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WEAK CONGRUENCE REPRESENTABILITY OF SUBORDERS AND DIRECT PRODUCTS

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Abstract. Here we treat the problem of representability of an algebraic lattice by the weak congruence lattice of an algebra, i.e. the lattice of all symmetric and transitive relations compatible with algebra. We prove that some suborders of the representable lattices are representable, and give conditions under which these suborders are also sublattices of the initial lattices. We also prove that the direct product of a set of representable lattices, slightly extended, is representable itself.

1. Introduction and preliminaries

We denote by $Sub\mathbb{A}$ the set of all subuniverses of an algebra $\mathbb{A}$, as well as the lattice it forms under inclusion. By $Con\mathbb{A}$ we denote the set of all congruences, as well as the corresponding lattice. We define a notion more general than congruence:

Definition 1.1. ([**4**]) Let $\mathbb{A}$ be an algebra, the support of which is A, and ρ a relation on A. We say that ρ is a weak congruence of $\mathbb{A}$ if it is symmetric, transitive and compatible with all the operations of $\mathbb{A}$, including nullary ones.

Compatibility with a nullary operation c means that $c\rho c$, so a weak congruence relation has the property of so called **weak reflexivity**, i.e. every nullary operation is in relation to itselves. The set of all weak congruences of $\mathbb{A}$ we denote by $Cw\mathbb{A}$. Notice the following: $\Delta = \{(x,x) \mid x \in A\}$ is a weak congruence, while for a subset B of A we have that $\Delta_B = \{(x,x) \mid x \in B\}$ is a weak congruence of $\mathbb{A}$ if and only if B is a subuniverse of $\mathbb{A}$.

Theorem 1.1. ([**5**]) *The collection $Cw\mathbb{A}$ of weak congruences on an algebra $\mathbb{A}$ is an algebraic lattice under inclusion.*

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The importance of the weak congruence lattice follows from the following theorem:

THEOREM 1.2. ([**9**]) *If* $Cw\mathbb{A}$ *is the lattice of weak congruences of an algebra* $\mathbb{A}$*, then:*

(i) *for every subalgebra* $\mathbb{B}$ *of* $\mathbb{A}$*,* $Con\mathbb{B}$ *is the interval sublattice* $[\Delta_B, B^2]$*, in particular* $Con\,\mathbb{A} = \uparrow\Delta$*;*

(ii) *the lattice* $Sub\,\mathbb{A}$ *of subuniverses of* $\mathbb{A}$ *is isomorphic with the principal ideal* $\downarrow\Delta$*, under* $\mathbb{B} \mapsto \Delta_B$*;*

(iii) *the map* $m_\Delta : \rho \mapsto \rho \wedge \Delta$ *is a homomorphism from* $Cw\mathbb{A}$ *onto* $\downarrow\Delta$*.*

So, the weak congruence lattice of an algebra contains the congruence lattice, as well as a lattice isomorphic to the subalgebra lattice of the same algebra. Those two lattices are a principal filter and a principal ideal generated by the same element, Δ.

The problem of the representability of an algebraic lattice by the weak congruence lattice of an algebra is a long standing, open problem. We say that an element a of a lattice L is Δ-suitable in L, if there is an algebra $\mathbb{A}$ whose weak congruence lattice is isomorphic to L under an isomorphism mapping Δ to a. We say that a lattice L and its element a are representable, if a is Δ-suitable in L.

In certain cases representability of a lattice imply the representability of another lattice. It could be a sublattice, or a suborder, or in another way related to the first lattice. We call the representability of a lattice which is derived from the representability of another lattice, the derived representability. In [**6**], several cases of derived representability are given. This paper contains another case when a suborder of a representable lattice, which is itself a lattice, is representable. Under some conditions this implies the derived representability of some sublattices. We also prove that, in some cases when we could expect that a sublattice of a representable lattice should be representable, this may not be the case.

Also, representability of a set of lattices may imply the representability of another lattice, derived from them in a way. We prove that the direct product of a set of representable lattices, slightly extended, is representable. This implies also that an extension of the direct product of a representable lattice and an arbitrary algebraic lattice is representable.

Some properties of the weak congruence lattice $Cw\mathbb{A}$ and its element Δ imply some necessary conditions for an element a of a lattice to be Δ-suitable. For example, Δ is codistributive, therefore any Δ-suitable element of a lattice is codistributive. A codistributive element of an algebraic lattice fulfills the following:

THEOREM 1.3. ([**5**]) *If an element* a *of an algebraic lattice* L *is codistributive, then for every* $b \in\downarrow a$*, the family* $\{x \in L \mid a \wedge x = b\}$ *has the top element.*

If L is an algebraic lattice and $x \in L$, we denote the top element of the family $\{y \in L \mid a \wedge y = a \wedge x\}$ by $\overline{x}$.

Some further conditions a codistributive element of an algebraic lattice must fulfill in order to be Δ-suitable are given in the following theorem, and they are based on the properties of the weak congruence lattice:

THEOREM 1.4. ([**6, 8, 9**]) *A Δ-suitable element $a \in L$ satisfies the following:*
(1) *if $x \wedge y \neq \mathbf{0}$ then $\overline{x \vee y} = \overline{x} \vee \overline{y}$;*
(2) *if $\overline{x} \neq \mathbf{0}$ and $\overline{x} < y$, then $\overline{y \wedge a} \neq y \wedge a$;*
(3) *If $\overline{x} \neq 0$ and $x < y \leqslant a$, then $[y \vee \overline{x}, \overline{y}) \setminus \bigcup_{z \in (x,y)} [y \vee \overline{z}, \overline{y})$ is either the empty set, or has the top element;*
(4) *If $\overline{x} \neq 0$, $x < y \leqslant a$, then there if a mapping $\varphi : [x, y) \to [y, \overline{y})$, such that:*
- for all $t \in [x, \overline{x}]$ and $u \in [x, y)$, the set $\{c \in \mathrm{Ext}^y(t) \mid c \leqslant \varphi(u)\}$ is either empty or has the top element, and
- for all $t \in [x, \overline{x}]$, the set $\{c \in \mathrm{Ext}^y(t) \mid (\forall u \in [x, y))(c \nleqslant \varphi(u))\}$ is an antichain (possibly empty), where

$$\mathrm{Ext}^y(t) := \{w \in [y, \overline{y}] \mid w \cap \overline{t} = t\}.$$

PROPOSITION 1.1. ([**6**]) *If a is a Δ-suitable element of a lattice L and x an element of L such that $x = \overline{x}$, then $a \wedge x$ is Δ-suitable in the lattice $\downarrow x$.*

THEOREM 1.5. ([**6**]) *If a is a Δ-suitable element of a lattice L and b a compact element of $\downarrow a$ and $d \in [b, \bar{b}]$, then a is a Δ-suitable element of the lattice $L' = L \setminus \cup\{(c, \bar{c}) \setminus [d \vee c, \bar{c}] | c \in [b, a]\}$, ordered by the order of L (L' is a subposet of L).*

2. Results

2.1. Representability of the suborder. The next theorem describes a case when the representability of a suborder of a lattice is deduced from the representability of that lattice.

THEOREM 2.1. *If a is a Δ-suitable element of a lattice L and $d \in \uparrow a$. Set $L' = \downarrow d \cup \{\bar{b} \mid b \leqslant a\}$ is a lattice under the order on L, element a being a Δ-suitable in L'.*

PROOF. Let $\mathbb{A} = (A, H)$ be an algebra such that $Cw\mathbb{A}$ is isomorphic to L in the isomorphism mapping Δ_A and ρ respectively to a and d. Let $\mathbb{O} = \{O_i \mid i \in I\}$ be the set of the congruence classes of ρ. For all $i \in I$ we define an operation of arity 3:

$$f_i(x, y, z) = \begin{cases} y & x \in O_i \\ z & x \notin O_i. \end{cases}$$

Now, let $\mathbb{A}' = (A, H \cup \{f_i \mid i \in I\})$. We prove that $Cw\mathbb{A}' \cong L'$:

Obviously, $Cw\mathbb{A}'$ is a subset of $Cw\mathbb{A}$, so it remains to see what weak congruences of $\mathbb{A}$ are also weak congruences of $\mathbb{A}'$. If $\tau \in Cw\mathbb{A}$, we have that $\tau \in Con\mathbb{B}$, for a subalgebra $\mathbb{B}$ of $\mathbb{A}$. If $\tau \subseteq \rho$, then τ is compatible with f_i for all $i \in I$, because if $x\tau x'$, $y\tau y'$ and $z\tau z'$, then $x\rho x'$, and we have that x and x' belong to the same class O_j, thus $f_j(x, y, z) = y\tau y' = f_j(x', y', z')$ and $f_i(x, y, z) = z\tau z' = f_i(x', y', z')$ for $i \neq j$, so $\tau \in Cw\mathbb{A}'$. If, on the contrary, $\tau \nsubseteq \rho$, then there exist x, x' such that $(x, x') \in \tau \setminus \rho$. If $x \in O_i$, then $x' \notin O_i$. If τ is compatible with operations f_i, then for any $y, z \in B$ we have $f_i(x, y, z)\tau f_i(x', y, z)$, i.e. $(\forall y, z \in B) y\rho z$, thus $\tau = B^2$.

Finally, if $\tau = B^2$, since B is a subuniverse of $\mathbb{A}$, it is also a subuniverse of $\mathbb{A}'$, so τ is obviously compatible with all the operations of $\mathbb{A}'$. $\square$

In the previous proof we did not have to prove that L', as it is defined in the theorem, is a lattice under the order on L, for it follows from the fact that poset $(L, \subseteq)$ is isomorphic to poset $(Cw\mathbb{A}', \subseteq)$. This lattice does not have to be a sublattice of the initial lattice, as it can be seen from the following example of the lattice and its suborder on Figure 1:

L $\qquad\qquad$ L'

Figure 1

Lattice L is representable by the algebra $\mathbb{A}$, where $\mathbb{A} = (\{a, b, c\}, b, g)$, where $g(a) = g(b) = a$; $g(c) = c$. Element d is represented by $\rho = \Delta \cup \{a, c\}^2$, so we add two relations f_1 and f_2 of arity 3 to get a representation of L. These relations are given as follows:

$f_1(a, x, y) = f_1(c, x, y) = x$, $f_1(b, x, y) = y$;

$f_2(a, x, y) = f_2(c, x, y) = y$, $f_2(b, x, y) = x$.

(Here it suffices to take either f_1 of f_2.)

Poset $(L', \leqslant)$ is a lattice under the order on L, but it is not a sublattice of L. However, under some conditions, set L' from the previous theorem will be closed under the operations in L, hence L' will be a sublattice of the initial lattice. Thus we come to a case when the representability of a sublattice is derived from the representability of the initial lattice.

Corollary 2.1. *Let a be Δ-suitable element of L and $d \in L$ an element such that $d \geqslant a$. Let L fulfill the following condition for any $c \in downarrowa$: if $b \in [c, a]$ and $b \vee \bar{c} < \bar{b}$, then $\bar{c} \leqslant d$. Then set $L' = \downarrow d \cup \{\bar{b} \mid b \leqslant a\}$ is closed under the operations of L, and element a is Δ-suitable in sublattice L' of L.*

Proof. On the basis of the previous theorem it would be enough to prove that set L' is closed under the operations in L.

Let $l, m \in L'$.

If $l \leqslant d$ and $m \leqslant d$, then $l \wedge m \leqslant l \vee m \leqslant d$, so that $l \wedge m$ and $l \vee m$ belong to L'.

If $l = \bar{c}$, $(c \leqslant a)$, let $b = m \vee l = m \vee \bar{c}$. The following holds:

$$\bar{b} \geqslant b \geqslant \bar{c} \vee (b \wedge a) \tag{2.1}$$

$$b \geqslant \bar{c} \geqslant c \Rightarrow (b \wedge a) \geqslant c \Rightarrow \bar{c} \vee (b \wedge a) \leqslant \bar{b}$$

Now, we have two subcases:

If $\bar{c} \vee (b \wedge a) = \bar{b}$, from (2.1) we get $\bar{b} \geqslant b \geqslant \bar{c} \vee (b \wedge a) = \bar{b} \Rightarrow b = \bar{b} \Rightarrow b \in L'$.

If, on the contrary, $\bar{c} \vee (b \wedge a) < \bar{b}$, then $\bar{c} \leqslant d$, by the given condition; if $m \leqslant d$, then $m \vee l = m \vee \bar{c} \leqslant d$; if $m \nleqslant d$, then $m = \bar{p}$, and by the same argument we get $b = \bar{b} \in L'$ or $\bar{p} \leqslant d$. Thus $m \vee l \in L'$ in any case.

On the other hand, $l \wedge m \leqslant d$ whenever $l \leqslant d$ or $m \leqslant d$. If $l \nleqslant d$ or $m \nleqslant d$, then $l = \bar{c}$ and $m = \bar{b}$, therefore $m \wedge l = \bar{n} \in L'$, $n = b \wedge c$. □

The following corollary is a more general result derived from the previous theorem and corollary, for it does not contain condition $d \geqslant a$.

Corollary 2.2. *If a is a Δ-suitable element of a lattice L and $d \in L$, then $d \wedge a$ is a Δ-suitable element of lattice $L' ={\downarrow} d \cup \{\bar{b} \mid b \leqslant d \wedge a\}$, whose order is the same as that on L. Lattice L' is also a sublattice of L, if the following implication holds in L: if $b \in [c, a \wedge d]$ and $b \vee \bar{c} < \bar{b}$, then $\bar{c} \leqslant d$.*

Proof. On the basis of Proposition 1.1, $d \wedge a$ is a Δ-suitable element of lattice $L_1 ={\downarrow}\bar{d}$. Now, applying Theorem 2.1 on lattice L_1 and its elements $d \wedge a$ and $d \geqslant d \wedge a$, we get that $d \wedge a$ is a Δ-suitable in lattice $L' = \{x \in L_1 \mid x \leqslant d\} \cup \{\bar{b} \mid b \leqslant d \wedge a\} = \{x \in L \mid x \leqslant d\} \cup \{\bar{b} \mid b \leqslant d \wedge a\}$, whose order is the same as that of lattices L_1 and L.

The second assertion of the corollary follows from the previous corollary. □

Combining Theorem 1.5 and Corollary 2.2 we get the following more general corollary:

Corollary 2.3. *If a is a Δ-suitable element of a lattice L and $b, c, d, e \in L$, such that $c \leqslant b \leqslant a$, $d \leqslant e$, $d \in [c, \bar{c}]$ and $e \in [b, \bar{b}]$, the set $L' = [d, e] \cup [c, b] \cup \{\overline{x} \mid x \in [c, b]\}$ is a lattice under the order on L, and b is a Δ-suitable element in the lattice. If the following implication holds: if $t \in [c, b]$ and $t \vee \bar{c} < \bar{t}$, then $\bar{c} \leqslant e$, then L' is a sublattice of L.*

Let L be a representable lattice and a its Δ-suitable element. Δ-suitability of an element b in a sublattice L' of L could be expected if the structure of the lattice under Δ-suitable element is preserved in the sublattice, i.e. if any equivalence class of $\sim$ defined on L' by $x \sim y \Leftrightarrow b \wedge x = b \wedge y$ is the intersection of a class of the equivalence ρ on L defined by $x \rho y \Leftrightarrow a \wedge x = a \wedge y$. Therefore a natural question arises whether the representability of a sublattice of L, consisting of some equivalence classes of ρ, could be derived from the representability of L. Given a lattice and a codistributive element a, we can get such a sublattice taking all the classes of any sublattice of ideal ${\downarrow} a$. We could expect that the Δ-suitability of element a in the initial lattice should imply its Δ-suitability in the sublattice, because the classes in this sublattice do not differ from those of the initial lattice, so that the whole structure is completely preserved. However, the representability of the initial lattice does not imply the representability of such a lattice in general. A "reason" for that is that the condition (4) of Theorem 1.4 may not be fulfilled in such a sublattice. This is illustrated in the following example.

EXAMPLE 2.1. Element a in the lattice in Figure 2 is Δ-suitable: let $\mathbb{A} = (A, \{f, g, h\})$, $A = \{a, b, c, d, e\}$ and let f, g, h be the unary operations given by the tables:

f	a	b	c	d	e
	c	c	a	c	e

g	a	b	c	d	e
	c	b	b	c	e

h	a	b	c	d	e
	a	b	c	d	d

We can notice that $Cw\mathbb{A}$ is isomorphic to the lattice in Figure 2, so a is a Δ-suitable element of the lattice L. But it is not Δ-suitable in the sublattice consisting of the classes of the top and the smallest element of $\downarrow a$, for the condition (4) of Proposition 1.4 is not fulfilled.

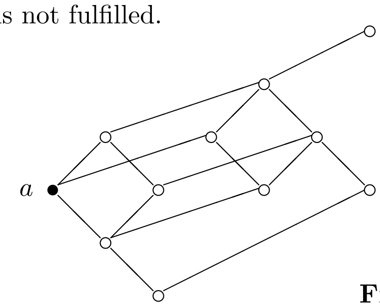

Figure 2

2.2. Representability connected with the direct product. Now, if we have a set of lattices, does the representability of all of them imply the representability of some other lattice, related to them? One such lattice is the direct product. We shall take any set of representable lattices, and from algebras representing them we shall form another algebra which will represent the direct product of those lattices, slightly enlarged.

Let $\Lambda = \{(L_i, a_i) \mid i \in I\}$ be a family of ordered pairs such that L_i is a lattice and a_i a Δ-suitable element of L_i, for every $i \in I$. Let L' be the lattice made of the lattice $L = \Pi L_i$ as follows:

(i) If there are at least two lattices L_i, L_j $(i, j \in I)$ that are, together with their elements a_i and a_j, represented by the weak congruence lattices of algebras, each having at least one constant, then for each element $b \in L, b \leqslant a$ we add an element b', such that $b' \wedge a = b$, b' is greater from all the elements of the set $\{x \in L \mid x \wedge a = b\}$ and the following inequalities hold:

$$x \leqslant b' \Leftrightarrow x \wedge a \leqslant b;$$
$$b' \leqslant x \Leftrightarrow (x = c' \text{ and } c \geqslant b).$$

(ii) If there is only one lattice L_j in the set $\{L_i \mid i \in I\}$ that is, together with its element a_j, representable by the weak congruence lattice of an algebra with at least one constant, then we take the lattice L' from (i) without the elements of the form l', where $l = (l_i)_{i \in I}$, $l_j \leqslant a_j$ and $l_i = 0$ whenever $i \neq j$.

(iii) If there is no lattice L_i representable, together with its element a_i, by the weak congruence lattice of an algebra having at least one constant, then we take lattice L', as in case (i) without the elements of the form l', where $l = (l_i)_{i \in I}$, $l \leqslant (a_i)_{i \in I}$ and there exists $j \in I$, such that $l_i = 0$ whenever $i \neq j$.

Lattice L' we call **extended direct product** of family Λ.

THEOREM 2.2. *If a_i is a Δ-suitable element of a lattice L_i, for all $i \in I$, then $a = (a_i)_{i \in I}$ is Δ-suitable in the extended direct product of the family of pairs $\{(L_i, a_i) \mid i \in I\}$.*

PROOF. Let $L_i \cong Cw\mathbb{A}_i$ for all $i \in I$ and $A_i \cap A_j = \emptyset$ whenever $i \neq j$. Let $A = \cup\{A_i \mid i \in I\}$.

Let f be an operation in the algebra $\mathbb{A}_i$, for an $i \in I$. In $f = c \in A_i$ is a constant, we take c to be the operation of A. If the arity of f is n, $n \geqslant 1$, we define an operation $\bar{f}$ on A in the following way:

$$\bar{f}(x_1, x_2, ..., x_n) = \begin{cases} f(x_1, x_2, ..., x_n), & x_1, x_2, ..., x_n \in A_i \\ x_1, & \text{else.} \end{cases}$$

Let g_i be an operation on A defined by the following equations:

$$g_i(x, y, z) = \begin{cases} y, & x \in A_i \\ z, & x \notin A_i. \end{cases}$$

Now, let $H = \{g_i \mid i \in I\} \cup \{\bar{f} \mid f \text{ be an operation in } \mathbb{A}_i, \text{for an } i \in I\}$ and let $\mathbb{A} = (A, H)$. We prove that $Cw\mathbb{A} \cong L'$:

For all $i \in I$ we have an isomorphism $\pi_i : Cw\mathbb{A}_i \to L_i$ mapping the diagonal relation Δ_{A_i} on a_i.

First, notice that any subuniverse B of $\mathbb{A}$ is a union of subuniverses B_i of $\mathbb{A}_i$ and vise versa, i.e. $Sub\mathbb{A} = \{\mathbb{B} \mid B = \cup_{i \in I} B_i, \mathbb{B}_i \leqslant \mathbb{A}_i\}$. Let $\mathbb{B}$ be a subalgebra, $B = \cup_{i \in I} B_i, \mathbb{B}_i \leqslant \mathbb{A}_i$. Notice that any $\rho \in Con\mathbb{B}$ is either equal to B^2 or equal to $\cup_{i \in I} \rho_i$, for some $\rho_i \in Con\mathbb{B}_i$. B^2 is itself of the form $\cup_{i \in I} \rho_i$, $\rho_i \in Con\mathbb{B}_i$, if and only if $B = B_j$, where B_j is a subuniverse of $\mathbb{A}_j$, for some $j \in I$ and algebras $\mathbb{A}_i$ are without constants for all $i \neq j$ (then $B_i = \emptyset$ and $\rho_i = \emptyset$ for $i \neq j$, $B_j = B$ and $\rho_j = B_j^2$). This will happen only if all the algebras $\mathbb{A}_i$ are without constants and B_j is a subuniverse of $\mathbb{A}_j$, or it is only algebra $\mathbb{A}_j$ that has a constant (at least one).

Now, we define $\pi : Cw\mathbb{A} \to L'$. If $\rho \in Con\mathbb{B}$, where $B = \cup_{i \in I} B_i, \mathbb{B}_i \leqslant \mathbb{B}$:

$$\pi(\rho) = \begin{cases} (\pi_i(\rho_i))_{i \in I}, & \rho = \cup_{i \in I} \rho_i \\ (\pi_i(\Delta_{b_i}))'_{i \in I} & \rho = B^2, \text{ where B isn't a subuniverse of } \mathbb{A}_i, \text{ for } i \in I. \end{cases}$$

Now, π is an isomorphism of lattices and

$$\pi(\Delta_A) = \pi(\cup_{i \in I} \Delta_{A_i}) = (\pi_i(\Delta_{A_i}))_{i \in I} = (a_i)_{i \in I} = a,$$

accordingly $Cw\mathbb{A} \cong L'$, and $a = (a_i)_{i \in I}$ is a Δ-suitable element of the lattice L'. $\square$

Since it's known that the least element of any algebraic lattice is Δ-suitable ([**5**]), we could derive the following corollary from the previous theorem:

COROLLARY 2.4. *Let a be a Δ-suitable element of a lattice L and L' an extension of L such that $L' = L \cup S$, where $S = \{s_b \mid b \leqslant a\}$ and*

$$x \leqslant s_b \Leftrightarrow x \wedge a \leqslant b;$$
$$s_b \leqslant x \Leftrightarrow (x = s_c \text{ and } b \leqslant c).$$

If M is an algebraic lattice, let $K = (L \times M) \cup \{(s,1) \mid s \in S\}$. K is a subuniverse of $L' \times M$. If L is represented by the weak congruence lattices of an algebra with at least one constant, then $(a,0)$ is Δ-suitable in the lattice K. If L is represented by the weak congruence lattices of an algebra without any constant, then $(a,0)$ is Δ-suitable in the lattice $K_1 = K \setminus \{(s_0,1)\}$, the order of which is the same as that of K.

In [**3**] a similar result was proved - that $L' \times M$ is representable, but it is not $(a,0)$ that was the corresponding Δ-suitable element, but $(a,1)$.

3. Conclusion

The results given here are certain contributions to the general, long standing open problem of representation of the lattices by the weak congruences:

Let L be an algebraic lattice and $a \in L$. Is there an algebra such that its weak congruence lattice is isomorphic with L, the diagonal relation being the image of a under the isomorphism?

Some necessary conditions for a lattice to be representable are given in [**10**], [**5**] and generalized in [**7**]. A sufficient condition is given in [**8**], and generalized in [**3**], where it was also proved that the class of atomic Boolean algebras is representable. Besides, we can prove representability in many particular cases. Starting from those representable lattices, using these results in derived representability we can get some other cases and some other classes of representable lattices. We can combine these few cases of derived representability and also some other cases which will be proved. We need not confine ourselves to the cases of suborders, sublattices or direct product of the lattices. We are interested in any construction which gives another representable lattice starting from a representable lattice, or a set of representable lattices, or even a set in which some of the lattices are representable.

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