated by some theoretical distribution. In reality, especially during market turmoil, empirical return distributions of financial assets are typically asymmetric, with fat tail(s) and cannot be accurately approximated by any theoretical distribution.

Further, in Danielsson et al. [12], the authors demonstrated that the set of feasible portfolios under VaR constraint need not to be connected or convex, while the number of local optima increases exponentially with the number of securities.

In general, optimal portfolio allocation in the VaR context is very complex, often unsolvable by using classical optimization methods. In order to overcome drawbacks of classical optimization methods, several researchers applied metaheuristics (either single or multiobjective) for solving portfolio optimization problems.

The focus of this paper is on the effectiveness of different GA techniques for static portfolio optimization when return and percentage historical VaR are set as optimization objectives. We employed standard single-objective technique and SPEA2 method as a fully multiobjective technique to derive mean return-historical VaR efficient frontier. In addition, we introduced a novel single-objective GA technique, adjusted to specific characteristics of VaR measure. With the aim to improve execution efficiency, a novel set of weights which are constant over time for a static portfolio is introduced. We compared the differences between analyzed techniques and identified their relative advantages regarding risk/reward characteristics, diversity of solutions along efficient frontier and computational time.

The remainder of this paper is organized as follows. Section 2 discusses related work. In Section 3 we introduce historical VaR model. In Section 4 decision variables are defined. In Section 5 portfolio optimization model is defined. The fundamentals of genetic algorithm operators are presented in Section 6. In Section 7 we provide the computational results. In Section 8 we discuss the results and conclude.

2. Related work

Depending on the applied risk model, portfolio optimization may be a highly nonlinear problem, very difficult to solve using deterministic methods. In practice, portfolio optimization problems become even more complex since they include a lot of additional constraints such as: cardinality constraints, transaction cost, trading limitation etc.

There have been various studies applying genetic algorithms for solving portfolio optimization problems based on different risk measures and/or additional constraints. One of the earliest attempts of GA portfolio optimization is given by Arnone, Loraschi, and Tettamanzi [4]. The authors considered bi-objective, mean return-risk, unconstrained portfolio optimization problem, regarding downside, variance based risk measures. They transformed bi-objective problem to a single-objective problem using trade-off coefficient. Lin and Liu [24] presented a study about portfolio optimization based on Markowitz model with the minimum transaction lots constraint. The authors generated mean-variance efficient frontiers by minimizing risk for given return levels using single-objective GA. Chang et al. [7] presented GA approach to mean return-risk portfolio optimization problems with the cardinality constraint. As in [4], the authors applied bi-objective to a single-objective problem transformation by using trade-off coefficient. As risk measures, the authors considered semi-variance, mean absolute deviation and variance with skewness. Branke et al. [6] considered mean-variance problem with maximum exposure constraint. The authors generated mean-variance efficient frontiers by using multiobjective evolutionary algorithms (MOEAs). Recently, Anagnostopoulos and Mamanis [3] presented an interesting study about effectiveness of five state-of-the-art MOEAs together with a steady state evolutionary algorithm on the mean-variance, cardinality constrained portfolio optimization problem. All enumerated papers examine Markowitz mean-variance model imposing different set of additional constraints.

Latest risk measurement practices trend toward quantifying and controlling risk of extreme quantiles, while VaR is a benchmark metric, as defined by regulators.

Accordingly, there have been some attempts to tackle portfolio selection problems which consider VaR as risk measure using GA. Dallagnol et al. [11], presented single objective optimization techniques for obtaining portfolio with minimal historical (nonparametric) VaR. The authors employed and compared two different techniques, GA and particle swarm optimization (PSO). They demonstrated that PSO was faster than GA, while GA appears to be less sensitive to the initial set of individuals (solution portfolios). Lin and Ko [23] used GA to construct a portfolio with the highest success rate of forecasting VaR. They estimated analytical (parametric) VaR by using extreme value theory models to parametrize the left tail of portfolio return distribution. Hocklie and Zuhail [18] applied a GA for solving portfolio optimization problem using downside risk and value at risk values of individual securities from the opportunity set.

We find that there is a need to explore into possibilities of using different GA techniques to obtain optimal trade-offs between return and risk of a portfolio when risk is defined as VaR, particularly in a market environment where portfolio VaR cannot be analytically (parametrically) estimated with appropriate accuracy. In this paper we examined bi-objective, mean return-risk, unconstrained portfolio optimization when

percentage historical VaR is used as a measure of risk. This is to the best of our knowledge the first attempt to do so in nonparametric VaR context by using GA approach.

3. VaR model

VaR can be interpreted as a loss that will be exceeded only in $\alpha \times 100\%$ of the time, for a given significance level α and time horizon t . Mathematically, VaR is defined as α -quantile of distribution. Expressed in value terms, VaR is the α -quantile of profit and loss distribution, while expressed as a percentage of portfolio's value it is the α -quantile of return distribution.

Formally, for the return r_α such that $p(r < r_\alpha) = \alpha$ percentage VaR is defined as:

$$VaR = -r_\alpha . \quad (1)$$

where α is significance level (i.e. $1-\alpha$ is confidence level) and $F^{-1}(\alpha)$ denotes α -quantile of return distribution r , that is, the inverse of distribution function at α . Minus sign is needed since VaR is defined as positive value. If the distribution function of returns, $F(\alpha)$, is known then α -quantile is calculated as $r_\alpha = F^{-1}(\alpha)$. When empirical distribution of portfolio returns is used VaR is referred to as historical VaR.

Historical VaR does not assume any parametric form of the distribution of risk factor returns (see [28] for more details on historical VaR and its variants). It is rather intuitive and easy to calculate measure at portfolio level. On the other hand, when using historical VaR there is potential risk to underestimate risk of future movements since historical VaR assumes that realized distribution would be repeated in the future. In addition, historical VaR estimates are dependent on a sample size and may result in conflicting results for different significance levels. Yet, Perignon and Smith [27] report that almost 75% of banks prefer to use historical VaR rather than alternative VaR models.

In general, historical VaR cannot be expressed as a function of underlying constituents' parameters. Thus, to perform portfolio optimization in VaR context, calculation of time series of portfolio returns is required. If using the definition Eq. (1), the estimate of historical VaR equals minus value of maximum of the subset containing α percentage of the lowest returns of considered portfolio (e.g. for time series consisted of 100 returns, 5% historical VaR would be minus value of 5-th lowest return).

4. Decision variables

By definition, percentage one-period portfolio return r_p at time t is:

$$r_{p,t} = \frac{P_t}{P_{t-1}} - 1 . \quad (2)$$

where P_t denotes total value of portfolio at time t :

$$P_t = \sum_{i=1}^N n_i p_{i,t} . \quad (3)$$

N is total number of assets, n_i is the number of shares of asset i and $p_{i,t}$ is the price per share of asset i at time t .

In [1] it is shown that percentage one-period portfolio return can be expressed as weighted sum of the asset returns:

$$r_{p,t} = \sum_{i=1}^N w_{i,t-1} r_{i,t} . \quad (4)$$

where $r_{i,t}$ denotes percentage one-period return of asset i at time t , while $w_{i,t}$ denotes proportion of capital invested in asset i at time t , given as:

$$w_{i,t} = \frac{n_i p_{i,t}}{P_t}, \quad i = 1, \dots, N . \quad (5)$$

Expression (4) is fundamental relationship in portfolio mathematics [1].

For static (buy-and-hold) portfolio, which is the focus of the paper, number of shares n_i remains the same for each asset i over the observed period. Consequently, the proportion of capital invested in each asset $w_{i,t}$ changes over time, whenever the price of any asset in portfolio changes. In typical portfolio optimization methods, starting with those applying Markowitz model, static portfolio is considered, while portfolio weights are defined as a proportion of capital invested (Eq. (5)) in each portfolio constituent. Basic assumption is that portfolio weights are constant over time. Assumption of constant weights implies frequent portfolio rebalancing which further implies transaction costs that affect overall portfolio performance.

The main drawback of using $w_{i,t}$ weight parameters for calculating time series of “real” static portfolio returns is the fact that it requires recalculations of their values for every time interval during an observation period (e.g. for daily time series of portfolio returns daily recalculations of $w_{i,t}$ weights are needed).

In order to overcome this drawback we introduce different set of weights \bar{w}_i . Here \bar{w}_i is defined as the proportion of shares held in asset i :

$$\bar{w}_i = \frac{n_i}{N_T} . \quad (6)$$

and $N_T = \sum_{i=1}^N n_i$.

It can be seen that \bar{w}_i weights are properly defined since their values lie between 0 and 1 for long-only portfolios (that is, for portfolios without short positions) and they sum up to 1.

Using Eq. (3) and Eq. (6) expression (2) becomes:

$$r_{p,t} = \frac{\sum_{i=1}^N n_i p_{i,t}}{\sum_{i=1}^N n_i p_{i,t-1}} - 1 = \frac{\sum_{i=1}^N \frac{n_i}{N_T} p_{i,t}}{\sum_{i=1}^N \frac{n_i}{N_T} p_{i,t-1}} - 1 = \frac{\sum_{i=1}^N \bar{w}_i p_{i,t}}{\sum_{i=1}^N \bar{w}_i p_{i,t-1}} - 1 . \quad (7)$$

Weights \bar{w}_i for static portfolio are, by definition, constant over observation period.

Using Eq.(5) and Eq. (6), proportion of capital $w_{i,t}$, which is of interest for an investor, is given as:

$$w_{i,t} = \frac{\bar{w}_i N_T p_{i,t}}{\sum_{i=1}^N \bar{w}_i N_T p_{i,t}} = \frac{\bar{w}_i p_{i,t}}{\sum_{i=1}^N \bar{w}_i p_{i,t}} . \quad (8)$$

In the portfolio optimization problem, presented in this paper, weights \bar{w}_i are adopted as decision variables.

5. Optimization model

Using definition of VaR (Eq. (1)),the portfolio optimization problem can be defined as follows:

$$\min VaR(\bar{\mathbf{w}}) = -r_\alpha . \quad (9)$$

$$\max \bar{r}_p(\bar{\mathbf{w}}) = \frac{\sum_{t=1}^T r_{p,t}}{T} . \quad (10)$$

$$\text{subject to} \quad \sum_{i=1}^N \bar{w}_i = 1 . \quad (11)$$

$$0 \leq \bar{w}_i \leq 1, \quad i = 1, \dots, N . \quad (12)$$

where

$\bar{\mathbf{w}}$ denotes the vector of weights \bar{w}_i , $VaR(\bar{\mathbf{w}})$ denotes the value at risk of a portfolio, $\bar{r}_p(\bar{\mathbf{w}})$ is the expected return of portfolio and T denotes the time series length.

Eq. (9) minimizes VaR of the portfolio. Eq. (10) maximizes the expected return of portfolio. Eq. (11) describes the standard budget constraint which requires that proportions (weights) must sum up to 1.

Eq. (12) describes the constraint that no short sales are allowed, which means that none of the weights can be negative.

The model presented above is a standard multiobjective optimization problem. Solutions of the presented optimization problem comprise the set of non-dominated or efficient portfolios. Non-dominated or efficient portfolio is every portfolio whose characteristics cannot be improved in terms of one objective function without

deteriorating in terms of the other one. Each efficient portfolio represents a point in the objective functions space. Hence, the set of efficient portfolios represents a curve in the return-risk space connecting the portfolio with maximum return and portfolio with minimum risk. Often, this curve is called the efficient frontier. The aim of the portfolio optimization presented by the model is to find portfolios along the efficient frontier.

The presented optimization problem is the bi-objective optimization problem and can be solved using two different approaches.

The first technique implies transformation of bi-objective problem to single-objective problem using trade-off coefficient λ ($0 \leq \lambda \leq 1$). The result is single-objective fitness function instead of two separate objective functions. In that case optimization problem (Eq. (9), Eq. (10)) becomes:

$$\max \quad f = \lambda \bar{r}_p(\bar{\mathbf{w}}) - (1 - \lambda) VaR(\bar{\mathbf{w}}) . \quad (13)$$

In Eq. (13) the case $\lambda = 1$ corresponds to portfolio with maximum expected return and $\lambda = 0$ corresponds to portfolio with minimum VaR (risk). Values of λ satisfying $0 < \lambda < 1$ correspond to the portfolios lying on the efficient frontier.

The advantage of presented approach is that a single-objective technique can be used. The additional advantage is that decision maker (investor) can predefine the importance of each objective. This approach is known as decision before search approach [20], [21].

The main drawback of this approach is that each value of coefficient λ produces only one efficient portfolio, i.e. only one point of the efficient frontier. Therefore, if efficient frontier is needed, the set of separate calculations with different λ is required, which can be very time consuming.

The second approach implies use of fully multiobjective techniques. In this paper, results obtained using the both approaches are presented.

6. Genetic Algorithms

The term evolutionary algorithms (EA) or evolutionary strategies, addresses a class of stochastic optimization methods which emulate the natural evolution. The origins of EAs can be found in the late 1950s, and since the 1970s several evolutionary methodologies have been proposed. This class of optimization methods addresses genetic algorithms, evolutionary programming, and evolution strategies [10].

In order to solve the portfolio optimization problems defined in previous sections, genetic algorithm is used.

Genetic algorithm is a stochastic optimization technique invented by Holland based on the Darwin principle that in the nature only “the fittest survive” [19]. The main idea of Holland’s theory is the application of the basic phenomena of the biological evolution such as inheritance, crossover and mutation, in order to find (generate) a solution that fits best. In the case of the portfolio optimization problems, term “the fittest” corresponds to the optimal portfolio.

So, in GA there is a set of individuals often called population. Each individual from population presents candidate solution of optimization problem. The individuals are

usually referred to as chromosomes. Each chromosome, i.e. candidate solution, represents a decision vector made of decision variables. In the vocabulary of genetic algorithms each decision variable in the chromosome is called a gene.

In this research, each individual (chromosome) presents one weight vector \bar{w} . Therefore, each gene corresponds to weight \bar{w}_i defined by expression (6)

Generally, genetic algorithm consists of the following steps:

1. Initialization of population with random individuals,

Initial population with the stated dimension of randomly chosen individuals is generated with the aim to uniformly cover the solution space.

2. Fitness evaluation of the individuals in the population,

Fitness value is assigned to each individual from the population defined by adopted fitness function.

3. Generation of a new population, using crossover and mutation,

Population of offspring is generated by applying crossover and mutation operators to population of parents.

4. Selection of individuals according to their fitness using some strategy (e.g. a Roulette wheel selection),

The aim of the selection process is to provide a set of offspring which “survived” and will be transmitted to the next generation.

5. Stop if terminating condition is satisfied (e.g., a fixed number of iterations), otherwise go to step 2.

In the following text the basic operators of genetic algorithm developed in this research are presented.

6.1. Initialization

First step of genetic algorithm is initialization of population. It is very important since initial population is supposed to cover the solution space in a best possible way. The second demand is to provide random and independent generation of individuals in order to ensure independence of each single run of GA. Therefore, each gene (weight parameter) within the individual has to be generated randomly and independently. At the same time, each individual constraint, defined by Eq. (12), must be satisfied.

In order to realize independence and randomness of gene generation the following procedure is used:

For each individual I in the population P :

1. Generate vector \mathbf{o} that contains randomly chosen indexes of weights in vector \bar{w} :

$$o[i] = rand_{int}[1, N], \quad i = 1, 2, \dots, N$$

where N denotes the number of assets held in portfolio; $rand_{int}()$ is random function with uniform distribution of integers within the range $[1, N]$. Vector \mathbf{o} defines the order of genes' initialization.

2. Generate weights in the sequence determined by \mathbf{o} using following expression:

$$\begin{aligned}\bar{w}_{o[i]} &= rand_{real}(0,1) \left(1 - \sum_{j=1}^{i-1} \bar{w}_{o[j]} \right), \quad i = 1, \dots, N-1 \\ \bar{w}_{o[N]} &= 1 - \sum_{j=1}^{N-1} \bar{w}_{o[j]}\end{aligned} \quad (14)$$

where $rand_{real}()$ denotes random function with uniform distribution of real numbers within the range $[0,1]$.

6.2. Crossover

In this research a basic (simple) crossover operator is implemented. The basic crossover operator involves two parents and produces two offspring (two new individuals). Idea is to divide both parents' chromosomes in two segments at dividing point (gene) and then to swap obtained segments. Operator is stochastic one, because the dividing point is chosen randomly each time operator is applied. Usually, newborn offspring does not satisfy constraint defined by Eq. (12) and hence additional normalization of offspring is required.

6.3. Mutation

The mutation operator is implemented as follows. First, a set of randomly chosen individuals that will mutate is generated. For each individual from this set two genes are randomly selected. Then, the value of the first gene (weight parameter) is increased by some predefined value (e.g. 0.1) and the value of the second gene is decreased by the same value. In order to ensure satisfaction of constraint defined by Eq. (12) the following principle is applied. If new gene (weight) value is <0 then it is set to be $=0$. Also, if the new gene value is >1 it is set to be $=1$. For both cases, normalization of chromosome must be applied.

6.4. Selection

The aim of selection process is to choose the individuals from the current population who will survive. According to the analogy with living beings, the individuals that fit best will survive and be transmitted to the next generation. The fitness of each individual is determined by using of adequate fitness function. The strategy which implies that only the best fitted individuals survive is known as Elitistic strategy. Generally, using this strategy, search algorithm would relatively quickly find the optimized solution. The major drawback of this approach is tendency to get stuck on a local extremum.

In order to prevent the search process from getting stuck on a local extremum, the combination of *Elitistic* and *Roulette wheel* selection is applied. Elitistic approach is realized in such a way that the best individual in each generation is isolated and

transferred to the next generation. In this way, the best individual (solution) obtained during the evolution process is kept. The rest of generation is processed throughout *Roulette wheel* selection.

Roulette wheel selection is partly stochastic strategy which is commonly used in GA approaches to portfolio optimization problems (see for example [22], [31]). Selection process is random but based on fitness value of individuals. If an individual is with better fitness value it has more chance of being selected and vice versa.

In order to apply Roulette wheel selection, relative fitness is introduced. Relative fitness of each individual is defined as

$$f_{r,k} = f_k - f_{\min}, \quad k = 1, \dots, M \quad (15)$$

where f_k is fitness value of k -th individual, f_{\min} is minimal fitness value obtained in the current generation and M is population size.

Using the relative fitness value, a fictive “*roulette wheel*” can be constructed. Each field of the fictive roulette wheel corresponds to one individual and the width of each field is proportional to relative fitness of that individual. The width of each roulette wheel field can be defined as range:

$$\begin{aligned} rwf_k &= \left(\sum_{i=1}^{k-1} f_{r,i}, \sum_{i=1}^k f_{r,i} \right], \quad k = 2, \dots, M \\ rwf_1 &= \left(0, f_{r,1} \right] \end{aligned} \quad (16)$$

Now, k -th individual in the population will be selected if following expression is satisfied:

$$rand\left(0, \sum_{i=1}^M f_{r,i}\right) \in rwf_k \quad (17)$$

where $rand_{real}()$ is random function with uniform distribution of real numbers within the defined range.

7. Data and research results

In this section, we present the computational results obtained by performing experiments on a historical data set. For VaR we considered 5% 1-day historical percentage VaR of static long-only linear portfolios. Calculations are based on daily returns of sample of 10 exchange-traded funds (ETFs): iShares Barclays 7-10, iShares eb.rexx 5.5-10.5, iShares iBoxx € Liquid Sovereigns Capped 5.5-10.5, db x-trackers iBoxx € Sovereigns 5-7, db x-trackers iBoxx € Sovereigns 7-10, Lyxor EuroMTS 5-7, Lyxor EuroMTS 7-10, iShares eb.rexx DE, iShares iBoxx € Liquid Sovereigns Capped 1.5-10.5 and db x-trackers iBoxx € Sovereigns overall. We used historical daily data for sample ETFs from February 2008 to December 2010. Returns are presented as annualized daily returns, as it is common in finance. The source of our data is the official websites of sample ETFs’ providers.

Underlying indices for sample ETFs are debt portfolios with exposure to euro zone sovereign debt. First 7 ETFs comprise representative sample of debt portfolios which target bonds from 5 to 10 years maturity segment¹ while the latter 3 ETFs target overall bond indices which include different maturity segments.

In times of market turmoil investors typically move towards less risky and more liquid financial products, such is sovereign debt. At the same time, current financial crisis strongly affected euro zone sovereign debt due to increased credit and liquidity risk concerns. Yield spreads rose sharply while correlations dropped. As a consequence, empirical return distributions of sovereign debt portfolios are more often characterized by extreme events. Investing in sovereign debt of euro zone countries through ETFs is arguably the most liquid way of getting desired exposure during the crisis while daily data are publicly available.² The motivation for choosing 5-10 maturity segment ETFs for the research is the fact that their return distributions are highly non-normal for chosen sample period (see more on characteristics of euro zone sovereign debt ETFs in [15] and [16]).

We compare risk/return characteristic of optimized portfolios to those of individual assets and to the 1/n portfolio. DeMiquel et al. [14] evaluated the out-of-sample performance of Markowitz model of optimal asset allocation and its various extensions (in total 14 different models). The authors demonstrated that naïve optimization rule 1/n of assets in portfolio is good proxy of optimal portfolio and can be challenged against more sophisticated portfolio designs. We employed static version of 1/n portfolio in order to be consistent with our basic assumption that all portfolios are buy-and-hold portfolios.

Table 1 shows 1-day historical VaR and average 1-day annualized return values for sample ETFs and naïve 1/n portfolio.

Table 1. 5% 1-day historical VaR and average 1-day annualized return values for sample ETFs and naïve 1/n portfolio

No.	Name	VaR(%)	Return(%)
1*	eb.rexx 5.5-10.5	0.515	6.238
2*	iShares Bar 7-10	0.553	6.120
3*	eb.rexx DE	0.352	5.325
4*	db 5-7	0.418	4.527
5*	Lyxor GY 5-7	0.411	4.410
6*	db 7-10	0.524	4.089
7*	db overall	0.421	4.066
8*	Lyxor GY 7-10	0.517	3.885
9*	iBoxx 5.5-10.5	0.507	3.711
10*	iBoxx 1.5-10.5	0.443	3.544
1/N	Naïve 1/n portfolio	0.398	4.570

¹ These are all European ETFs targeting 5-10 year maturity segment of euro zone sovereign debt with available time series of data long enough for this research.

² Sovereign debt markets are typically over-the-counter markets where data are not publicly available. In contrast, ETFs are traded on organized exchanges which provide daily time series of data.

Genetic algorithm, described in the previous sections, is coded in C# and run on personal computer with Intel Core(TM)2 Duo CPU E7300 2.66GHz processor and 2GB of RAM.

For solving optimization problem Eq. (9)-(12) three different techniques are applied (denoted as A1, A2 and A3, respectively). Therefore, results refer to three different sets of optimized portfolios, that is, on three different efficient frontiers.

Techniques A1 and A2 are based on the transformation of bi-objective optimization problem into single-objective problem using trade-off coefficient λ (Eq. (13)). In the A1 technique we examined equidistant values of trade-off coefficient λ . Within A2 technique we applied strategy of searching for optimized portfolios with predefined levels of return. For single-objective techniques we used following parameters: initial population size equals 200, number of generations equals 500 per single run, crossover rate equals 0.9, mutation rate equals 0.1.

Technique A3 employs fully multiobjective optimization genetic algorithm, the SPEA2 method (Strength Pareto Evolutionary Algorithm 2).

7.1. Single-objective technique A1 - Equidistant values of trade-off parameter

In this section we present the results of solving portfolio optimization problem for transformed VaR model (Eq. (13)) using 21 equidistant values of trade-off parameter λ within the range [0,1].

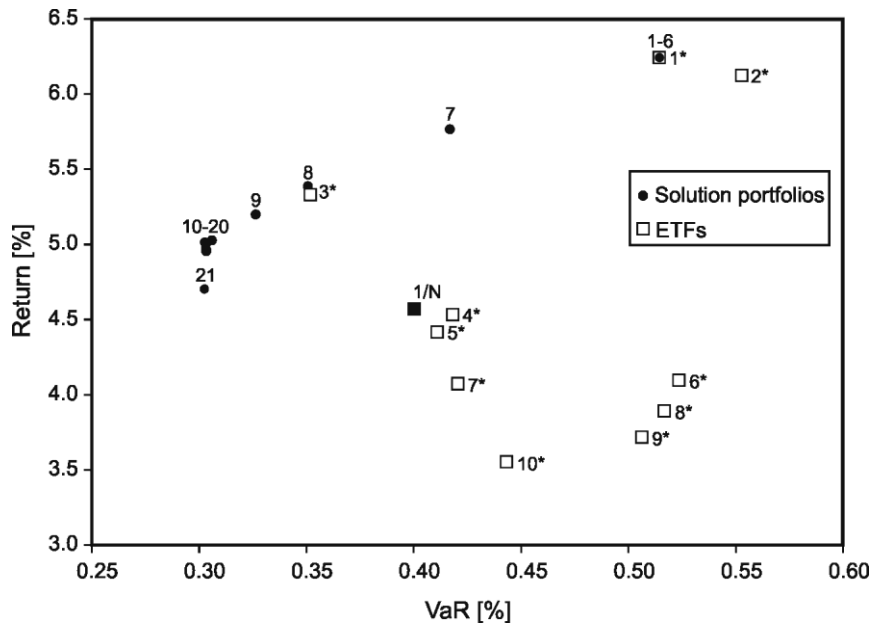


Fig. 1. Mean-VaR efficient frontier for sample ETFs-A1 technique

Optimized portfolios are plotted in return and risk (VaR) coordinates are presented with filled dots (Fig. 1). Single asset values are presented with squared dots. Each filled dot in the following graph is obtained by execution of genetic algorithm using one single value of parameter λ . Different values of parameter λ correspond to different relative importance of merged objective functions (return and risk). Efficient frontier is generated by repeating optimization procedure for each predefined value of λ . Optimized portfolios are numerated in decreasing order by return. 5% 1-day historical VaR and average 1-day annualized return values for optimized portfolios are presented in the Table 2. Asterisk numeration corresponds to sample ETFs presented in Table 1.

Table 2. 5% 1-day historical VaR and average 1-day annualized return values for optimized portfolios obtained using single-objective technique A1

No.	VaR(%)	Return(%)	No.	VaR(%)	Return(%)
1	0.515	6.238	12	0.304	5.004
2	0.515	6.238	13	0.303	4.997
3	0.515	6.238	14	0.302	4.975
4	0.515	6.238	15	0.304	4.970
5	0.515	6.238	16	0.303	4.961
6	0.515	6.238	17	0.302	4.954
7	0.417	5.761	18	0.303	4.954
8	0.351	5.381	19	0.303	4.949
9	0.326	5.192	20	0.303	4.949
10	0.306	5.021	21	0.304	4.705
11	0.303	5.007			

It can be seen from Fig. 1 that A1 technique results in portfolios with superior risk/reward characteristics compared to the individual assets from the sample and to the naïve 1/n portfolio. The results motivate the need for portfolio diversification and bring some evidence about the effectiveness of chosen algorithm. However, the plots presented in the graph show that equidistant values of trade-off parameter do not provide uniform distribution of solution portfolios along the resulting efficient frontier. It should be emphasized that diversity of points along the efficient frontier is crucial in portfolio optimization. Higher level of diversity implies more alternative portfolios, suited for the investors with different risk-return profiles.

7.2. Single-objective technique A2 - Imposing return levels

In order to improve the diversity of solution portfolios we introduce the A2 technique. The objective of this technique is to generate solution portfolios with predefined return levels. To keep the predefined return level we search for a trade-off parameter λ which corresponds to the given return level. We considered 21 levels of return in order to maintain the dimensionality of solution set. Levels are predefined within the return range $(\bar{r}_{p\min}, \bar{r}_{p\max})$ using following expression:

$$l_i = \bar{r}_{p \min} + i \frac{\bar{r}_{p \max} - \bar{r}_{p \min}}{20}, \quad i = 0, 1, \dots, 20 \quad (18)$$

$$\bar{r}_{p \min} = \bar{r}_p \Big|_{\lambda=0}, \quad \bar{r}_{p \max} = \bar{r}_p \Big|_{\lambda=1}$$

where l_i denotes i -th return level.

For each return level an optimization process is executed. Each execution represents bisection iterative method implemented to determine λ corresponding to the considered return level. Finally, each iteration of the bisection method implies one single run of genetic algorithm which results in one solution portfolio.

Since it is impossible to reach the exact value of requested return level, for each execution of bisection iterative method we applied the following stop criterion:

$$1) \quad \bar{r}_p^n \in [l_i - tol, l_i + tol], \quad tol = 0.1 \frac{\bar{r}_{p \max} - \bar{r}_{p \min}}{20} \quad (19)$$

or

$$2) \quad n > 30$$

where n denotes number of iterations, \bar{r}_p^n denotes return of the portfolio obtained in n -th iteration.

In the Fig. 2 we present mean-VaR efficient frontier for sample ETFs obtained using A2 technique. Corresponding 5% 1-day historical VaR and average 1-day annualized return values for optimized portfolios are presented in the Table 3.

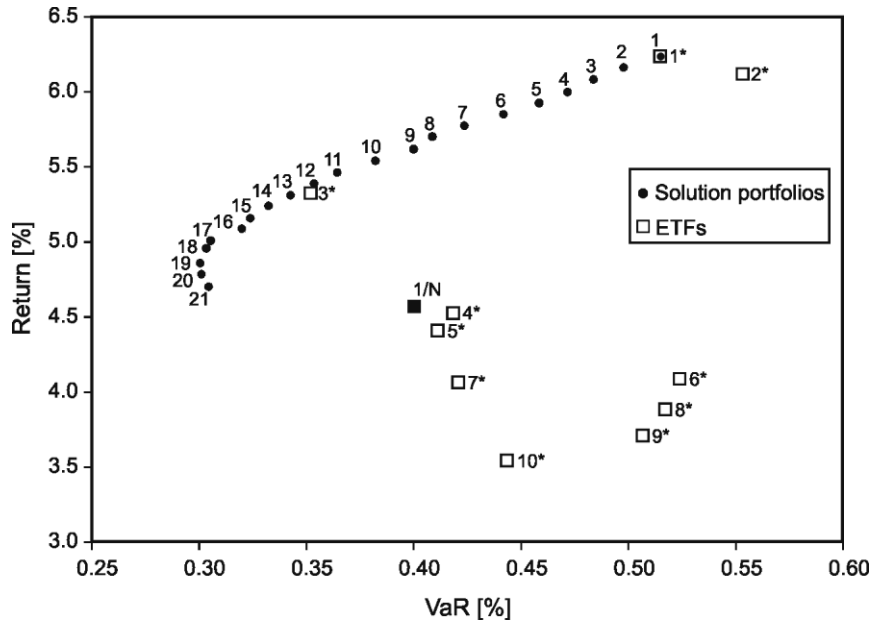


Fig. 2. Mean-VaR efficient frontier for sample ETFs-A2 technique

Table 3. 5% 1-day historical VaR and average 1-day annualized return values for optimized portfolios obtained using single-objective technique A2

No.	VaR(%)	Return(%)	No.	VaR(%)	Return(%)
1	0.515	6.238	12	0.353	5.392
2	0.497	6.165	13	0.343	5.314
3	0.483	6.086	14	0.332	5.245
4	0.471	6.002	15	0.324	5.162
5	0.458	5.929	16	0.320	5.091
6	0.442	5.854	17	0.305	5.012
7	0.423	5.778	18	0.303	4.962
8	0.408	5.704	19	0.300	4.862
9	0.400	5.622	20	0.301	4.788
10	0.382	5.543	21	0.304	4.705
11	0.364	5.465			

Fig. 2 shows that A2 technique results in portfolios that are better distributed along the efficient frontier compared to solutions computed by A1 technique. Also, it should be noticed that for all, except for one predefined return level, proposed iterative procedure reached portfolio with return within targeted range.

Since VaR is not a coherent risk metric, it is very important to emphasize that solution portfolios obtained by single-objective techniques are not necessarily efficient portfolios in terms of given definition. It can be noticed from Fig. 2 that the solution portfolios corresponding to the two lowest return levels are not efficient portfolios since they are dominated. These points are not excluded in order to demonstrate the specificity of VaR.

7.3. Multiobjective technique A3 - SPEA2 method

In this section, we present the results obtained using multiobjective evolutionary algorithm (MOEA). Several different MOEA techniques can be found in the modern literature such as: Strength Pareto evolutionary algorithm 2 (SPEA2) [32], nondominated sorting genetic algorithm II (NSGA-II) [13], Pareto envelope-based evolutionary algorithm (PESA) [8], Niche Pareto genetic algorithm 2 (NPGA2) [9], e-multiobjective evolutionary algorithm (e-MOEA) [17].

In this research, we applied Strength Pareto evolutionary algorithm 2 (SPEA2). For more details see [32].

Common to all MOEAs techniques is the performance of full multiobjective optimization without transformation of multiobjective problem into single-objective.

Therefore, the input optimization problem for SPEA2 method is defined by Eq. (9)-(12).

Initial population size is set equal to 200, number of generations is set equal to 10,000, crossover rate is set equal to 0.9 and mutation rate is set equal to 0.1.

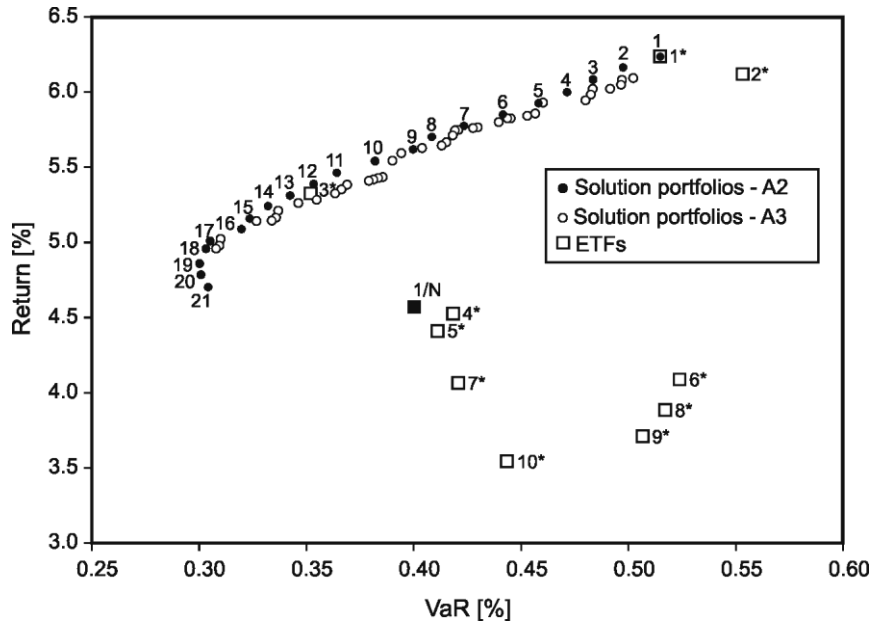


Fig. 3. Mean-VaR efficient frontier for sample ETFs-A3 technique

Table 4. 5% 1-day historical VaR and average 1-day annualized return values for optimized portfolios obtained using multiobjective technique A3 (portfolios are numerated in decreasing order by return, M stands for multiobjective technique)

No.	VaR(%)	Return(%)	No.	VaR(%)	Return(%)
1M	0.503	6.092	21M	0.404	5.625
2M	0.498	6.080	22M	0.395	5.591
3M	0.497	6.047	23M	0.390	5.541
4M	0.492	6.021	24M	0.386	5.433
5M	0.484	6.020	25M	0.384	5.427
6M	0.483	5.982	26M	0.382	5.416
7M	0.480	5.944	27M	0.379	5.407
8M	0.461	5.929	28M	0.369	5.382
9M	0.457	5.856	29M	0.367	5.351
10M	0.453	5.841	30M	0.364	5.323
11M	0.446	5.824	31M	0.355	5.281
12M	0.444	5.823	32M	0.347	5.260
13M	0.440	5.799	33M	0.337	5.208
14M	0.430	5.763	34M	0.336	5.163
15M	0.428	5.758	35M	0.334	5.142
16M	0.421	5.747	36M	0.327	5.139
17M	0.420	5.744	37M	0.310	5.020
18M	0.419	5.711	38M	0.310	4.976
19M	0.416	5.664	39M	0.308	4.956
20M	0.413	5.643			

Fig. 3 shows mean-VaR efficient frontier for sample ETFs obtained using SPEA2 method (no filled dots). In order to compare A2 and A3 techniques we keep the results presented in the Fig. 2 (A2 efficient frontier and sample ETFs). A3 optimized portfolios are not numerated in the figure for clarity reasons, but 5% 1-day VaR and average 1-day annualized return values are given in the Table 4.

Fig. 3 shows that A3 technique results in portfolios that are well distributed along efficient frontier. At the same time solution portfolios are slightly dominated by solution portfolios computed by A2 technique. Opposing the A2 technique, efficient frontier provided by A3 technique, lacks solution portfolios around maximum return, as well as, around minimum risk.

8. Concluding remarks

In this paper we presented different GA techniques for the optimization of static portfolios in the context of historical VaR. In general, historical VaR cannot be expressed as a function of underlying constituents' parameters. Thus, to perform portfolio optimization in VaR context, calculation of time series of portfolio returns is required. If considering static (buy-and-hold) portfolio, commonly used portfolio weights, given as proportion of capital invested in individual assets, are function of time. Therefore, when applying these weights to calculate daily time series of returns, one needs daily recalculations of weights. In order to avoid daily recalculations, for decision variables we adopted alternative portfolio weights which are based on proportion of shares of individual assets. For static portfolios, proposed weights are constant during the sample period. We showed that commonly used weights can be easily expressed as a function of the weights that we adopted.

In general, optimal portfolio allocation in the VaR context is computationally very complex and, very often, alternatives to existing exact optimization methods are required. In order to generate mean-VaR efficient frontier we used genetic algorithm. We tested two single-objective GA techniques and one multiobjective technique.

We first applied single-objective technique which uses the set of equidistant trade-off parameter values. It is clear from Fig. 1 that solution portfolios offer much better risk/reward characteristics compared to individual assets from our sample. However, results also showed that chosen set of trade-off parameter does not provide uniform distribution of portfolios along the resulting efficient frontier. The second shortfall of this technique is that it is time consuming. Namely, for each value of trade-off parameter single run of genetic algorithm is needed. Execution time of single genetic algorithm was approximately 15 seconds. Hence, for portfolio optimization with 21 different values of trade-off parameter total execution time was approximately 5 minutes.

The second technique is an improvement of the first technique. In order to overcome poor diversity of portfolios along efficient frontier, we implemented iterative search (bisection method) over trade-off parameter. The aim of this technique was to generate efficient frontier consisting of portfolios which returns correspond to predefined return

levels. As a result, we achieved the same level of diversification as was the case for the former technique, and, at the same time, returns of solution portfolios are well distributed along efficient frontier and satisfy predefined return levels. However, adding bisection method in the optimization algorithm further increased time needed for determination of each solution portfolio. In this research approximately 25 iterations of bisection method were needed. Therefore, execution time of second technique is on average 25 times longer than execution time of first technique. In particular, for portfolio optimization with 21 predefined levels of portfolio return execution time was approximately 125 min.

We emphasize that, formally speaking, single-objective techniques applied to VaR based portfolio optimization do not necessarily result in a set of efficient portfolios. The reason is that VaR is not a coherent risk measure while single-objective techniques are based on a sequence of independent executions.

As an alternative to single-objective approach we applied multiobjective evolutionary algorithm – SPEA2 method. Results show that solution portfolios are well distributed along efficient frontier, but it should be noticed that solution portfolios are slightly dominated by solution portfolios computed by A2 technique. On the other hand, when using multiobjective evolutionary algorithm we are not able to impose return levels of solution portfolios. The main advantage of SPEA2 method is computation time which in this research was approximately 3 minutes.

From an investor's point of view, eligible portfolio optimization technique should provide her with:

1. efficient portfolios with returns distributed within the given range,
2. possibility to obtain portfolio with minimum risk for desired level of return,
3. acceptable computation time.

We showed that presented single-objective technique with imposed return levels (A2) satisfy conditions 1 and 2, while multiobjective technique (A3) satisfies conditions 1 and 3.

Financial implications of presented results would be of interest for all those investors who employ historical analysis for portfolio structuring, which is imposed by constraints in the context of VaR measure. In this respect, out-of-sample performance analysis of VaR optimized portfolios would be an interesting topic for further research.

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