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SOMBOR INDEX – ONE YEAR LATER

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A b s t r a c t. The Sombor index (SO) *is a vertex–degree–based graph invariant, invented in the Summer of 2020, and made publicly available in early 2021. In less than one year, a remarkable number (almost fifty) research papers on this topological index were produced. In the present article, we summarize the results achieved so far, and offer a few more.*

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1. *Introduction*

Let G be a simple graph, that is a graph without directed, weighted or multiple edges, and without self loops. Let $\mathbf{V}(G)$ and $\mathbf{E}(G)$ be the vertex set and the edge set of G, and let $|\mathbf{V}(G)| = n$, $|\mathbf{E}(G)| = m$. The vertices of G are labeled by $v_1, v_2, \ldots, v_n$. The edge connecting the vertices v_i and v_j will be denoted by e_{ij}.

The degree (= number of first neighbors) of the vertex v_i is denoted by d_i. As well known,

$$\sum_{i=1}^{n} d_i = 2m$$

which implies that $2m/n$ is the average value of the vertex degrees.

In the mathematical and chemical literature, several dozens of vertex–degree–based graph invariants (usually referred to as "topological indices") have been introduced and extensively studied [10, 16, 7, 15]. Their general formula is

$$TI = TI(G) = \sum_{e_{ij} \in \mathbf{E}(G)} \Phi(d_i, d_j) \tag{1.1}$$

where $\Phi(x, y)$ is some function with the property $\Phi(x, y) = \Phi(y, x)$.

In Summer 2020, the present author proposed a new such TI, named it "Sombor index", and defined as [6]

$$SO = SO(G) = \sum_{e_{ij} \in \mathbf{E}(G)} \sqrt{d_i^2 + d_j^2}\,. \tag{1.2}$$

In fact, in the paper [6] several variants of the Sombor index were considered, of which we mention here the "average Sombor index"

$$SO_{avr} = SO_{avr}(G) = \sum_{e_{ij} \in \mathbf{E}(G)} \sqrt{\left(d_i - \frac{2m}{n}\right)^2 + \left(d_j - \frac{2m}{n}\right)^2} \tag{1.3}$$

and the "reduced Sombor index"

$$SO_{red} = SO_{red}(G) = \sum_{e_{ij} \in \mathbf{E}(G)} \sqrt{(d_i - 1)^2 + (d_j - 1)^2}\,. \tag{1.4}$$

As a kind of (pleasant) surprise, the ideas outlined in the paper [6] were almost immediately accepted, further elaborated, and generalized by numerous other scholars. In less than one year, almost 50 papers on the Sombor index and its variants were produced. Those containing "Sombor index" in their titles, published before the completion of the present paper, are [17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 22]. More are in press, and more deal with Sombor index, but does not mention this in the title. The authors of these papers are from China, Colombia, Croatia, Great Britain, Hungary, India, Iran, Mexico, Mongolia, Montenegro, Norway, Pakistan, Saudi Arabia, Slovenia, South Korea, Spain, Turkey, United States, Vietnam, and – of course – from Serbia.

Such great research activity in such a short time is remarkable, not often encountered in contemporary mathematics and its applications. It may be an indicator of the quality and originality of the paper [6].

2. *The geometric route to Sombor index*

The fundamental novelty in the concept of Sombor index [6] is in the re-interpretation of the term $\sum\limits_{e_{ij}\in\mathbf{E}(G)}$ in Eq. (1.1). Instead of considering degrees of adjacent vertices, emphasis is given on summation over all edges, where each edge is characterized by a *pair* of degrees of its end-vertices. In other words, the contribution of an edge $e = e_{ij} \in \mathbf{E}(G)$ depends on the pair (x_e, y_e) where $x_e = d_i$ and $y_e = d_j$. This point of view enables to use geometric arguments, as seen from the following definitions.

Definition 2.1. The ordered pair (x_e, y_e), where $x_e = d_i$, $y_e = d_j$, $x_e \leq y_e$, is the *degree-coordinate* of the edge $e = e_{ij} \in \mathbf{E}(G)$. For brevity, this edge will be referred to as an (x_e, y_e)-edge. In the (2-dimensional) coordinate system, it pertains to a point called the *degree-point* of the edge e.

Definition 2.2. The point with coordinates (y_e, x_e) is the *dual* of the degree-point of the edge e.

Definition 2.3. The distance between the degree-point (x_e, y_e) and the origin of the coordinate system is the *degree-radius* of the edge e, denoted by $r(x_e, y_e)$.

Based on elementary geometry (using Euclidean metrics) [31], we have

$$r(x_e, y_e) = \sqrt{x_e^2 + y_e^2}\,. \tag{2.1}$$

From Eq. (2.1), we immediately see that a degree-point and its corresponding dual have equal degree-radii.

The considerations summarized in Definitions 2.1–2.3 provide the motivation for the introduction of the graph invariant SO of the type (1.1), defined as

$$SO(G) = \sum_{e\in\mathbf{E}(G)} r(x_e, y_e)$$

which, of course, is equivalent to Eq. (1.2).

3. *Mathematical properties of the Sombor index*

As already mentioned, the publication of the paper [6] triggered an unprecedentedly vigorous research of the mathematical properties and chemical applications of the Sombor index. Chemical applications are reported in [18, 46, 51], whereas molecular graphs are considered in [19, 20, 26, 28, 34, 36, 37, 38, 40]. Other classes of graphs were studied in [30, 35, 41, 54, 55, 22]. An application of Sombor index in network theory has recently been attempted [17].

Numerous inequalities between Sombor index and other vertex–degree–based indices of the form (1.1) were reported [23, 29, 49, 48, 50, 53]. Of these we list here a few simplest.

We first recall the definitions of the relevant vertex–degree–based graph invariants of the form (1.1). The first and second Zagreb indices are [9, 8]

$$M_1(G) = \sum_{e_{ij} \in \mathbf{E}(G)} (d_i + d_j) = \sum_{v_i \in \mathbf{V}(G)} d_i^2$$

and

$$M_2(G) = \sum_{e_{ij} \in \mathbf{E}(G)} d_i \cdot d_j \,.$$

The forgotten index is [4]

$$F(G) = \sum_{e_{ij} \in \mathbf{E}(G)} (d_i^2 + d_j^2) = \sum_{v_i \in \mathbf{V}(G)} d_i^3 \,,$$

whereas the Albertson irregularity index is [1]

$$Alb(G) = \sum_{e_{ij} \in \mathbf{E}(G)} |d_i - d_j| \,. \tag{3.1}$$

Recall that $Alb(G)$ is the oldest and most extensively studied measure of graph irregularity. For details on this matter see the recent papers [3, 5, 13] and the references cited therein.

Let G be a simple graph with n vertices and m edges. In order to avoid trivialities, we assume that G is connected and that $n \geq 2$.

Theorem 3.1 ([29, 31, 49])**.**

$$\frac{1}{\sqrt{2}} M_1(G) \leq SO(G) < M_1(G) \,.$$

Equality on the left–hand side holds if and only if the graph G is regular.

Theorem 3.2 ([29])**.**

$$SO(G) \geq \frac{\sqrt{2}}{n-1} M_2(G) \,.$$

Equality holds if and only if $G \cong K_n$, where K_n is the complete graph of order n.

Theorem 3.3 ([23])**.**

$$SO(G)^2 \leq \left(\frac{\Delta}{\delta} + \frac{\delta}{\Delta}\right) m\, M_2(G),$$

where δ and Δ are, respectively, the minimum and maximum vertex degrees.

Theorem 3.4 ([23, 29, 49])**.**

$$\begin{aligned} SO(G) &\leq \sqrt{m\,F(G)}, \\ SO_{avr}(G) &\leq \sqrt{m\,F(G) - \frac{4m}{n}\,M_1(G) + \frac{8m_2}{n^2}}, \\ SO_{red}(G) &\leq \sqrt{m\,F(G) - 2M_1(G) + 2m}\,. \end{aligned}$$

Equality in the first relation holds if and only if the graph G is regular.

Theorem 3.5 ([49])**.**

$$\begin{aligned} SO(G) &\leq \frac{1}{\sqrt{2}}\left[M_1(G) + Alb(G)\right], \\ SO_{avr}(G) &\leq \frac{1}{2}\left[M_1(G) + Alb(G) - \frac{4m^2}{n}\right], \\ SO_{red}(G) &\leq \frac{1}{\sqrt{2}}\left[M_1(G) + Alb(G) - 2m\right], \end{aligned}$$

with equality if the graph G is regular.

Theorem 3.6 ([49])**.** *Let $\overline{G}$ be the complement of the graph G. Then*

$$SO(G) + SO(\overline{G}) \leq \frac{1}{\sqrt{2}}\, n(n-1)^2\,.$$

Equality hods if and only if n is odd and G is regular of degree $(n-1)/2$.

Theorem 3.7 ([49])**.**

$$SO(G) \cdot SO(\overline{G}) \geq \frac{2m^2\,[n(n-1) - 2m]^2}{n^2}\,.$$

Equality holds if and only if G is regular. If the graph G is unicyclic, then

$$SO(G) \cdot SO(\overline{G}) \geq 2n^2(n-3)^2\,.$$

Graphs with extremal values of the Sombor index were characterized in [6, 20, 21, 24, 25, 26, 33, 34, 45, 52]. An elementary result along these lines is

Theorem 3.8 ([6]). *Let K_n, P_n, and S_n denote the complete graph, the path, and the star of order n. If G is a graph of order n, then*

$$SO(\overline{K_n}) \leq SO(G) \leq SO(K_n)\,.$$

Among connected graphs,

$$SO(P_n) \leq SO(G) \leq SO(K_n)\,.$$

If T is a tree of order n, then

$$SO(P_n) \leq SO(T) \leq SO(S_n)\,.$$

In [50], the graphs having the maximum Sombor index are characterized for the classes of connected unicyclic, bicyclic, tricyclic, tetracyclic, and pentacyclic graphs of a fixed order, and a conjecture is stated concerning the maximum Sombor index of graphs with higher cyclicity (sse also [24]).

In [27], the graphs whose Sombor indices are integers were established.

In line with the general theory of degree–based matrices [2, 12, 14], the "Sombor matrix" is defined as a symmetric square matrix of order n, whose (i,j)-element is

$$\mathbf{A}_{SO}(G)_{ij} = \begin{cases} \sqrt{d_i^2 + d_j^2} & \text{if } e_{ij} \in \mathbf{E}(G), \\ 0 & \text{if } e_{ij} \notin \mathbf{E}(G), \\ 0 & \text{if } i = j\,. \end{cases}$$

Spectral properties of this matrix, in particular its spectral radius and energy, were examined in [32, 42].

A variety of graph invariants were proposed, related to the Sombor index, or aimed at generalizing it. Of these we mention the "reduced Sombor index" [6, 47], Eq. (1.4), and its generalization [26]

$$\sum_{e_{ij} \in \mathbf{E}(G)} \sqrt{(d_i - \gamma)^2 + (d_j - \gamma)^2},$$

with γ being some real number (usually a positive integer), the "Banhatti–Sombor index" [35, 37, 39, 43]

$$\sum_{e_{ij} \in \mathbf{E}(G)} \sqrt{\frac{1}{d_i^2} + \frac{1}{d_j^2}}\,.$$

the "δ-Sombor index" [38]

$$\sum_{e_{ij}\in\mathbf{E}(G)} \sqrt{(d_i-\delta+1)^2+(d_j-\delta+1)^2},$$

with δ being the minimum vertex degree of the graph G, and the Nirmala index [11]

$$\sum_{e_{ij}\in\mathbf{E}(G)} \sqrt{d_i+d_j}\,.$$

In [52], the Minkowski distance was proposed to be ussed instead of Euclidean, that would result in the following modification of the Sombor index:

$$\sum_{e_{ij}\in\mathbf{E}(G)} \left(d_i^p+d_j^p\right)^{1/p},$$

where $p>0$. Evidently, for $p=1$ and $p=2$, this Minkowski–distance–based expression reduces, respectively, to the first Zagreb index and to the Sombor index.

It is easy to realize that infinitely many analogous Sombor–index–like graph invariants could be "invented". However, none of these (except the Minkowski–variant) has its origin in geometry–based reasoning, which is the true essence of the Sombor–index concept.

4. *More geometry–based considerations*

Let a and b be two edges of the graph G. These correspond to the degree-points A and B with coordinates (x_a,y_a) and (x_b,y_b). We now study the area of the triangle ABO formed by A, B, and the origin O, see Fig. 1.

Theorem 4.1. *The area of the triangle formed by the degree-points A and B with coordinates (x_a,y_a) and (x_b,y_b), and the origin O (cf. Fig. 1), satisfies the relation*

$$\mathcal{A}rea(ABO)=\frac{1}{2}\left|x_a\,y_b-x_b\,y_a\right|. \tag{4.1}$$

PROOF. Consider Fig. 2. Assuming that $y_a\geq y_b$, we see that

$$\begin{aligned}\mathcal{A}rea(ABO) &= \mathcal{A}rea(ABB'A')+\mathcal{A}rea(AA'O)-\mathcal{A}rea(BB'O)\\ &= \frac{1}{2}(x_b-x_a)(y_a+y_b)+\frac{1}{2}x_a\,y_a-\frac{1}{2}x_b\,y_b\\ &= \frac{1}{2}(x_b\,y_a-x_a\,y_b)\,.\end{aligned}$$

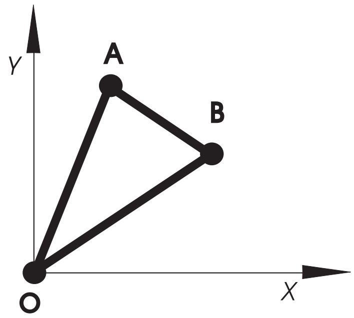

Figure 1: A triangle whose area leads to the design of new geometry–motivated vertex–degree–based graph invariants.

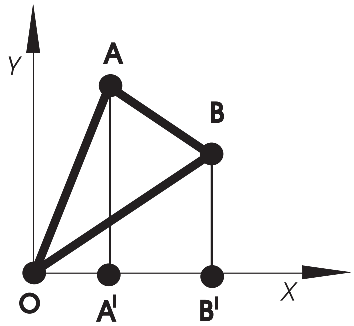

Figure 2: A diagram used in the proof of Theorem 4.1.

If $y_a < y_b$, then an analogous reasoning would give $\mathcal{A}rea(ABO) = \frac{1}{2}\,(x_a\,y_b - x_b\,y_a)$. In both cases, Eq. (4.1) holds.

Note that if B is the dual of the degree-point A, then the right–hand side of Eq. (4.1) reduces to $\frac{1}{2}\,|x_a^2 - y_a^2|$, which will lead to the below Corollary 4.1. If B is the average degree-point, with coordinates $(2m/n, 2m/n)$, then the right–hand side of (4.1) is equal to $\frac{2m}{n}\,|x_e - y_e|$. This will lead to Corollary 4.2 below.

In [6], it was shown that the sum over all edges $e \in \mathbf{E}(G)$ of the distance between a degree-point $A = (x_e, y_e)$ and its dual $\overline{A} = (y_e, x_e)$ is equal to $\sqrt{2}\,Alb(G)$, where $Alb(G)$ is the Albertson index, Eq. (3.1).

Corollary 4.1. *Using the above specified notation, let $A\overline{A}O$ be the triangle pertaining to the edge $e = e_{ij}$. Then*

$$\sum_{e \in \mathbf{E}(G)} \mathcal{A}rea(A\overline{A}O) = \frac{1}{2} \sum_{e_{ij} \in \mathbf{E}(G)} \left| d_i^2 - d_j^2 \right| . \tag{4.2}$$

The right–hand side of Eq. (4.2) should be compared with Eq. (3.1). As seen, it is a modification of the Albertson index, and may be viewed an a novel measure for graph irregularity, cf. [3, 5, 13].

In [6], the sum over all edges $e \in \mathbf{E}(G)$ of the distance between a degree-point $A = (x_e, y_e)$ and the average degree-point $A_{avr} = (2m/n, 2m/n)$ is defined as the average Sombor index, Eq. (1.3). In parallel to this, we now have:

Corollary 4.2. *Using the above specified notation, let $AA_{avr}O$ be the triangle pertaining to the edge $e = e_{ij}$. Then*

$$\sum_{e \in \mathbf{E}(G)} \mathcal{A}rea(AA_{avr}O) = \frac{1}{2}\,\frac{2m}{n} \sum_{e_{ij} \in \mathbf{E}(G)} |d_i - d_j| = \frac{m}{n}\,Alb(G),$$

i.e.,

$$Alb(G) = \frac{n}{m} \sum_{e \in \mathbf{E}(G)} \mathcal{A}rea(AA_{avr}O)\,.$$

The latter equality in Corollary 4.2 offers a new interpretation of the Albetrson index.

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