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CONJUGACY IN PATTERN RECOGNITION AND CHOICE PROBLEMS

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Abstract: Common approaches to the construction and analysis of the conjugate problems of decision variant choice and of diagnosing such variants are proposed and investigated here. Practical applications of these approaches are provided. New models of pattern recognition and decision making problems are given. The models are based on immersional, existential and logical conjugacy for the initial problem of diagnostics and effective choice in the general form. The model of the multi-valued interpretation of contradictory information is also investigated.

Keywords: Conjugacy, duality, choice, diagnostics, interpretation, stability, immersion, pattern recognition.

1. INTRODUCTION

In this paper we consider general approaches to the construction of the conjugate problems of decision making and mathematical diagnostics. These approaches can produce the conjugate problems of classification and pattern recognition, and of decision variant choice, which may be useful for non-formalised and improper problems (such as contradictory decision making problems).

Conjugacy in decision making has become a more profound and substantional phenomenon than could be expected a priori. Conjugacy or duality traditionally arose from geometry (separability of sets) and mechanics (methods of variations and optimality conditions, and stability of mechanical objects). However, the conjugacy concept may be examined on more abstract logical principles (in particular, the negation operation principle). That the concept of conjugacy is fundamental can be seen from important results in mathematical programming and pattern recognition produced by algorithms for the analysis of conjugate choice and diagnostic problems.

2. THE COMMON IDEA OF CONJUGACY

Let Z be a diagnostic or choice problem, which can be formulated as: find an element $x \in M$ of some set M, that has as its particular cases the mathematical programming problem or pattern recognition problem, and let Z^* be a conjugate problem. We shall study the following properties of Z and Z^*:

- noncontradictoriness of the problem settings Z and Z^*;
- consistency of the constraint systems of these problems;
- solvability of problems Z and Z^*;
- stability of their solutions with respect to small variations in information or in problem setting;
- uniqueness of the solutions or boundedness of the solution sets.

If $Arg\,Z = 0$, $Arg\,Z^* = 0$, the correlation between generalised solution sets is investigated. For example, if Z is the problem of finding at least one solution of a linear inequality system, then, in the case of its inconsistency, relations between terminal (maximal consistent and minimal inconsistent) subsystems of problems Z and Z^* are considered [1].

As examples of the pair Z, Z^* we may consider classical dual problems of mathematical programming as well as the dual problems of pattern recognition proposed by the author.

This paper presents a short review of models of conjugate problems of effective choice of decision variants and their diagnosis in operations research.

3. GENERAL APPROACHES TO THE CONSTRUCTION OF GENERALIZED CONJUGATE PROBLEMS

We consider the following approaches to conjugation relations:

(i) Immersional approach

This approach originates in the nonstationary processes of decision making, modelling the technical, economic and natural systems, mathematical programming, and pattern recognition [2]. It consists of the immersion of problem Z into a neighbourhood $V(Z)$ of similar problems and the investigation of the properties of problem Z and its solution in this neighbourhood. For example, in paper [6], a formula is given for finding the derivative of $Arg\,Z$ along the direction of changing information in problem Z.

For example, let the problem of pattern recognition be reduced to the following linear programming problem:

$$L : \max \{ \langle c,x \rangle : Ax \le b, x \ge 0 \},\ x \in R^n.$$

The dual problem is:

$$L^* : \min \{ \langle b,u \rangle : A^T u \ge c, u \ge 0 \}.$$

We assume:

$$M = Arg\,L \ne \varnothing,\ M^* = Arg\,L^* \ne \varnothing,$$

$$opt\,(L) : \max \{ \langle c,x \rangle \} : Ax \le b,\ x \ge 0 \} = m(A,b,c).$$

Then the derivative of $m(A,b,c)$ along the direction $[A,b,c]$ is:

$$\frac{dM(A,b,c)}{d[A,b,c]} = \max_{x\in M}\ \min_{u\in M^*} \left[\, \langle c,x\rangle + \langle u, b-Ax\rangle \,\right].$$

(ii) Existential approach

This approach consists of establishing the conditions of dependence of some restrictions upon others, such as

- conditions of dependence and nonnegative linear dependence of left-hand sides of equations and inequalities [3];
- conditions of logical dependence, i.e. conditions for a relation to follow from a system of some other relations [4].

For example, let us consider the following pattern recognition problem: separate the sets $A=\{x:\langle c_j,x\rangle \le b_j,\ j=1,\dots,m\}$, $B=\{x:\langle d_i,x\rangle \le h_i,\ i=1,\dots,p\}$, by the function $f(x)=\langle c,x\rangle - a$. Then the inequality $\langle c,x\rangle - a \le 0$ depends on the system $\langle c_j,x\rangle - b_j \le 0,\ j=1,\dots,m,$ and the inequality $\langle -c,x\rangle + a \le 0$ depends on the system $\langle d_i,x\rangle - h_i \le 0,\ i=1,\dots,p$. On the basis of the Farkas theorem for linear inequalities, we may construct the following dual problem:

$$\langle c,x\rangle - a = \sum_{i=1}^{m} u_i\left[\, \langle c_i,x\rangle - h_i^{\cdot} \,\right] - u_0,$$

$$-\langle c,x\rangle + a = \sum_{i=1}^{p} v_i\left[\, \langle d_i,x\rangle - h_i \,\right] - v_0,$$

$$u_j \ge 0 \quad (j=1,\dots,m), \quad v_i \ge 0 \quad (i=0,1,\dots,p).$$

(iii) Logical conjugacy

This is a more general form of conjugacy: problem Z^* comprises all or the essential part of the formal consequences of statements of problem Z. Hence, the general logical scheme of conjugacy is as follows:

Problem Z: If $\exists x: A(x)$. This is a system of statements including variable vector $x=[x';x'']\in L = L'\times L''$. Problem Z^* : if $\exists x'': B(x'')=\phi(A(x)), x\in X, \phi\in\Phi$. Here $\phi(A)$ is a consequence of A. $A \Rightarrow \phi(A)$.

This scheme realizes the idea of quantor elimination of the contraction of all or part of the variables [5]. In the next section we present realizations in which the above-mentioned approaches are applied.

4. HEREDITARY EXISTENCE

Here conditions for the existence of solutions (as well as generalized solutions) to choice and diagnostic problems are considered.

The choice problem in the general form: for feasible set M of decision making variants we must find an element $x\in M$.

The diagnostic problem in the general forms is: if $M = \bigcup_{i=1}^{m} M_i, M_i \cap M_j = \varnothing \, (i \neq j)$, and classes M_i, are unknown, but we know the precedent sets $N_i \subset M_i \quad (i \in \{1,...,m\})$, then for $x \in M$ we must indicate the class M_i, with the property $x \in M_i$.

The conditions for the existence of solutions (generalized solutions) can be expressed as follows:

- in terms of linear dependence, nonnegative linear dependence or some other type of dependence;
- in terms of the consistency of subsystems as in Helly's theorem.

These conditions have been generalized to nonlinear systems and abstract restrictions of the form $x \in M_i$.

For example, in the case of choice problem $x \in M$ we may use two dual forms of describing the set M: if $M \subset R^n$ is the convex polyhedral set, then there exist vectors $c_j \quad (j \in \{1,...,m\})$ the numbers $b_j \quad (j \in \{1,...,m\})$ and the finite sets A, B such that:

$$M = coA + coneB = \{x : \langle c_j, x\rangle - b_j \leq 0, \quad (j \in \{1,...,m\})\}$$

Another example illustrating the case of generalized solutions for abstract restrictions is the p-committee [1]:

The p-committee (where $0 \leq p < 1$) for the system $x \in M_i, (i \in \{1,...,m\})$ on the class Y is a finite set $C \subset Y$ such that:

$$|\, C \cap M_i \,| > p|C| \; (\forall i),$$

where $|C|$ denotes the number of members of set C. The p-committee exists by the proposition that arbitrary s sets M_i have a non-empty intersection, and $\frac{s}{m} > p$.

5. SEQUENCE CONDITIONS

In this section the conditions which provide that some inequalities are the consequences of the system of some other inequalities are considered. The general form of conditions that provide $A \Rightarrow B$ is as follows: $B \in covA$, where cov symbolizes some type of a hull. From here, models of infinite sets discrimination and models of discriminating informally described sets can be obtained.

Let the feasible set $D \subset R^n$ be divided into two classes: $D = M \cup N$, where classes M and N are unknown or are described informally, but the precedent sets $M' \subset M, N' \subset N$ are given. Then we can establish the following relations:

$$x \in M' \Rightarrow f(x) > 0; f(y) > 0 \Rightarrow y \in M; x \in N' \Rightarrow f(x) < 0; f(y) < 0 \Rightarrow y \in N.$$

Here inequality $f(x) > 0$, as a consequence of inclusion $x \in M'$, can be expressed in terms of the conjugate problem, the form of which depends on the representation of M'. It can be easily done if M' is finite. In the case

$$M' = \{x : f_i(x) > 0 \quad (i \in I)\},$$

where I is a finite index set, the dependence of the inequality $f(x)>0$ on the system $f_i(x)>0 \quad (i \in I)$ can be expressed as

$$f(x)=\sum_{i \in I} u_i f_i(x), \quad u_i \geq 0 \quad (i \in I).$$

6. DISCRIMINATION OF SETS DEFINED PARAMETRICALLY AND THE CORRESPONDING CONJUGACY

In this section, a conjugacy scheme constructed with the help of an analog of the Lagrange function

$$\Phi(f,p)=f(x(p)),\ f=argDA(A,B),\ A=\{x(p): p \in P\},\ B=\{x(q): q \in Q\},$$

is considered. Here $DA(A,B,F)$ is the problem of discriminating sets A, B by a function belonging to class F. In this case the pair of conjugate problems is

$$\max_{f \in F} \ \min_{p \in P, q \in Q} \frac{f(x(p))-f(x(q))}{\|f\|}, \quad \min_{p,q} \ \max_{f} \frac{f(x(p))-f(x(q))}{\|f\|},$$

where the choice of the norm $\|f\|$ depends on the accepted substantial interpretation of the problem.

The links between problems of this type are considered in [6]: the saddle-point of function $\Phi(f,p)$ exists under some regularity conditions.

The problem of finding useful feature estimates can be reduced to linear programming as follows. Let

$$x=(x_1,\ldots,x_n) \in R^n$$

be the original vector describing recognised or classified objects, and let $\Phi_j(x)$ be the j-th possible secondary feature ($j \in \{1,\ldots,m\}$).

Now assume that a_{ij} represents the measurement costs for the i-th original feature (primary feature) under the condition that the j-th secondary feature Φ_j is used with the unit intensity. Here by intensity we mean a rather general notion which is interpreted in one way or another in each specific situation. In some class of applied problems it might be e.g. the frequency of applying the j-th feature.

Now, let b_i be the amount of resources needed for determination of the i-th primary feature. Finally, let c_j be the measurement costs for the j-th secondary feature on the condition that it is used with the unit intensity. Then we obtain the linear program:

minimize $c_1x_1+\ldots+c_mx_m$

subject to $a_{i1}x_1+\ldots+a_{im}x_m \leq b_i \quad (i \in \{1,\ldots,n\}), \quad x_1 \geq 0,\ldots,x_m \geq 0.$

Then the solution to the dual problem gives useful (from the point of view of contained information) estimates for the primary features, and the solution to the primal problem gives estimates for the secondary features.

In the next section we shall shortly discuss the application of the contraction method. Let the problem $DA(A,B,F)$ be reduced to the linear inequality system:

$$a_{j1}Z_1+\ldots+a_{jn}Z_n > 0 \quad (j \in \{1,\ldots,p\}),$$

where Z_i are the coefficients of the separating function. Then the contraction method can be used to find the committee solutions of the discriminant analysis problem.

7. NUMERICAL EXAMPLE

Now we will give a numerical example of finding the deadlock (minimal inconsistent and maximal consistent) subsystems of linear inequalities by the contraction method.

Let us consider the following system:

$$-x_1 + 2x_2 - 4x_4 < 1, \tag{1}$$
$$x_1 + x_2 + x_3 + x_4 < -2, \tag{2}$$
$$-5x_2 - x_4 < -6 \tag{3}$$
$$3x_1 + x_3 - x_4 < 0, \tag{4}$$
$$x_1 - x_2 + x_3 + 4x_4 < 3, \tag{5}$$
$$-x_3 < -1, \tag{6}$$
$$-x_1 - x_2 - x_4 < 0. \tag{7}$$

This system may be corresponded to the following discriminant analysis problem: Construct the discriminant (separating) function $f(a) = f(a_1,\ldots,a_4) = \sum_{i=1}^{4} x_i a_i$ for the vectors $a^1 = (-1,2,0,-4)$; $a^2 = (-1,-1,-1,-1)$; $a^3 = (0,5,0,1)$; $a^4 = (3,0,1,-1)$; $a^5 = (-1,1,-1,-4)$; $a^6 = (0,0,1,0)$; $a^7 = (-1,-1,0,-1)$.

For the discriminant function we require:

$$f(a^1) < 1, f(a^4) < 0, f(a^7) < 0, f(a^2) > 2, f(a^3) > 6, f(a^5) > -3, f(a^6) > 1.$$

It is, we separate the set $\{a^1, a^4, a^7\}$ from the set $\{a^2, a^3, a^5, a^6\}$, and we get the system (1) - (7).

Let us eliminate some variable, for example x^3. Then we obtain the x^3-contraction. To get this, we combine each pair of inequalities for which the signs of x^3 are opposite. A combination is obtained as follows: we multiply the inequalities with such positive numbers, that after the addition we obtain an inequality with coefficient of x^3 equal to zero. We enter inequalities with coefficient of x_3 equal to zero into x^3-contraction without change. The index on the left side indicates which inequalities were combined in order to get contraction inequalities.

Should the union of indices of two inequalities contain an index of some other inequality, such inequalities are not combined and the contraction is called a fundamental contraction.

Note that the contraction requires the solution of the dual system:

$$u_1(1)+\ldots+u_7(7)=0 \quad (\forall x_3),$$
$$u_i \geq 0 \quad (i \in \{1,\ldots,7\}),$$
$$u=(u_1,\ldots,u_7) \neq 0.$$

We obtain the fundamental x^3-contraction:

$$\begin{aligned}
-x_1+2x_2-4x_4 &< 1, && (1)\\
x_1+x_2+x_4 &< -3, && (2,6)\\
-5x_2-x_4 &< -6, && (3)\\
3x_1-x_4 &< -1, && (4,6)\\
x_1-x_2+4x_4 &< 2, && (5,6)\\
-x_1-x_2-x_4 &< 0. && (7)
\end{aligned}$$

If we eliminate x_1 from the x_3- contraction, then we obtain the (x_1,x_3)-fundamental contraction:

$$\begin{aligned}
3x_2-3x_4 &< -2, && (1,2,6)\\
-5x_2-x_4 &< -6, && (3)\\
6x_2-13x_4 &< 2, && (1,4,6)\\
x_2 &< 3, && (1,5,6)\\
0 &< -3, && (2,6,7)\\
-3x_2-4x_4 &< -1, && (4,6,7)\\
-2x_2+3x_4 &< 2. && (5,6,7)
\end{aligned}$$

After the elimination of x_4 we obtain the (x_1,x_3,x_4)-contraction:

$$\begin{aligned}
17x_2 &< -16, && (3,5,6,7)\\
x_2 &< 3, && (1,5,6)\\
0 &< -3, && (2,6,7)\\
-17x_2 &< 5. && (4,5,6,7)
\end{aligned}$$

After the elimination of x_2 we obtain the full fundamental contraction:

$$\begin{aligned}
0 &< 35, && (1,3,5,6,7)\\
0 &< -3, && (2,6,7)\\
0 &< 56. && (1,4,5,6,7)
\end{aligned}$$

And now we may use the Chernikov theorem [3]: the indices correspond to contradictory inequalities of a full fundamental contraction if and only if they correspond to minimal inconsistent subsystems. In our case we have the contradictory inequality 0<-3 with index (2,6,7). Hence, there exists only one minimal inconsistent subsystem with numbers (2), (6), (7) of inequalities of the initial system.

If $I=\{i_1,\ldots,i_q\}$ is the index set of a maximal consistent subsystem, then $I \not\supset \{2,6,7\}$, $I \subset \{1,\ldots,7\}$, and I is the maximal set with this property. In our example the index sets of maximal consistent subsystems are: $\{1,2,3,4,5,6\}$; $\{1,2,3,4,5,7\}$; $\{1,3,4,5,6,7\}$.

8. COMMITTEE CONJUGACY IN NONFORMALIZED PROBLEMS

The majority committee (a p-committee with p=1/2) for the system $f_j(x) \le b_j$, $(j \in \{1,\ldots,m\})$ is the set $C=\{x^1,\ldots,x^q\}$ with the property: each inequality of the system is satisfied by the majority of members of C. The following discrimination problem with separated sets given by precedents is considered: The sets $A' \subset A$, $B' \subset B$ are given. Determine A and B.

A discriminant committee is a set of separating functions; $\{f_1,\ldots,f_q\} \subset F$ is the committee of the system $f(a)>0\ (a \in A')$, $f(b)<0\ (b \in B')$, $f \in F$. The problem $DA(A,B,COM)$, where COM is the class of committee decision rules, is replaced by the problem $DA(A',B',COM)$. The conjugate system for the latter problem can be written as

$$\sum_{a \in A'} u_a \langle a,x\rangle - \sum_{b \in B'} u_b \langle b,x\rangle = 0, \quad u \ge 0.$$

When new vectors a,b are added to the sample precedent set, new variables u_a, u_b appear in the conjugate system which can be taken into account automatically as in the simplex method. The conjugate system makes it possible to find maximal noncontradictory subproblems for problems $DA(A,B,LIN)$ or $DA(A,B,AFF)$. Here LIN is the class of linear functions, and AFF is the class of affine functions. The solutions to these subsystems are used to construct the committee of linear or affine functions that makes the decisions on belonging to the class through the majority "voting" of these functions [1].

For example, the sets $A,B \subset R^2$:

$$A = \{(0,0),(1,1),(2,4),(6,0)\}, \quad B = \{(2,5/2),(0,7/2),(4,0),(4,7/2)\}.$$

are separated by the affine committee

$$C = \{\ \ (x) = -x_1+1, f_2(x) = -x_2+3, f_3(x) = x_1+x_2-5\}.$$

9. CONJUGACY IN THE SEPARATION OF INFINITE SETS

This conjugacy scheme is based on the ability to find the separation hyperplane equation

$$\langle c,x\rangle = b$$

as an implication of the inequality system defining the separation problem:

$$x \in M \Rightarrow \langle c,x\rangle < b;$$
$$x \in N \Rightarrow \langle c,x\rangle > b.$$

Consider the primal and conjugate interaction of sets in the problem of their separation by a certain function. Let E be a linear space over the real number field R:

$$E = E(R).$$

Elements $x \in E$ can be interpreted as object (or phenomenon or arbitrary process) state vectors. Let D be the set of feasible states in space E: $D \subset E$ which is characterized by the fact that if $x \notin D$, $x \in E$, then in the given specified problem vector x does not correspond to any actually possible state. This characterization makes it possible to describe D roughly through the use of discrimination analysis, i.e. by means of constructing a plane separating observed objects from D from observed objects from $E \setminus D$.

Suppose that there exists (but is not realized analytically) a subdivision of the feasible set into the classes: $\mathcal{D} = \mathcal{A} \cup \mathcal{B} \cup \mathcal{C}$. Here $\mathcal{A}$ and $\mathcal{B}$ are the basic classes, $\mathcal{C}$ is the boudary set separating A and B. Let $A \in \mathcal{A}$, $B \in \mathcal{B}$, $C \in \mathcal{C}$. Here A, B and C are actual or precedent sets (observation material).

Let us introduce an interaction function for the precedent sets as follows:

$$U(p,A,B,C,x)$$

is a display of the interaction of the sets A, B, C at some point $x \in E$. Here p is a vector parameter determining the specific choice of the interaction made from the class

$$\{U(p,A,B,C,x) : p \in P\}.$$

We assume that the interaction has no display on the boundary set C (i.e. it is neither positive nor negative here):

$$\exists p \in P : U(p,A,B,C,x) = 0 \quad (\forall x \in C). \tag{1}$$

This is a system with respect to p. Here in fact the problem of finding (identifying and adjusting) interaction function U is considered.

A particular case of system (1) is the model

$$DA(A,B,C,D,E,F)$$

of discriminating the sets A, B, C by the class F of separating functions (in this particular case the class of potential functions is meant). In the case

$$U(p,A,B,C,x)=\sum_{a\in A}u(p,a,x)-\sum_{b\in B}u(p,b,x)+\varepsilon\sum_{c\in C}u(p,c,x),\quad \varepsilon\geq 0.$$

Here $u(p,a,x)$ is interpreted as the potential created by point a at point x; ε is a sufficiently small number (it may be that $\varepsilon=0$); $u(p,a,x)\geq u(p,a,a)\geq 0$. In that case, by finding p in (1) we identify the function u.

If we assume that the unknown functions u in this problem are of the form

$$u(p,a,x)=v(a)f(p,a,x)=v(a)\phi_{a,p}(x), v(a)>0,$$

then it would appear natural to take the system

$$\exists x: f(p,a,x)>0 \quad (a\in A), f(p,b,x)<0 \quad (b\in B), p\in P \tag{2}$$

as a conjugate model. This is the problem

$$DA(A,B,F), F=\{f(p,a,x): p\in P, x\in X\}.$$

It is conjugated to problem (1) of finding interactions: in (1) p is found, i.e. u as a function of p, and in (2) x is found (adjusted).

By analogy with this example, in the general case the problem of finding x such as:

$$\frac{1}{v(c)}=U(p,A,B,C,x)\begin{cases}>0 & (a\in A),\\ <0 & (b\in B).\end{cases}$$

will be called the conjugate of (1). Here left-hand sides are functions of a and b.

So, (1) is the original problem:

Find $p\in P: U(p,A,B,C,x)=0 \quad (\forall x\in C)$. Here the potential created at each point x is adjusted. Let us agree to use symbol $\exists$ in place of the word "find" (which is in agreement with the gist of the matter):

$$\exists p\in P, \forall x\in C: U(p,A,B,C,x)=0. \tag{3}$$

Here the interaction or the potential created at each point $x\in E$ is adjusted (through the choice of vector parameter p).

The conjugate problem is

$$\exists x \in X, \forall \tilde{p} \in \tilde{P} \subset P : \frac{1}{v(a)} U(\tilde{p}, A, B, C, x) \begin{cases} > 0 & (a \in A), \\ < 0 & (b \in B). \end{cases} \tag{3*}$$

Here a point x is chosen and sets A and B are separated according to the interaction value at x in the given region $\tilde{P}$.

This is the interpretation of conjugated discrimination problems (3) and (3*) by means of interaction functions.

10. CONJUGACY IN THE PROBLEM OF THE CHOICE OF FEATURE SPACE

Here conjugate models are constructed through reduction to dual linear programming problems and through the contraction method, which is connected with projecting on a subspace. For example, consider the discrimination analysis problem

$$f \in F, f(a) \geq 0 \ (a \in A), f(b) \leq 0 \ (b \in B), a \in R^n, b \in R^n.$$

We may use the transformation ϕ of space $R^n : a \to a\phi, b \to b\phi$, with the aim of translating all the objects in a "good" class $\mathcal{A} \supset A : \mathcal{A} = \{ x : f(x) \geq 0 \}$.

$$\left. \begin{array}{l} f(a\phi) \geq 0 \ (a \in A), \quad f(b\phi) \geq 0 \ (b \in B), \\ f \in F, \phi \in \Phi \end{array} \right\}$$

But this problem may be contradictory and we use the contraction method (connected with the dual problem) to find the maximal solvable subproblems.

11. CONJUGATE MODELS IN TAXONOMY AS DUAL INEQUALITY SYSTEMS

Here the taxonomy problem for the set D is solved; it is the problem of representation

$$D = U_{i \in I} D_i, D_i \cap D_j = \varnothing \quad (i \neq j),$$

where D_i is the i-th class or taxon, and all the elements of the i-th class are near to each other. Consider the system

$$f(x) \leq 0 \ (x \in D), f \in F,$$

where the set $\{ x : f(x) \leq 0 \}$ is interpreted as a taxon, corresponded to the function f.

We may use the conjugate system to find the minimal inconsistent subsystems:

$$f(x) \leq 0 \ (x \in D_i'), \ D_i' \subset D, i \in \{1, \ldots, q\}.$$

The first taxon D_1 corresponds to the maximal subsystem that does not include these minimal inconsistent subsystems. Then this procedure is repeated for $D \setminus D_1$, etc.

12. EXISTENTIAL CONJUGACY

We shall discuss diagnostic and choice problems in the abstract form

$$x \in M, (i \in I), x \in D. \tag{4}$$

For this problem, conditions for the existence of solutions and generalized solutions (such as committees) are expressed in terms of consistency of subsystems. Examples are Helly's theorem for convex sets and its generalization to committees.

In a more specific form, the problem takes the form of the system

$$f_i(x) > 0 \; (i \in I), \tag{5}$$

which in general does not have to be consistent. For this system, conditions for the existence of solutions are expressed in terms of nonnegative linear combinations. This is also true for the committees. We can linearize the system (5):

- by expansion with respect to the basis:

$$\sum_{j \in J} u_{ij} \, g_j(x) > 0 \; (i \in I), x \in A; \tag{6}$$

- through the gradients, for example,

$$< f_i'(0), x > 0 \; (i \in I). \tag{7}$$

Then the conjugate system can be written down. For (7) the conjugate system is

$$\sum_{i \in I} v_i f_i' = 0.$$

For (6) the conjugate system can be present as follows: if A is a finite set, then $g_j(a)$, $a \in A$, are known coefficients. We obtain a linear inequality system with respect to u_{ij}.

In greater detail consider affine

$$\langle \, c_i, x \, \rangle > b_i \quad (i \in I) \tag{8}$$

or even the linear system

$$\langle \, c_i, x \, \rangle > 0 \quad (i \in I) \tag{9}$$

Their dual systems are associated with the elimination of unknowns and with the Farkas theorem.

Consider the system that is s-conjugated to (9) with respect to the set D:

$$\left.\begin{aligned} &\textstyle\sum_{i\in I} u_i \langle c_i, x \rangle = 0 \qquad (\forall x \in D), \\ &u = (u_i : i \in I) \geq 0, \quad 1 \leq \textstyle\sum_{i\in I} sgn\ u_i \leq s. \end{aligned}\right\} \tag{10}$$

Each solution u of this system must have not more than s nonzero components. Here

$$sgn\, h = \begin{cases} 1, h > 0; \\ 0, h = 0. \end{cases}$$

Consider some relations between systems (9) and (10). Let system (10) be inconsistent. Then for each $I' \subset I$, $|I'| \leq s$, the system

$$\textstyle\sum_{i\in I} u_i \langle c_i, x \rangle = 0 \quad (\forall x \in D), u = (u_i : i \in I) \geq 0$$

in inconsistent. This means that the system

$$\langle c_i, x \rangle > 0 \quad (i \in I'), \quad (x \in D) \tag{11}$$

is consistent. Thus, if system (10) is inconsistent, then each system (11) with $|I'| \leq s$ is inconsistent. This fact can be applied:

- to the question of existence of p-committee [1];
- to the choice of informative features;
- to the contraction method.

Here the p-committee for system (11) is the set $C \subset D$:

$$\forall i \in I' : \left| \{x \in C : \langle c_i, x \rangle > 0 \} \right| > p|C|.$$

In the case $s = 1$: it is required that each inequality in system (10) has a solution. In that case there exists a p-committee ($\forall\, p < \frac{1}{2}$) (see [1].

Consider nonlinear system (5):

$f_i(x) > 0\ (i \in I), x \in L$. Let $s \leq |I|, D \subset L,$ where L is a linear space. Consider (s, D) - conjugate of system (5):

$$\left.\begin{aligned} &\textstyle\sum_{i\in I} u_i f_i(x) \qquad (\forall x \in D), \\ &u = (u_i : i \in I) \geq 0, \quad 1 \leq \textstyle\sum_i \operatorname{sgn} u_i \leq s. \end{aligned}\right\} \tag{12}$$

Theorem 1. Let $I' \subset I$. If the system

$$f_i(x)>0 \quad (i \in I'), \quad (x \in D) \tag{13}$$

is consistent, then the system

$$\sum_{i \in I} u_i f_i(x)=0 \quad (x \in D), \quad u=(u_i : i \in I) \geq 0 \tag{14}$$

is inconsistent.

Proof. Let system (13) be consistent and let $\bar{x}$ be its solution. Then $f_i(x)>0$ $(i \in I')$ and system (14) is inconsistent. In fact, if on the contrary $\exists \bar{u} \geq 0,\ \sum_{I'} u_i f_i(x)=0,\ \ 0>0$, which is impossible.

The inverse statement is true for $f \in LIN$ (where LIN is the class of linear functions).

Theorem 2. Let each collection of s sets from M_i have a nonempty intersection that, in its turn, has a nonempty intersection with D. Then for $\frac{s}{m} > p$ there exist a p-committee $C \subset D$ for system (4), where $m=|I|$.

Theorem 3. Let $f_i(0)=0$ $(\forall i)$ and let f_i be differentiable. There exists a committee of the system

$$< f_i'(0), x > 0 \quad (i \in \{1,\dots,m\}) \Leftrightarrow \textit{its}\ (2, R^n) - \text{conjugate system is inconsistent.}$$

For a comparison see Helly's theorem.

13. THE MODEL OF AMBIGUOUS INTERPRETATION OF CONTRADICTORY INFORMATION

Any real information presented in the form of a symbolic text suggests the existence of a latent part (that of which the author of the information was aware but failed to express in the text).

This situation can be modelled as follows: a real message is the projection of message x (that was intended to be conveyed in reality) on space U that is accessible to us:

$$x=(u,v) \in X = U\text{x}V.$$

In place of a real transparent 3-dimensional cube we can represent only its projection on a plane. In that case reconstruction of the original message x is ambiguous. Let us see how it can be done.

Let $D \subset X$, where D is the set of admissible messages. Let Y' denote the original message that is desired to be obtained when interpreting its accessible projection S on subspace U.

Suppose that S has the following structure:

$$S = U_{i=1}^{k} M_i = (U_{i=1}^{k} D_i)\phi,$$

where ϕ is the projection operator on subspace U and M_i is the set of message elements in order i (i.e. the message elements are hierarchically ordered).

The set $I(M) = Y$ is assumed to be ordered, where I is the interpretation operator.

The interpretation $I(M) = Y$ is found in the following form:

$$Y = \{q = (x, y(x)) \in X : x \in M\}.$$

Let $y = (y(x) : x \in M)$ be the vector defining interpretations. All the other interpretations are defined sequentially: $I(M_{i+1})$ is defined via $I(M_i)$.

The following method is put forward. Let $q^i = (x^i, y(x^i))$ and let an interpretation predicate p be given that assigns a meaning to the piece $\{q^1, ..., q^t\}$ of the message:

$$p[q^1, ..., q^t](y) = \begin{cases} 1, \text{ if } \{q^1, ..., q^t\} \text{ is an intelligent independent block of the message} \\ 0, \text{ otherwise.} \end{cases}$$

Then, the interpretation of message Y will mean the maximal consistent subsystem of the system

$$P[q^1, ..., q^t](y) = 1 \quad (\forall T = \{x^1, ..., x^t\} \subset S).$$

Note that the projection block T is thus put into correspondence with the message block

$$\{q^1 = (x^1, y(x^1)), ..., q^t = (x^t, y(x^T))\}$$

In some cases p can be defined by means of linear inequalities. So, if the expansion of element x in terms of the block

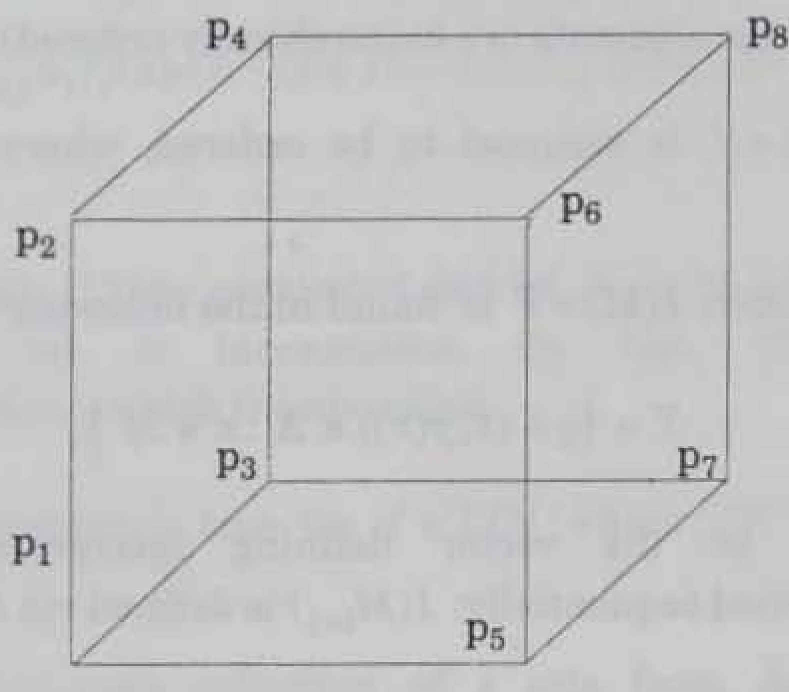
$$x = \sum_{i=1}^{t} Z_i(x) x_i$$

is found, it might be natural to assume that

$$P_q(y) = 1 \Leftrightarrow y(x) \geq \sum_{i=1}^{s} Z_i(x) y(x^i).$$

For example, let us consider the Neckers cube:

$$M_p = \{ p_1, \ldots, p_8 \},$$



where

$p_1 = (0,0);\ p_2 = (0,1);\ p_3 = (\frac{1}{4}, \frac{1}{2});\ p_4 = (\frac{1}{4}, \frac{3}{2});\ p_5 = (1,0);\ p_6 = (1,1);\ p_7 = (\frac{5}{4}, \frac{1}{2});\ p_8 = (\frac{5}{4}, \frac{3}{2}).$

The edged set is:

$M_p^2 = \{ p_1p_2, p_1p_3, p_1p_5, p_2p_4, p_2p_6, p_3p_4, p_3p_7, p_4p_8, p_5p_6, p_5p_7, p_6p_8, p_7p_8$;
$| M_p^2 | = 12.$

The point $p_i = (x_1', x_2')$ corresponds to the vertex $q_i = (x_1', x_2', y_i)$ of a polyhedron in 3-dimensional space. Note that the edges $p_1p_2, p_2p_4, p_4p_8, p_7p_8, p_5p_7, p_1p_5$ are visible for any interpretation.

Let us compose the system of equations:

$$\left.\begin{array}{ll}
1) & q_1 \in q_2q_5q_6 \Rightarrow y_1 - y_2 - y_5 + y_6 = 0; \\
2) & q_5 \in q_6q_7q_8 \Rightarrow y_5 - y_6 - y_7 + y_8 = 0; \\
3) & q_3 \in q_4q_7q_8 \Rightarrow y_3 - y_4 - y_7 + y_8 = 0; \\
4) & q_1 \in q_2q_3q_4 \Rightarrow y_1 - y_2 - y_3 + y_4 = 0; \\
5) & q_1 \in q_3q_5q_7 \Rightarrow y_1 - y_3 - y_5 + y_7 = 0; \\
6) & q_2 = q_4q_6q_8 \Rightarrow y_2 - y_4 - y_6 + y_8 = 0.
\end{array}\right\} \qquad (15)$$

The visibility of the edges is described by the following inequalities.

1) Edge q_1q_3 may be shielded only by the face $q_1q_2q_6q_5$. The relation $p_3 = \frac{1}{4}p_1 + \frac{1}{2}p_2 + \frac{1}{4}p_5$ implies the inequality

(1) $$y_3 < \frac{1}{4} y_1 + \frac{1}{2} y_2 + \frac{1}{4} y_5.$$

2) Analogously, for edge $q_2 q_6$:

(2) $$y_6 < -2y_1 - y_2 + 4y_3;$$

(3) $$y_2 < \frac{5}{4} y_4 + \frac{1}{2} y_7 - \frac{3}{4} y_8;$$

(4) $$y_6 < \frac{1}{4} y_4 + \frac{1}{2} y_7 + \frac{1}{4} y_8.$$

3) Consider the case: edge $q_3 q_4$, faces $q_1 q_2 q_6 q_5$, $q_2 q_4 q_8 q_6$, relations $p_3 = \frac{1}{4} p_1 + \frac{1}{2} p_2 + \frac{1}{4} p_5$; $p_4 = \frac{-3}{4} p_1 + \frac{3}{2} p_2 + \frac{1}{4} p_5$; $p_3 = \frac{3}{2} p_2 - p_4 + \frac{1}{2} p_6$. It follows:

(5)=(1) $$y_3 < \frac{1}{4} y_1 + \frac{1}{2} y_2 + \frac{1}{4} y_5;$$

(6) $$y_4 < \frac{-3}{4} y_1 + \frac{3}{2} y_2 + \frac{1}{4} y_5;$$

(7) $$y_3 < \frac{3}{2} y_2 - y_4 + \frac{1}{2} y_6.$$

4) Consider the case: edge $q_3 q_7$, faces $q_1 q_2 q_6 q_5$, $q_5 q_6 q_8 q_7$, relations $p_3 = \frac{1}{4} p_1 + \frac{1}{2} p_2 + \frac{1}{4} p_5$; $p_3 = 2p_5 + 2p_6 - 3p_7$; $p_7 = \frac{-3}{4} p_1 - \frac{1}{2} p_2 + \frac{5}{4} p_5$. It follows:

(8)=(1) $$y_3 < \frac{1}{4} y_1 + \frac{1}{2} y_2 + \frac{1}{4} y_5;$$

(9) $$y_3 < 2y_5 + 2y_6 - 3y_7;$$

(10) $$y_7 < \frac{-3}{4} y_1 + \frac{1}{2} y_2 + \frac{5}{4} y_5.$$

5) Consider the case: edge $q_5 q_6$, faces $q_1 q_3 q_7 q_5$, $q_3 q_4 q_8 q_7$, relations $p_5 = \frac{1}{4} p_4 + \frac{3}{2} p_7 - \frac{3}{4} p_8$; $p_6 = \frac{1}{4} p_4 + \frac{1}{2} p_7 + \frac{1}{4} p_8$; $p_6 = \frac{-3}{2} p_1 + 2p_3 + \frac{1}{2} p_5$.

It implies the inequalities:

(11) $$y_5 < \frac{1}{4} y_4 + \frac{3}{2} y_7 - \frac{3}{4} y_8;$$

(12)=(4) $$y_6 < \frac{1}{4} y_4 + \frac{1}{2} y_7 + \frac{1}{4} y_8;$$

(13) $$y_6 < \frac{-3}{2} y_1 + 2y_3 + \frac{1}{2} y_5.$$

6) Consider the case: edge $q_6 q_8$, face $q_3q_4q_8q_7$, relation $p_6 < \frac{1}{4}p_4 + \frac{1}{2}p_7 + \frac{1}{4}p_8$. It implies:

(14)=(4)
$$y_6 < \frac{1}{4}y_4 + \frac{1}{2}y_7 + \frac{1}{4}y_8;$$

Equations (15) and the inequalities are equivalent to the following system

$$\begin{array}{lll} (I)h<0, & (II)-h<0, & (III)-h<0, \\ (IV)-h<0, & (V)h<0, & (VI)h<0, \\ (VII)h<0, & (VIII)h<0, & (IX)-h<0, \\ (X)-h<0, & & \end{array}$$

where $h = -y_1 - 2y_2 + 4y_3 - y_5$. This system has only two maximal solvable subsystems:

1) I, V, VI, VII, VIII.
2) II, III, IV, IX, X.

Case 1) gives the interpretation where edges q_1q_3, q_3q_4, q_3q_7 are visible. Case 2) gives the interpretation where edges q_2q_6, q_5q_6, q_6q_8 are visible.

14. APPLICATIONS TO THE DESCRIPTION OF COMPLEX SYSTEMS

Here some examples of applications to modelling natural systems are given.

The following models for metasomatosis problems are put forward [7].

1) Mathematical programming model which includes a pattern recognition block to deal with nonformalised restrictions and criteria. In this model, some restrictions are imposed upon the state vectors (balance restrictions, conservation laws, interaction relationships, etc.). Criteria (objective functions) are related to thermodynamic state functions (potential or entropy). Conservation laws can be taken into account in the relaxed form, as bilateral inequalities; they can be nonlinear.

Another characteristic feature of this model is: the corresponding optimization problem can be not only unstable but also contradictory. And then the model defines not an ordinary equlibrium but more general quasi-equilibrium constructions. Moreover, the function defining the problem can be discontinuous and nonformalized. The model includes the means to identify its blocks.

2) Models of discriminant analysis, taxonomy and choice of informative features. These models are used to: identify the classes of thermodynamically equivalent states, construct surfaces dividing homogenouos regions and identify the state of natural systems.

Since the problems in this sphere are significantly nonformalized, corresponding pattern recognition models are, as a rule, contradictory. Then the formal tools of dealing with sub-definite problems should be used. In particular, a collective decision rule (i.e. a more complex decision rule than in the noncontradictory case) should be constructed, as it is regularly done in the committee method.

15. NUMERICAL EXAMPLES OF DUAL PROBLEMS OF DIAGNOSIS AND CHOICE

First let us consider conjugacy relations in the common form for problems with different kinds of representation of sets.

The initial choice problem is:

- find the element $x \in covA$ where x is an effective element corresponding to binary relation r, and cov is some hull.

The conjugated choice problem is:

- find the element x as the effective correspondent of binary relation r satisfying the system

$$f_i(x) \leq 0 \quad (i \in I).$$

Here the form of hull cov corresponds to the class of functions f_i.

We also consider initial and dual pattern recognition problems (discriminant analysis). The initial discriminant problem for sets M, N and class F of separating functions is: $DA(M,N,F)$, where $M = covA$, $N = covB$.

The dual problem is: $DA(M,N,F)$, where

$$M = \{x : f_i(x) \leq 0\,(i \in I)\}, \quad N = \{x : g_j(x) \leq 0\,(j \in J)\}.$$

The relation r can be represented through the dual description of the corresponding set

$$R = \{(x,y) : xry\}$$

More precisely, let the initial model be the following problem Z:

$$(1) \qquad -x_2 \leq 1,$$

$$(2) \qquad x_1 - x_2 \leq 0,$$

$$(3) \qquad x_1 - x_2 \leq -1,$$

$$(4) \qquad -2x_1 + x_2 \leq -1.$$

This system may be interpreted as the problem of the choice of parameters x_1, x_2 of a technical object or economic system. It may also be the problem of learning diagnosing, where we must construct the separating function $f(x) = x_1 z_1 + x_2 z_2$ for sets (precedent classes)

$$A = \{ a^1 = (0,-1),\ \ a^2 = (1,1) \};$$
$$B = \{ b^1 = (-1,1),\ \ b^2 = (2,-1) \}.$$

The separating conditions are:

$$f(a^1) \le 1,\ f(a^2) \le 0,\ f(b^1) \ge 1,\ f(b^2) \ge 1.$$

Here the surface separating A and B is described by the equality $f(z) = 1$.

The dual problem Z^* is: for nonnegative combinations of inequalities (1) - (4) we require:

if a linear combination of left-hand sides is identically equal to zero, then the same combination of right-hand sides is nonnegative. Let v_i be the coefficient of the i-th inequality in this combination. Then the problem Z^* has the form:

$$v_1 \ge 0, v_2 \ge 0, v_3 \ge 0, v_4 \ge 0, \qquad (0^*)$$
$$v_2 + v_3 - 2v_4 = 0, \qquad (1^*)$$
$$-v_1 + v_2 - v_3 + v_4 = 0, \qquad (2^*)$$
$$v_1 - v_3 - v_4 \ge 0, \qquad (3^*)$$

and (3^*) is the consequence of (0^*), (1^*), (2^*).

The solution sets $ArgZ$ and $ArgZ^*$ give the evaluations of noncontradictoriness stability of problems Z and Z^*.

The stability of solutions under small variations of problem statements in this inconsistent case is reduced to finding consistent subproblems and then investigating the stability of subproblem solutions.

The solutions sets of maximal consistent subsystems of system (1) - (4) are shown on Fig. 1 as shaded regions. We show here the vertices and the directions of infinite edges of polyhedral solutions sets.

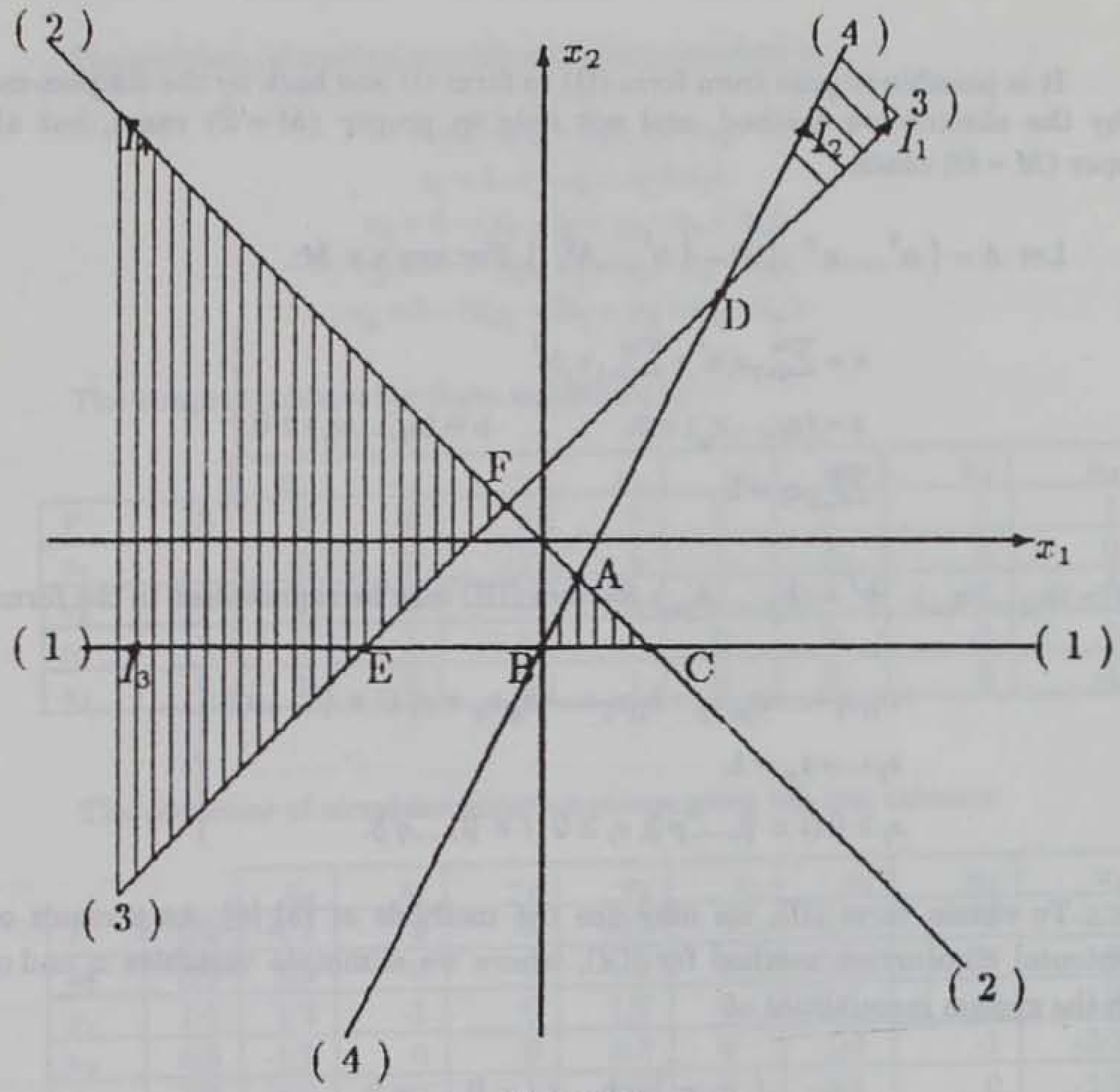


Figure 1.

From the geometric point of view, duality is connected with the dual representations of sets:

- predicative form, when the sets are given through the restrictions;
- structural form, when the sets are given as the hull of some points and directions.

Such geometric representations are important for optimal choice problems and for pattern recognition; for the convex polyhedral sets such connections are well known (see, for example, [1], [2], [3], [4]).

So, a convex polyhedral set M may be represented by dual forms:

$$M = coA + coneB; \qquad (I)$$

$$M = \{x \in R^n : < c_j, x > \leq b_j \quad (j \in \{1,...,m\})\}. \qquad (II)$$

Here A and B are some finite sets in space R^n; $x, c_j \in R^n$; $\langle c_j, x \rangle$ is the scalar product, *co* is the convex hull, *cone* is the convex cone hull.

It is possible to pass from form (II) to form (I) and back by the simplex-method and by the elimination method, and not only in proper ($M \neq \varnothing$) cases, but also in inproper ($M = \varnothing$) cases.

Let $A = \{ a^1,...,a^p \}$, $B = \{ b^1,...,b^q \}$. For any $x \in M$:

$$\left.\begin{array}{ll} x = \sum_{i=1}^{n} z_i a^i + \sum_{j=1}^{q} v_j b^j & \\ z = (z_1,...,z_q) \geq 0, & v = (v_1,...,v_q) \geq 0, \\ \sum_{i=1}^{p} z_i = 1. & \end{array}\right\} \qquad (III)$$

Let $a^i = (a_{i1},...,a_{in})$, $b^j = (b_{j1},...,b_{jn})$. System (III) may be represented in the form:

$$\left.\begin{array}{l} a_{1i}z_1 + ... + a_{pi}z_p + b_{1i}v_1 + ... + b_{qi}v_q = x_i \ (i \in \{1,...,n\}), \\ z_1 + ... + z_p = 1, \\ z_i \geq 0 \ (i \in \{1,...,p\}), v_i \geq 0 \ (i \in \{1,...,q\}). \end{array}\right\} \qquad (IV)$$

To obtain form (II), we may use the methods of [3], [6]. As a result of the fundamental elimination method for (IV), where we eliminate variables z_i and v_i, we obtain the system inequalities of

$$\langle c_j, x \rangle \leq b_j \quad (j \in \{1,...,m\}). \qquad (V)$$

If $M \neq \varnothing$, then $M = \{x : (V)\}$, and

$$M \neq \varnothing \Leftrightarrow A \cup B \neq \varnothing$$

But if system (V) is inconsistent, then the pass $(II) \rightarrow (I)$ may be realized in different ways. For example, we may find the maximal corrections of the right-hand sides of system (V) to obtain an inconsistent system, and then apply the simplex method to obtain form (I).

And another way is possible. System (V) has a finite set of maximal consistent subsystems, and then we have to obtain the representation of any maximal consistent subsystem in the form (I). Finding maximal consistent subsystems can be done using the fundamental elimination.

And now let us go back to the numerical example (see Fig. 1). We first transform the problem (1) - (4) to the canonical form using representations $x_i = y_i - z_i$, $y_i \geq 0$, $z_i \geq 0$ and slack variables $u_j \geq 0$:

$$\left.\begin{array}{r} -y_2 + z_2 + u_1 = 1 \\ y_1 - z_1 + y_2 - z_2 + u_2 = 0 \\ -y_1 + z_1 + y_2 - z_2 - u_3 = 1 \\ 2y_1 - 2z_1 - y_2 + z_2 - u_1 = 1 \end{array}\right\}$$

The problem of minimizing the summary residual is:

$$
\begin{aligned}
\min F &= v_1 + v_2 + v_3 + v_4 \\
v_1 &= 1 - (-y_2 + z_2 + u_1), \\
v_2 &= 0 - (y_1 - z_1 + y_2 - z_2 + u_2), \\
v_3 &= 1 - (-y_1 + z_1 + y_2 - z_2 - u_3), \\
v_4 &= 1 - (2y_1 - 2z_1 - y_2 + z_2 - u_4).
\end{aligned}
$$

The simplex-tableau for these equations is:

		y_1	z_1	y_2	z_2	u_1	u_2	u_3	u_4
F	3	2	-2	0	0	1	1	1	1
v_1	1	0	0	-1	1	1	0	0	0
v_2	0	1	-1	1	-1	0	1	0	0
v_3	1	-1	1	1	-1	0	0	-1	0
v_4	1	2	-2	-1	1	0	0	0	-1

The sequence of simplex-transformations gives the last tableau:

		u_2	z_1	y_2	v_4	v_1	v_2	u_3	u_4
F	5/3	-1/3	0	0	-2/3	-1	-1/3	-1	-2/3
u_1	2/3	2/3	0	0	-1/3	1	2/3	0	1/3
y_1	1/3	1/3	-1	0	1/3	0	1/3	0	-1/3
v_3	5/3	-1/3	0	0	2/3	0	-1/3	-1	-2/3
z_2	1/3	-2/3	0	-1	1/3	0	-2/3	0	-1/3

For this tableau we have: $v_1 = v_2 = v_4 = 0$. These equations correspond to the maximal consistent subsystem with the index set $I_1 = \{1,2,4\}$. From the tableau we find the vertex of the solution set for this subsystem: $A = (\frac{1}{3}, -\frac{1}{3})$.

The transformation of the simplex-tableau corresponds to going to a neighbouring vertex on the edge. Such steps can give all the vertices. Working with the subsystem I_1, we obtain the vertices $B = (0,-1)$, $C = (-1,1)$.

The solutions set of the subsystem is co(A,B,C). Using the complements to I_1, we find other maximal consistent subsystems:

$$
\begin{array}{ccc}
 & I_1 = \{1,2,4\} & \\
 & \downarrow & \\
 & \{3\} \text{ (complement to } I_1 & \\
\swarrow & & \searrow \\
I_2 = \{1,3,4\} & & I_3 = \{1,2,3\}
\end{array}
$$

By the simplex-method we obtain the representations for solutions sets of these subsystems:

$$D+cone\ \{s_1.s_2\};$$
$$co(E.F)+cone\ \{s_3,s_4\}.$$

In general, such examples can be obtained from the following construction: Let the choice or pattern recognition problem be reduced to the system

$$Z:\langle c_j,x\rangle>0 \quad (j\in\{1,\dots,m\}).$$

The dual problem is

$$Z^*:\sum v_jc_j=0,\quad v\geq 0,$$

and the connection between Z and Z^* is as follows:

The problem Z is inconsistent $\Leftrightarrow$ the set of fundamental solutions of problem Z^* defines the set of minimal inconsistent subsystems of problem Z.

16. CONCLUSIONS

Conjugacy in pattern recognition has specific characteristics as compared to that in mathematical programming but it yields the same practical results, making it possible to estimate the stability of solutions to choice and diagnosing problems and of corresponding decision rules, as well as to resolve the contradictions in the observed data and in the corresponding mathematical models.

REFERENCES

[1] Mazurov, Vl. D., *Committee Method in Optimization & Classification Problems*, Nauka, Moscow, 1990 (in Russian).

[2] Eremin, I.I., Mazurov, Vl. D., *Nonstationary Processes of Mathematical Programming*, Nauka, Moscow, 1979 (in Russian).

[3] Chernikov, S. N., *Linear Inequalities*, Nauka, Moscow, 1968 (in Russian).

[4] Mazurov, Vl. D., "Duality in pattern recognition and operation research", *Pat. Rec. and Im. Anal.*, 1 (1991) 375-384.

[5] Eaves, B. C., Rothblum, U. G., "Dines-Fourier-Motzkin quantifier elimination and an application of corresponding transfer principles over ordered fields", *Mathematical Programming*, 53 (1992) 307-321.

[6] Eremin, I.I., *Contradictory Models of Optimal Planning*, Nauka, Moscow, 1988 (in Russian).

[7] Mazurov, Vl. D., "Duality issues in discriminant analysis, Contradictory optimization problems", Ural Branch of the USSR Acad. of Sci., Sverdlovsk, 1987 (in Russian).

[8] Gorban, A.N., *Bypassing Equilibrium*, Nauka, Novosibirsk, 1984 (in Russian).