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Anti fuzzy ideal extension of $\Gamma-$semiring

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Abstract. In this paper the concept of anti fuzzy prime ideal, anti fuzzy semi prime ideal ,anti fuzzy ideal extension in a $\Gamma-$semiring have been introduced. We obtain a characterization of a prime ideal of a $\Gamma-$semiring in terms of anti fuzzy ideal extension of complement of its characteristic function.

1. Introduction

The notion of semiring was introduced by H. S. Vandiver [**14**] in 1934.The notion of $\Gamma-$ring was introduced by N. Nobusawa [**10**] as a generalization of ring in 1964. M. K. Sen [**13**] introduced the notion of $\Gamma-$semigroup in 1981. The notion of ternary algebraic system was introduced by Lehmer [**5**] in 1932, Lister [**6**] introduced ternary ring. Dutta & Kar [**1**] introduced the notion of ternary semiring which is a generalization of ternary ring and semiring. In1995, M. Murali Krishna Rao [**7**] introduced the notion of $\Gamma-$semiring which is a generalization of $\Gamma-$ring, ternary semiring and semiring. After the paper [**7**] published, many mathematicians obtained interesting results on $\Gamma-$semiring. L. A. Zadeh [**16**] introduced the notion of a fuzzy subset μ of a set X as a function from X into [0,1]. The concept of fuzzy subgroup was introduced by A. Rosenfeld [**11**]. In 2001, X. Y. Xie [**15**] introduced the notion of extension of fuzzy ideal in semigroups. In 2009, M. Shabir and Y. Nawaz [**12**] M. Khan and T. Asif [**4**] introduced the notion of an anti fuzzy ideal in semigroups. Zhan, Dudek , Jun contributed a lot of theory of fuzzy semiring. In 2011, T. K. Dutta et .al [**2**] introduced the notion of fuzzy ideal extension in a $\Gamma-$semiring . In this paper the concept of anti fuzzy prime ideal , anti fuzzy semiprime ideal, anti fuzzy ideal extension in a $\Gamma-$semiring have been introduced and obtained a characterization of a prime ideal of a $\Gamma-$semiring in terms of anti fuzzy ideal extension of complement of its characteristic function. Anti fuzzy prime

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ideal, anti fuzzy semi prime ideal and anti fuzzy $k-$ideal are preserved by anti fuzzy ideal extension.

2. Preliminaries

In this section we recall some of the fundamental concepts and definitions, which are necessary for this paper.

DEFINITION 2.1. A set R together with two associative binary operations called addition and multiplication (denoted by $+$ and $\cdot$ respectively) will be called a semiring provided

(i). Addition is a commutative operation
(ii). Multiplication distributes over addition both from the left and from the right .
(iii). There exists $0 \in R$ such that $x+0=x$ and $x \cdot 0 = 0 \cdot x = 0$ for each $x \in R$

DEFINITION 2.2. Let M and Γ be additive abelian groups. If there exists a mapping $M \times \Gamma \times M \to M$ (images to be denoted by $x\alpha y, x, y \in M, \alpha \in \Gamma$) satisfying the following conditions for all $x, y, z \in M, \alpha, \beta \in \Gamma$

(i). $x\alpha(y\beta z) = (x\alpha y)\beta z$
(ii). $x\alpha(y+z) = x\alpha y + x\alpha z$
(iii). $x(\alpha+\beta)y = x\alpha y + x\beta z$
(iv). $(x+y)\alpha z = x\alpha z + y\alpha z$

Then M is called a $\Gamma-$ring.

DEFINITION 2.3. Let $(M, +)$ and $(\Gamma, +)$ be commutative semigroups. Then we call M as a $\Gamma-$semiring, if there exists a mapping $M \times \Gamma \times M \to M$ is written (x, α, y) as $x\alpha y$ such that it satisfies the following axioms for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$

(i). $x\alpha(y\beta z) = (x\alpha y)\beta z$
(ii). $x\alpha(y+z) = x\alpha y + x\alpha z$
(iii). $x(\alpha+\beta)y = x\alpha y + x\beta z$
(iv). $(x+y)\alpha z = x\alpha z + y\alpha z$

We illustrate the definition of $\Gamma-$semiring by the following examples.

EXAMPLE 2.1. Every semiring M is a $\Gamma-$semiring with $\Gamma = M$ and ternary operation as the usual semiring multiplication.

EXAMPLE 2.2. Let M be the additive seimgroup of all $m \times n$ matrices over the set of non negative rational numbers and Γ be the additive semigroup of all $n \times m$ matrices over the set of non negative integers, then with respect to usual matrix multiplication M is a $\Gamma-$semiring.

EXAMPLE 2.3. Let S be a semiring and $Mp; q(S)$ denote the additive abelian semigroup of all $p \times q$ matrices with identity element whose entries are from S. Then $Mp, q(S)$ is a $\Gamma-$semiring with $\Gamma = Mp, q(S)$ ternary operation is defined by $x\alpha z = x(\alpha')z$, with respect to usual matrix multiplication, where α' denote the transpose of the matrix α, for all x, y and $\alpha \in Mp, q(S)$.

EXAMPLE 2.4. Let X and Y be abelian semigroups with identity element. Let $M = Hom(X,Y), \Gamma = Hom(Y,X)$ and $\forall a,b \in M, \alpha \in \Gamma$. Define $a\alpha b$ be the usual composition map. Then M is a $\Gamma-$semiring.

DEFINITION 2.4. A $\Gamma-$semiring M is said to have zero element if there exists an element $0 \in M$ such that $0 + x = x = x + 0$ and $0\alpha x = x\alpha 0 = 0, \forall x \in M$.

DEFINITION 2.5. A $\Gamma-$semiring M is said to be a commutative $\Gamma-$semiring if $x\alpha y = y\alpha x, \forall x,y \in M$ and $\alpha \in \Gamma$.

DEFINITION 2.6. A subset A of $\Gamma-$semiring M is a left (right) ideal of M if A is an additive semigroup of M and the set $M\Gamma A = \{x\alpha y \mid x \in M, \alpha \in \Gamma, y \in A\}(A\Gamma M)$ is contained in A. If A is both left and right ideals then A is an ideal of M.

DEFINITION 2.7. An ideal I of a $\Gamma-$semiring M is called a $k-$ideal, if $b \in M, a + b$ and $a \in I$ then $b \in I$.

DEFINITION 2.8. Let S be a nonempty set, a mapping $f : S \to [0,1]$ is called a fuzzy subset of S.

DEFINITION 2.9. Let A be a nonempty subset of S. The characteristic function of A is a fuzzy subset of S is defined by

$$\chi_A(x) = \begin{cases} 1, & \text{if } x \in A; \\ 0, & \text{if } x \notin A. \end{cases}$$

The complement of characteristic function is denoted by χ_A^c.

DEFINITION 2.10. Let f be a fuzzy subset of S, for $t \in [0,1]$ the set $f_t = \{x \in S \mid f(x) \geqslant t\}$ is called level subset of S with respect to f.

DEFINITION 2.11. A fuzzy subset μ of a $\Gamma-$semiring M is called fuzzy left(right) of M if it satisfies

$$\mu(x+y) \geqslant min\{\mu(x), \mu(y)\}, \mu(x\alpha y) \geqslant \mu(y)\{\mu(x\alpha y) \geqslant \mu(x)\}, \forall x,y \in M, \alpha \in \Gamma.$$

If μ is a fuzzy left (right) ideal of $\Gamma-$semiring M then $\mu(0) \geqslant \mu(x), \forall x \in M$.

DEFINITION 2.12. A fuzzy subset f of $\Gamma-$semiring M is called fuzzy ideal of M, if $\forall x,y \in M, \alpha \in \Gamma$,

$$f(x+y) \geqslant min\{f(x), f(y)\}, f(x\alpha y) \geqslant max\{f(x), f(y)\}$$

DEFINITION 2.13. A fuzzy subset μ of a $\Gamma-$semiring M is called an anti fuzzy ideal left (right) ideal of M if,

$$\mu(x+y) \leqslant max\{\mu(x), \mu(y)\}, \mu(x\alpha y) \leqslant \mu(y)\{\mu(x\alpha y) \leqslant \mu(x)\}, \forall x,y \in M, \alpha \in \Gamma.$$

If μ is an anti fuzzy left (right) ideal of $\Gamma-$semiring M then $\mu(0) \leqslant \mu(x), \forall x \in M$.

DEFINITION 2.14. A fuzzy subset μ of a $\Gamma-$semiring M is called an anti fuzzy ideal of M if μ is both an anti fuzzy left and anti fuzzy right ideal of M.

DEFINITION 2.15. The complement of a fuzzy subset μ of a $\Gamma-$semiring M is denoted by μ^c and is defined as $\mu^c(x) = 1 - \mu(x), \forall x \in M$.

Definition 2.16. A fuzzy ideal f of a $\Gamma-$semiring M with zero 0 is said to be a $k-$fuzzy ideal of M if $f(x+y) = f(0)$ and $f(y) = f(0) \Rightarrow f(x) = f(0), \forall x, y \in M$.

Definition 2.17. A fuzzy ideal f of a $\Gamma-$semiring M is said to be a fuzzy $k-$ideal of M if $f(x) \geqslant min\{f(x+y), f(y)\}, \forall x, y \in M$.

Definition 2.18. Let M be a $\Gamma-$semiring. A fuzzy ideal μ of M is said to be an anti fuzzy-$k-$ideal of M if $\mu(x) \leqslant max\{\mu(x+y), \mu(y)\}$.

Definition 2.19. Let M be a $\Gamma-$semiring. Let μ be an anti fuzzy ideal of $\Gamma-$semiring M, for any $t \in [0,1], \mu_t$ is defined by $\mu_t = \{x \in M \mid \mu(x) \leqslant t\}$ then μ_t is called an anti level subset.

3. Main results:

In this section the concept of anti fuzzy prime ideal, anti fuzzy semi prime ideal, anti fuzzy ideal extension in a $\Gamma-$semiring have been introduced.

Definition 3.1. Let μ be a fuzzy subset of a $\Gamma-$semiring M and $x \in M$. Then the fuzzy subset $\langle x, \mu \rangle : M \to [0,1]$ is defined by $\langle x, \mu \rangle(y) = \sup_{\alpha \in \Gamma} \mu(x\alpha y)$, for all $y \in M$, is called an extension of μ by x.

Definition 3.2. Let μ be a fuzzy subset of a $\Gamma-$semiring M. Then μ is called an anti fuzzy prime ideal if $\mu(x\alpha y) = min\{\mu(x), \mu(y)\}$, for all $x, y \in M, \alpha \in \Gamma$.

Definition 3.3. Let μ be an anti fuzzy ideal of a $\Gamma-$semiring M. Then μ is called an anti fuzzy semi prime ideal if $\mu(x) \leqslant \mu(x\alpha x)$, for all $x \in M$.

Definition 3.4. Let μ be a fuzzy subset of a $\Gamma-$semiring M. We define anti support $\mu = \{x \in M \mid \mu(x) < 1\}$.

Definition 3.5. Let M be a $\Gamma-$semiring $A \subseteq M.x \in M$. We define

$$\langle x, A \rangle = \{y \in M \mid x\alpha y \in A, \forall \alpha \in \Gamma\}.$$

Theorem 3.1. *Let A be a non empty subset of a $\Gamma-$semiring M. If a fuzzy subset μ in M such that*

$$\mu(x) = \begin{cases} 0, & if\ x \in A \\ 1, & if\ x \notin A\ . \end{cases}$$

Then μ is an anti fuzzy ideal of M if and only if A is an ideal of M.

Proof. Suppose μ is an anti fuzzy ideal of $\Gamma-$semiring M.

Let $x, y \in A$.

$\Rightarrow \mu(x) = 0, \mu(y) = 0, \mu(x+y) \leqslant max\{\mu(x), \mu(y)\} = 0$

$\Rightarrow x + y \in A$.

Let $x, y \in M, \alpha \in \Gamma$.

$\Rightarrow \mu(x\alpha y) \leqslant min\{\mu(x), \mu(y)\} = 0$

$\Rightarrow x\alpha y \in A$.

Hence A is an ideal of $\Gamma-$semiring M.

Conversely let $x, y \in M, \alpha \in \Gamma, A$ be an ideal of M.

Case(1): If $x, y \in A, \mu(x) = 0, \mu(y) = 0, \mu(x+y) = 0, x\alpha y \in A \Rightarrow \mu(x\alpha y) = 0$, then $\mu(x+y) \leqslant max\{\mu(x), \mu(y)\}, \mu(x\alpha y) \leqslant min\{\mu(x), \mu(y)\}$

Case(2): If $x, y \notin A$ then $x + y \notin A. \mu(x) = 1, \mu(y) = 1, \mu(x+y) = 1, x\alpha y \in A \Rightarrow \mu(x\alpha y) = 0$ then $\mu(x+y) \leqslant max\{\mu(x), \mu(y)\}, \mu(x\alpha y) \leqslant min\{\mu(x), \mu(y)\}$

Case(3): If $x \in A, y \notin A$ then $x + y \in A, \mu(x) = 0, \mu(y) = 1, \mu(x+y) = 1, x\alpha y \in A \Rightarrow \mu(x\alpha y) = 0$ then $\mu(x+y) \leqslant max\{\mu(x), \mu(y)\}, \mu(x\alpha y) \leqslant min\{\mu(x), \mu(y)\}$

Case(4): If $y \in A, x \notin A$ then $x + y \notin A, \mu(x) = 1, \mu(y) = 0, \mu(x+y) = 0, x\alpha y \in A \Rightarrow \mu(x\alpha y) = 0$ then $\mu(x+y) \leqslant max\{\mu(x), \mu(y)\}, \mu(x\alpha y) \leqslant min\{\mu(x), \mu(y)\}$

Therefore μ is an anti fuzzy ideal of M. □

THEOREM 3.2. *Let μ be an anti fuzzy ideal of a commutative $\Gamma-$semiring M. Then the following are equivalent*

(1). *μ is an anti fuzzy semi prime ideal*

(2). *$\mu(x) = \mu(x\alpha x)$ for all $x \in M, \alpha \in \Gamma$*

PROOF. (2) $\Rightarrow$ (1) is obvious. Suppose μ is an anti fuzzy semi prime ideal. By definition 3.3 , we have $\mu(x) \leqslant \mu(x\alpha x)$ for all $x \in M, \alpha \in \Gamma$, since μ is an anti fuzzy ideal of a $\Gamma-$semiring M. $\mu(x\alpha x) = \mu(x)$. Hence $\mu(x) = \mu(x\alpha x)$ for all $x \in M, \alpha \in \Gamma$. Hence the theorem. □

The following proof of the theorem is a straight forward verification.

THEOREM 3.3. *Let μ be a non empty fuzzy subset of $\Gamma-$semiring M. Then μ is an anti fuzzy prime ideal of a $\Gamma-$semiring M if and only if $\mu_t$is a prime ideal of a $\Gamma-$semiring M for any $t \in Im(\mu)$ where μ_t is defined by $\mu_t = \{x \in M \mid \mu(x) \leqslant t\}$.*

THEOREM 3.4. *Let μ be an anti fuzzy right ideal of a $\Gamma-$semiring M. Then $\langle x, \mu\rangle$ is an anti fuzzy right ideal of M.*

PROOF. Let $z, y \in M, \alpha \in \Gamma$. Then

$$\begin{aligned}
\langle x, \mu\rangle(y+z) &= \sup_{\alpha\in\Gamma} \mu\big(x\alpha(y+z)\big) \\
&= \sup_{\alpha\in\Gamma} \mu(x\alpha y + x\alpha z)) \\
&\leqslant \sup_{\alpha\in\Gamma} max\{\mu(x\alpha y), \mu(x\alpha y)\} \\
&= max\left\{\sup_{\alpha\in\Gamma} \mu(x\alpha y), \sup_{\alpha\in\Gamma} \mu(x\alpha y)\right\} \\
&= max\left\{\langle x, \mu\rangle y, \langle x, \mu\rangle z)\right\} \\
\langle x, \mu\rangle(y\alpha z) &= \sup_{\beta\in\Gamma} \mu\big(x\beta(y\alpha z)\big) \\
&= \sup_{\beta\in\Gamma} \mu\big((x\beta y)\alpha z)\big) \\
&\leqslant \sup_{\beta\in\Gamma} \mu\big(x\beta y\big) = \langle x, \mu\rangle y.
\end{aligned}$$

Hence $\langle x, \mu\rangle$ is an anti fuzzy right ideal of $\Gamma-$semiring M. □

Corollary 3.1. *Let μ be an anti fuzzy ideal of a commutative $\Gamma-$semiring M and $x \in M$. Then the extension $\langle x, \mu\rangle$ is an anti fuzzy ideal of $\Gamma-$semiring M.*

Theorem 3.5. *Let μ be an anti fuzzy prime ideal of a $\Gamma-$semiring M and $x \in M$. Then $\langle x, \mu\rangle$ is an anti fuzzy prime ideal of M.*

Proof. Let $x, y \in M, \beta \in \Gamma$. Then

$$\begin{aligned}
\langle x, \mu\rangle(y\beta z) &= \sup_{\alpha\in\Gamma} \mu\big(x\alpha(y\beta z)\big) \\
&= \sup_{\alpha\in\Gamma} min\,\{\mu(x), \mu(y\beta z)\} \\
&= \sup_{\alpha\in\Gamma} min\big\{\mu(x), min\{\mu(y), \mu(z)\}\big\} \\
&= \sup_{\alpha\in\Gamma} min\big\{min\{\mu(x), \mu(y)\},\ min\{\mu(x), \mu(z)\}\big\} \\
&= \sup_{\alpha\in\Gamma} min\big\{\mu(x\alpha y), \mu(x\alpha z)\big\}, \\
&= min\big\{\sup_{\alpha\in\Gamma} \mu(x\alpha y), \sup_{\alpha\in\Gamma} \mu(x\alpha z)\big\}, \\
&= min\big\{\langle x, \mu\rangle y,\ \langle x, \mu\rangle z\big\}.
\end{aligned}$$

Hence $\langle x, \mu\rangle$ is an anti fuzzy prime ideal of $\Gamma-$semiring M. □

Theorem 3.6. *Let μ be a fuzzy subset of a commutative $\Gamma-$semiring M and $x \in M$ such that the extension $\langle x, \mu\rangle = \mu$ for every $x \in M$. Then μ is a constant function.*

Proof. Let μ be a fuzzy subset of a commutative $\Gamma-$semiring M and $x, y \in M$.

$$\begin{aligned}
\mu(x) &= \langle y, \mu\rangle x \\
&= \sup_{\alpha\in\Gamma} \mu(y\alpha x) \\
&= \sup_{\alpha\in\Gamma} \mu(x\alpha y) \\
&= \langle x, \mu\rangle y = \mu(y).
\end{aligned}$$

Hence $\mu(x) = \mu(y)$. Therefore μ is a constant fuzzy function. □

Theorem 3.7. *Let μ be a fuzzy subset of a commutative $\Gamma-$semiring M. Then for every $t \in Im(\mu), \langle x, \mu_t\rangle = \langle x, \mu\rangle_t$ for every $x \in M$.*

Proof.

$$\begin{aligned}
\text{Let } y \in \langle x, \mu\rangle_t &\Leftrightarrow \langle x, \mu\rangle y \leqslant t \\
&\Leftrightarrow \sup_{\alpha\in\Gamma} \mu(x\alpha y) \leqslant t \\
&\Leftrightarrow \mu(x\alpha y) \leqslant t \\
&\Leftrightarrow x\alpha y \in \mu_t \\
&\Leftrightarrow y \in \langle x, \mu_t\rangle,\ \text{by definition 3.5.}
\end{aligned}$$

Hence the theorem. □

THEOREM 3.8. *Let μ be an anti fuzzy semi prime ideal of a commutative $\Gamma-$semiring M and $x \in M$. Then $\langle x, \mu\rangle$ is an anti fuzzy semi prime ideal of a $\Gamma-$semiring M.*

PROOF. Let μ be an anti fuzzy semi prime ideal of a $\Gamma-$semiring M and $x, y \in M, \beta \in \Gamma$. By corollary3.1, the extension $\langle x, \mu\rangle$ is an anti fuzzy ideal of M. Then

$$\begin{aligned}
\langle x, \mu\rangle(y\beta y) &= \sup_{\alpha\in\Gamma} \mu(x\alpha y\beta y)\\
&\geqslant \sup_{\alpha\in\Gamma} \mu(x\alpha y\beta y\alpha x)\\
&= \sup_{\alpha\in\Gamma} \mu(x\alpha y\beta x\alpha y)\\
&\geqslant \sup_{\alpha\in\Gamma} \mu(x\alpha y)\\
&= \langle x, \mu\rangle y
\end{aligned}$$

Hence $\langle x, \mu\rangle$ is an anti fuzzy semi prime ideal of a $\Gamma-$semiring M. □

THEOREM 3.9. *Let M be a commutative $\Gamma-$semiring $\{S_i\}_{i\in I}x$ a non empty family of semi prime ideals of M and $A = \{\cap S_i\}_{i\in I} \neq \phi$ then $\langle x, \chi_A^C\rangle$ is an anti fuzzy semi prime ideal of M, for all $x \in M$ where χ_A^C is the complement of characteristic function of A.*

PROOF. Let $x \in M, \alpha \in \Gamma$ then $x\alpha x \in A \Rightarrow x\alpha x \in S_i$ for all $i \in I \Rightarrow x \in S_i$ for all $i \in I \Rightarrow x \in A$. Hence A is a semi prime ideal of a $\Gamma-$semiring M. By theorem 3.1, χ_A^c is an anti fuzzy semi prime ideal of a $\Gamma-$semiring M. Therefore by theorem 3.8, $\langle x, \chi_A^c\rangle$ is an anti fuzzy semi prime ideal of a $\Gamma-$semiring M. □

THEOREM 3.10. *Let μ be an anti fuzzy prime ideal of a $\Gamma-$semiring M and $x \in M$ such that $\mu(x) = \sup_{y\in M} \mu(y)$. Then $\langle x, \mu\rangle = \mu$.*

PROOF. Let μ be an anti fuzzy prime ideal of a $\Gamma-$semiring M and $x \in M$ such that $\mu(x) = \sup_{y\in M} \mu(y)$.

$$\begin{aligned}
\text{Let } z \in M &\Rightarrow \mu(x) \geqslant \mu(z)\\
&\Rightarrow min\{\mu(x), \mu(z)\} = \mu(z)\\
&\Rightarrow \sup_{\alpha\in\Gamma} \mu(x\alpha z) = \mu(z), \forall z \in M\\
&\Rightarrow \langle x, \mu\rangle z = \mu(z).
\end{aligned}$$

Therefore $\langle x, \mu\rangle = \mu$. □

THEOREM 3.11. *Let μ be an anti fuzzy ideal of a commutative $\Gamma-$semiring M and $x \in M$. Then we have the following*

(i). $\mu \supseteq \langle x, \mu\rangle$
(ii). $\langle (x\alpha)^n x, \mu\rangle \supseteq \langle (x\alpha)^{n+1} x, \mu\rangle, \forall x \in M, \alpha \in \Gamma$.
(iii). *If $\mu(x) < 1$ then anti supp $\langle x, \mu\rangle = M$.*

PROOF.

(i). Let $y \in M$. Then $\langle x, \mu\rangle(y) = \sup\limits_{\alpha\in\Gamma} \mu(x\alpha y) \leqslant \mu(y)$. Hence $\mu \supseteq \langle x, \mu\rangle$.

(ii). $\langle (x\alpha)^{n+1}x, \mu\rangle y = \sup\limits_{\beta\in\Gamma} \mu(x\alpha)^{n+1}x\beta y) \leqslant \sup\limits_{\alpha\in\Gamma} \mu(x\alpha)^{n}x\beta y$.
Hence $\langle (x\alpha)^{n}x, \mu\rangle \supseteq \langle (x\alpha)^{n+1}x, \mu\rangle$, for all $x \in M$.

(iii). Let $y \in M$. We have $\langle x, \mu\rangle(y) = \sup\limits_{\alpha\in\Gamma} \mu(x\alpha y) \leqslant \mu(x) < 1$, for all $y \in M$.
$\Rightarrow y \in$ anti supp $\langle x, \mu\rangle$, by definition 3.4. Hence anti supp $\langle x, \mu\rangle = M$. $\square$

THEOREM 3.12. *Let μ be an anti fuzzy prime ideal of a commutative $\Gamma-$semiring M. If μ is not constant then μ is not a minimal anti fuzzy prime ideal of a commutative $\Gamma-$semiring M.*

PROOF. By theorems 3.5 , 3.11, for each $x \in M, \langle x, \mu\rangle$ is an anti fuzzy prime ideal of M and $\langle x, \mu\rangle \subseteq \mu$. Since μ is not constant fuzzy subset by theorem 3.6, there exists $y \in M$ such that $\langle y, \mu\rangle$ is a proper subset of μ. Hence μ is not a minimal anti fuzzy prime ideal of a commutative $\Gamma-$semiring M. $\square$

THEOREM 3.13. *I is a prime ideal of $\Gamma-$semiring M if and only if χ_I^c is an anti fuzzy prime ideal of $\Gamma-$semiring M.*

PROOF. Suppose I is a prime ideal of $\Gamma-$semiring M and χ_I^c is the characteristic function of I. By theorem 3.1, χ_I^c is an anti fuzzy ideal of $\Gamma-$semiring M. Let $x, y \in M, \alpha \in \Gamma$ and $x\alpha y \in I$. Then $\chi_I^c(x\alpha y) = 0$. Since I is a prime ideal of $\Gamma-$semiring M. We have $x \in I$ or $y \in I, \Rightarrow \chi_I^c(x) = 0$ or $\chi_I^c(y) = 0$. Hence $\chi_I^c(x\alpha y) = min\{\chi_1^c(x), \chi_1^c(y)\} = 0$. Let $x\alpha y \notin I$. Since I is a prime ideal of $\Gamma-$semiring M. We have $x \notin I$ and $y \notin I$. $\chi_I^c(x) = 1, \chi_1^c(y)\} = 1, \chi_1^c(x\alpha y) = 1$. Hence $\chi_I^c(x\alpha y) = min\{\chi_I^c(x), \chi_I^c(y)\}$ Hence χ_I^c is an anti fuzzy prime ideal of $\Gamma-$semiring M.

Conversely χ_I^c is an anti fuzzy prime ideal of $\Gamma-$semiring M. Then χ_I is an fuzzy ideal of $\Gamma-$semiring $M \Rightarrow I$ is an ideal of $\Gamma-$semiring M. Let $x, y \in M, \alpha \in \Gamma$ such that $x\alpha y \in I$. Then $\chi_1^c(x\alpha y) = 0$. Suppose $x \notin I$ and $y \notin I, \chi_1^c(x\alpha y) = min\{\chi_1^c(x), \chi_1^c(y)\} = min\{1, 1\} = 1$. This is a contradiction to our assumption. Hence $x \in I$ or $y \in I$. Thus I is a prime ideal of $\Gamma-$semiring M. $\square$

THEOREM 3.14. *Let μ be an anti fuzzy ideal of a commutative $\Gamma-$semiring M. If for $y \in M, \mu(y)$ is not minimal in $\mu(M)$ and $\langle x, \mu\rangle = \mu$ then μ is an anti fuzzy prime ideal of a commutative $\Gamma-$semiring M.*

PROOF. Let $a, b \in M, \alpha \in \Gamma$ then $\mu(a\alpha b) \leqslant \mu(a)$ and $\mu(a\alpha b) \leqslant \mu(b)$.
Case(1): Let $\mu(a)$ be minimal in $\mu(M)$.
$\Rightarrow \mu(a\alpha b) = \mu(a)$ and $\mu(a\alpha b = \mu(a) = min\{\mu(a), \mu(b)\}$
Case(2): Neither $\mu(a)$ nor $\mu(b)$ is a minimal in $\mu(M)$ then by hypothesis $\langle a, \mu\rangle = \mu$

and $\langle b, \mu\rangle = \mu$. Hence $\langle a, \mu\rangle(b) = \mu(b)$ and $\langle b, \mu\rangle(a) = \mu(a)$

$$\Rightarrow \sup_{\alpha\in\Gamma} \mu(a\alpha b) = \mu(b) \text{ and } \sup_{\alpha\in\Gamma} \mu(b\alpha a) = \mu(a)$$
$$\Rightarrow \mu(b) \geqslant \mu(a\alpha b) \text{ and } \mu(a) \geqslant \mu(b\alpha a) = \mu(a\alpha b)$$
$$min\{\mu(a), \mu(b)\} \geqslant \mu(a\alpha b) \leqslant min\{\mu(a), \mu(b)\}.$$

Hence $\mu(a\alpha b) = min\{\mu(a), \mu(b)\}$. Therefore μ is an anti fuzzy prime ideal of a commutative $\Gamma-$semiring M. □

THEOREM 3.15. *I is a prime ideal of a $\Gamma-$semiring M if and only if $\langle x, \chi_1^c\rangle = \chi_1^c$ for $x \in M$ with $x \notin I$, where χ_1^c is the complement of the characteristic function of I.*

PROOF. Let I be a prime ideal of a commutative $\Gamma-$semiring M. By theorem 3.13, χ_1^c is an anti fuzzy prime ideal of $\Gamma-$semiring M for $x \in M$ with $x \notin I$, we have $\chi_1^c(x) = 1 = \sup_{x\in M} \chi_1^c(x)$. Hence theorem 3.10, $\langle x, \chi_1^c\rangle = \chi_1^c$.

Conversely suppose that $\langle x, \chi_1^c\rangle = \chi_1^c$ for $x \in M$ with $x \notin I$. We have χ_1^c is an anti fuzzy ideal of $\Gamma-$semiring M. Let $y \in M$ such that $\chi_1^c(y)$ is not minimal in $\chi_1^c(M)$ then $\chi_1^c(y) = 1 \Rightarrow y \notin I$. Hence, if $\langle y, \chi_1^c\rangle = \chi_1^c$ by theorem 3.14, χ_1^c is an anti fuzzy prime ideal of $\Gamma-$semiring M. □

THEOREM 3.16. *Let μ be an anti fuzzy $k-$ ideal of a commutative $\Gamma-$semiring M and $z \in M$. Then $\langle z, \mu\rangle$ is an anti fuzzy $k-$ ideal of a commutative $\Gamma-$semiring M.*

PROOF. Let μ be an anti fuzzy $k-$ ideal of a commutative $\Gamma-$semiring M and $x, y, z \in M, \alpha \in \Gamma$. Since μ is an anti fuzzy $k-$ ideal of a commutative $\Gamma-$semiring M. By corollary 3.1, the extension $\langle z, \mu\rangle$ is an anti fuzzy ideal of M.

We have $\mu(x) \leqslant max\{\mu(x + y), \mu(y)\}$, for all $x, y \in M$

$$\Rightarrow \mu(z\alpha x) \leqslant max\{\mu(z\alpha x + z\alpha y), \mu(z\alpha y)\}, \text{ for all } x, y \in M, \alpha \in \Gamma$$
$$\Rightarrow \sup_{\alpha\in\Gamma} \mu(z\alpha x) \leqslant max\left\{\sup_{\alpha\in\Gamma} \mu(z\alpha x + z\alpha y), \sup_{\alpha\in\Gamma} \mu(z\alpha y)\right\}$$
$$\Rightarrow \langle z, \mu\rangle(x) \leqslant max\{\langle z, \mu\rangle(x + y), \langle z, \mu\rangle(y)\}.$$

Therefore $\langle z, \mu\rangle$ is an anti fuzzy $k-$ideal of a commutative $\Gamma-$semiring M. □

References

[1] Dutta TK and Kar S., *On regular ternary semirings*, Advances in algebra proc. of the ICM Satellite conference in algebra and related topics, world sci. publ., Singapore; 2003; 205-213.

[2] Dutta TK, Sardar SK, Goswami S., *Fuzzy ideal extension in a $\Gamma-$semiring*, Int. Math. Forum, 6(18)(2011), 857-866.

[3] Jun Y, Hong SM, and Meng J., *Fuzzy interlior ideals in semigroups* , Ind. J. of Pure Appl. Math., 26(9)(1995), 859-863.

[4] Khan M, and Asif T., *Characterization of semigroups by their anti fuzzy ideals*, J. of Math. Reasearch, 2(3)(2010), 134-143.

[5] Lehmer H., *A ternary analogue of Abelian Groups*, American J. of Math., 59(1932), 329-338.

[6] Lister G. , *Ternary rings*, Trans. of American Math. Soc., 154(1971), 37-55.

[7] Murali Krishna Rao M., $\Gamma-$*semirings-I*, Southeast Asian Bull. of Math., 19(1)(1995), 49-54.
[8] Murali Krishna Rao M., $\Gamma-$*semirings-II*, Southeast Asian Bull. of Math., 21(3)(1997), 281-287.
[9] Murali Krishna Rao M., *The Jacobson radical of* $\Gamma-$*semiring*, Southeast Asian Bull. of Math., 23(1999), 127-134.
[10] Nobusawa N., *On a generalization of the Ring Theory*, Osaka J. Math., 1(1994), 81-89.
[11] Rosenfeld A., *Fuzzy groups*, J. Math. Appl., 1971; 35; 512-517.
[12] Shabir M, and Nawaz Y ., *Semigroups characterised by the properties of their anti fuzzy ideals*, J. of Advanced Research in pure math., 3(2009), 42-59.
[13] Sen MK., *On* $\Gamma-$*semigroup*, Proc. of International Conference of algebra and its application, Decker Publicaiton, New York, 1981 (pp. 301-308).
[14] Vandiver HS., *Note on a Single Type of Algebra in which the cancellation law of addition does not hold*, Bull. Amer. Math., 40(1934), 914-920.
[15] Xie XY., *Fuzzy ideals extension of semigroups*, Soochow J. Math., 27(2001), 125-138.
[16] Zadeh LA., *Fuzzy Sets*, Information control, 8(1965), 338-353.

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