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Theoretical study of chemical reaction effects on vertical oscillating plate with variable temperature

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Abstract

An exact solution to the flow of a viscous incompressible unsteady flow past an infinite vertical oscillating plate with variable temperature and mass diffusion is presented here, taking into account of the homogeneous chemical reaction of first-order. Both the plate temperature and the concentration level near the plate are raised linearly with respect to time. The dimensionless governing equations has been obtained by the Laplace transform method, when the plate is oscillating harmonically in its own plane. The effects of velocity and concentration are studied for different parameters like phase angle, chemical reaction parameter, thermal Grashof number, mass Grashof number, Schmidt number and time are studied. The solutions are valid only for small values of time t. It is observed that the velocity increases with decreasing phase angle ωt or chemical reaction parameter.

Keywords: chemical reaction, oscillating, vertical plate, heat and mass transfer

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Nomenclature

A	constant
C'	species concentration in the fluid $mol.m^{-3}$
C	dimensionless concentration
C_p	specific heat at constant pressure $J.kg^{-1}.K^{-1}$
D	mass diffusion coefficient $m^2.s^{-1}$
Gc	mass Grashof number
Gr	thermal Grashof number
g	acceleration due to gravity $m.s^{-2}$
k	thermal conductivity of the fluid $J.m^{-1}.K^{-1}$
K_l	chemical reaction parameter J
K	dimensionless chemical reaction parameter
Pr	Prandtl number
Sc	Schmidt number
T	temperature of the fluid near the plate K
t'	time s
t	dimensionless time
u	velocity of the fluid in the x'-direction $m.s^{-1}$
u_0	velocity of the plate $m.s^{-1}$
U	dimensionless velocity
x	spatial coordinate along the plate
y'	coordinate axis normal to the plate m
y	dimensionless coordinate axis normal to the plate

Greek symbols

β	volumetric coefficient of thermal expansion K^{-1}
β^*	volumetric coefficient of expansion with concentration K^{-1}
μ	coefficient of viscosity $Pa.s$
ν	kinematic viscosity $m^2.s^{-1}$
ρ	density of the fluid $kg.m^{-3}$
τ	dimensionless skin-friction
θ	dimensionless temperature
ωt	phase angle
η	similarity parameter
$erfc$	complementary error function

Subscripts

ω	conditions at the wall
∞	conditions in the free stream

1 Introduction

Chemical reactions can be codified as either heterogeneous or homogeneous processes. It depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous if it takes place at an interface, and homogeneous, if it takes place in solution. On the other hand, in most cases of real chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of first order, if the rate of reaction is directly proportional to concentration. In many chemical engineering processes, there is the chemical reaction between a foreign mass and the fluid. These processes take place in many industrial applications such as food processing, manufacturing of ceramics and polymer production.

Chambre and Young [1] have analyzed a first order chemical reaction in the neighborhood of a stationary horizontal plate. Ramanamurthy and Govindarao [2] have made an analysis of flow around a cylindrical catalyst pellet with first-order chemical reaction. Apelblat [3] studied analytical solution for mass transfer with a chemical reaction of the first order. Das et al.[4] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. The flow of a viscous, incompressible fluid past an infinite isothermal vertical plate, oscillating in its own plane, was solved by Soundalgekar [5]. The effect on the flow past a vertical oscillating plate due to a combination of concentration and temperature differences was studied extensively by Soundalgekar and Akolkar [6]. The effect of mass transfer on the flow past an infinite vertical oscillating plate in the presence of constant heat flux has been studied by Soundalgekar et al. [7].

It is proposed to study the effects of homogeneous chemical reaction of first order and free convection on the oscillating infinite vertical plate with variable temperature and mass diffusion. The dimensionless governing equations are solved using the Laplace transform technique. The solutions are in terms of exponential and complementary error function.

2 Governing Equations

First order chemical reaction effects on unsteady flow of a viscous incompressible fluid past an infinite vertical oscillating plate with variable temperature and mass diffusion is studied. It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first order chemical reaction between the diffusing species and the fluid. The x-axis is taken along the plate in the vertically upward direction and the y$^{'}$-axis is taken normal to the plate. Initially, the plate and the fluid are of the same temperature $T\infty$ and concentration $C^{'}_{\infty}$. At time $t^{'} > 0$, the plate starts oscillating in its own plane with frequency $\omega^{'}$ and the temperature of the plate is raised linearly with respect to time t and the concentration level is also raised to t. Then by usual Boussinesqs' approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta^*(T - T_\infty) + g\beta(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y'^2}, \tag{1}$$

$$\rho C_p\frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y'^2}, \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - K_l C', \tag{3}$$

with the following initial and boundary conditions:

$$\begin{array}{llll} t' \leq 0: & u = 0, & T = T_\infty, & C' = C'_\infty \ \text{ for all } \ y\prime \\ t' > 0: & u = u_0\cos\omega' t', & \begin{array}{l} T = T_\infty + \\ (T_\omega - T_\infty)At', \end{array} & \begin{array}{l} C' = C'_\infty + \\ (C'_\omega - C'_\infty)At' \ \text{ at } \ y = 0, \end{array} \\ & u = 0, & T \rightarrow T_\infty, & C' \rightarrow C'_\infty \text{ as } y\prime \rightarrow \infty, \end{array} \tag{4}$$

where $A = \dfrac{u_0^2}{\nu}$.

On introducing the following non-dimensional quantities:

$$U = \frac{u}{u_0}, \quad t = \frac{t' u_0^2}{\nu}, \quad y = \frac{y\prime u_0}{\nu}, \quad \theta = \frac{T - T_\infty}{T_\omega - T_\infty},$$

$$Gr = \frac{g\beta\nu(T_\omega - T_\infty)}{u_0^3}, \quad C = \frac{C' - C'_\infty}{C'_\omega - C'_\infty}, \quad Gc = \frac{\nu g\beta^*(C'_\omega - C'_\infty)}{u_0^3}, \tag{5}$$

$$\Pr = \frac{\mu C_p}{k}, \quad Sc = \frac{\nu}{D}, \quad K = \frac{\nu K_l}{u_0^2}, \; \omega = \frac{\omega' \nu}{u_0^2}$$

in equations (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial y^2}, \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial y^2}, \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} - K\,C. \tag{8}$$

The initial and boundary conditions in non-dimensional form are

$$\begin{array}{lllllll} t \leq 0: & U = 0, & \theta = 0, & C = 0 & \text{for all} & y, \\ t > 0: & U = \cos\omega t, & \theta = t, & C = t & \text{at} & y = 0, \\ & U = 0, & \theta \to 0, & C \to 0 & \text{as} & y \to \infty. \end{array} \tag{9}$$

All the physical variables are defined in the nomenclature. The solutions are obtained for hydrodynamic flow field in the presence of first order chemical reaction.

3 Solution Procedure

The dimensionless governing equations (6) to (8), subject to the boundary conditions (9), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\theta = t\left[(1 + 2\eta^2 \Pr)\ erfc(\eta\sqrt{Pr}) - 2\eta\sqrt{\frac{\Pr}{\pi}}\exp(-\eta^2 Pr)\right] \tag{10}$$

$$
\begin{aligned}
C &= \frac{t}{2}\Big[\exp(2\eta\sqrt{KtSc})\ erfc(\eta\sqrt{Sc}+\sqrt{Kt})+ \\
&\qquad \exp(-2\eta\sqrt{KtSc})\ erfc(\eta\sqrt{Sc}-\sqrt{Kt})\Big] \\
&-\frac{\eta\sqrt{Sc}\sqrt{t}}{2\sqrt{K}}\Big[\exp(-2eta\sqrt{KtSc})erfc(\eta\sqrt{Sc}-\sqrt{Kt})- \\
&\qquad -\exp(2\eta\sqrt{KtSc})\ erfc(\eta\sqrt{Sc}+\sqrt{Kt})\Big], \qquad (11)
\end{aligned}
$$

$$
\begin{aligned}
U &= \frac{\exp(i\omega t)}{4}\Big[\exp(2\eta\sqrt{i\omega t}\ erfc(\eta+\sqrt{i\omega t})+ \\
&\qquad \exp(-2\eta\sqrt{i\omega t}\ erfc(\eta-\sqrt{i\omega t})\Big]+\frac{\exp(-i\omega t)}{4} \\
&\qquad \Big[\exp(2\eta\sqrt{-i\omega t}\ erfc(\eta+\sqrt{-i\omega t})+ \\
&\qquad \exp(-2\eta\sqrt{-i\omega t}\ erfc(\eta-\sqrt{-i\omega t})\Big]+ \\
&derfc(\eta)-\frac{ct^2}{6}\Bigg[\big(3+12\eta^2+4\eta^4\big)\,erfc(\eta)-\frac{\eta}{\sqrt{\pi}} \\
&\big(10+4\eta^2\big)\exp(-\eta^2)-\big(3+12\eta^2\,\mathrm{Pr}+4\eta^4\,\mathrm{Pr}^{\ 2}\big)\,erfc(\eta\sqrt{\mathrm{Pr}})+ \\
&\frac{\eta\sqrt{\mathrm{Pr}}}{\sqrt{\pi}}\big(10+4\eta^2\,\mathrm{Pr}\big)\exp(-\eta^2\,\mathrm{Pr})\Bigg]-\frac{d}{2}\Big[\exp(2\eta\sqrt{at}\ erfc(\eta+\sqrt{at})+ \\
&\qquad \exp(-2\eta\sqrt{at}\ erfc(\eta-\sqrt{-at})\Big]-\frac{d\,(1+at)}{2} \\
&\qquad \Big[\exp(-2\eta\sqrt{KtSc})erfc(\eta\sqrt{Sc}+\sqrt{Kt})+ \\
&\qquad \exp(-2\eta\sqrt{KtSc})erfc(\eta\sqrt{Sc}-\sqrt{Kt})\Big]+ \\
&ad\frac{\eta\sqrt{Sc}\sqrt{t}}{2\sqrt{K}}\Big[\exp(-2\eta\sqrt{KtSc})erfc(\eta\sqrt{Sc}-\sqrt{Kt})- \\
&\qquad \exp(2\eta\sqrt{KtSc})erfc(\eta\sqrt{Sc}+\sqrt{Kt})+tad\Big]
\end{aligned}
$$

$$\left[(1+2\eta^2)\ erfc(\eta)-\sqrt{\frac{2\eta}{\pi}}\exp(-\eta^2)\right]+$$

$$\frac{d\exp(at)}{2}\left[\exp(-2\eta\sqrt{Sc(a+K)t})\ erfc(\eta\sqrt{Sc}-\sqrt{(K+a)\,t})\right] \tag{12}$$

where

$$\eta=\frac{y}{2\sqrt{t}}\quad\text{and}\quad a=\frac{KSc}{1-Sc},\ c=\frac{Gr}{\Pr-1},\ d=\frac{Gc}{a^2(1-Sc)}.$$

In order to get the physical insight into the problem, the numerical values of U have been computed from (12). While evaluating this expression, it is observed that the argument of the error function is complex and, hence, we have separated it into real and imaginary parts by using the following formula:

$$erf(a+ib)=erf(a)+\frac{\exp(-a^2)}{2a\pi}[1-\cos(2ab)+i\sin(2ab)]+$$

$$\frac{2\exp(-a^2)}{\pi}\sum_{n=1}^{\infty}\frac{\exp(-n^2/4)}{n^2+4a^2}\left[f_n\left(a,b\right)+ig_na,b\right]+\epsilon\left(a,b\right)$$

where

$$f_n=2a-2a\ \cosh(nb)\cos(2ab)+n\ \sinh(nb)\sin(2ab)$$

and

$$g_n=2a\ \cosh(nb)\sin(2ab)+n\ \sinh(nb)\cos(2ab),$$

$$|\epsilon\left(a,b\right)|\approx10^{-16}\left|erf\left(a+ib\right)\right|$$

4 Results and discussion

In order to get a physical view of the problem numerical calculations are carried out for different values of the phase angle, chemical reaction parameter, Schmidt number and time. The purpose of the calculations given here is to asses the effect of different $\omega t, K, Sc$ and t upon the nature of the flow and transport. The Laplace transform solutions are in terms of exponential and complementary error function.

The velocity profiles for different phase angles ($\omega t=0,\pi/4,\pi/3,\pi/2$), $K=2, Gr=2, Gc=2, Sc=0.6, Pr=7.0$ and $t=0.2$ are shown in Figure 1. It is observed that the velocity increases with decreasing phase angle ωt. Figure 2 illustrates the effect of the velocity for different values of the reaction parameter $(K = 0.2, 7, 20), \omega t = \pi/3, Gr = 5, Gc = 10, Sc = 0.6, Pr = 7.0$ and $t = 0.3$. The trend shows that the velocity increases with decreasing chemical reaction parameter.

The effect of velocity profiles for different time $(t = 0.2, 0.3, 0.4), K = 0.2, \omega t = \pi/3, Gr = 5, Gc = 5, Pr = 7, Sc = 0.6$ are shown in Figure 3. In this case, the velocity increases gradually with increasing values of time t.

Figure 4 demonstrates the effect of the concentration profiles for different values of the chemical reaction parameter $(K = 2, 5, 10), Sc = 0.6$ and time $t = 0.4$. It is observed that the concentration increases with decreasing chemical reaction parameter. The concentration profiles for different Schmidt number $(Sc = 0.16, 0.3, 0.6, 2.01), K = 2$ at time $t = 0.4$ are shown in Figure 5. The effect of concentration is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the Schmidt number.

The temperature profiles are calculated for different values of time $(t = 0.2, 0.4, 0.6, 1)$ are shown in Figure 6. for water $(Pr = 7.0)$. The effect of thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with increasing time t.

5 Conclusions

In this paper, the governing equations are first linearized and then, an exact solution of a viscous incompressible unsteady flow past an infinite vertical oscillating plate with variable temperature and mass diffusion is found, taking into account of the homogeneous chemical reaction of first order. Both the plate temperature and concentration level near the plate are raised linearly with respect to time. The linearized dimensionless equations are solved using Laplace transform technique when the plate is oscillating harmonically in its own plane. The fields of velocity and concentration are analyzed for different parameters like phase angle, chemical reaction parameter, mass Grashof number, thermal Grashof

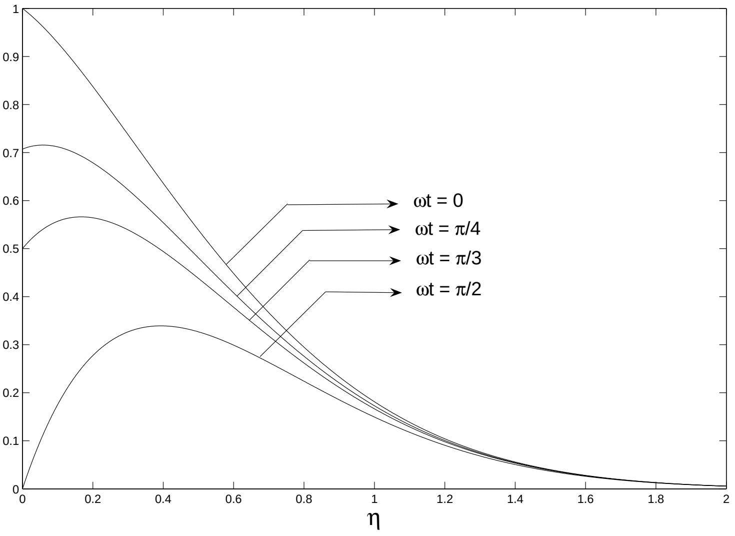

Figure 1: Velocity profiles for different $\omega\, t$

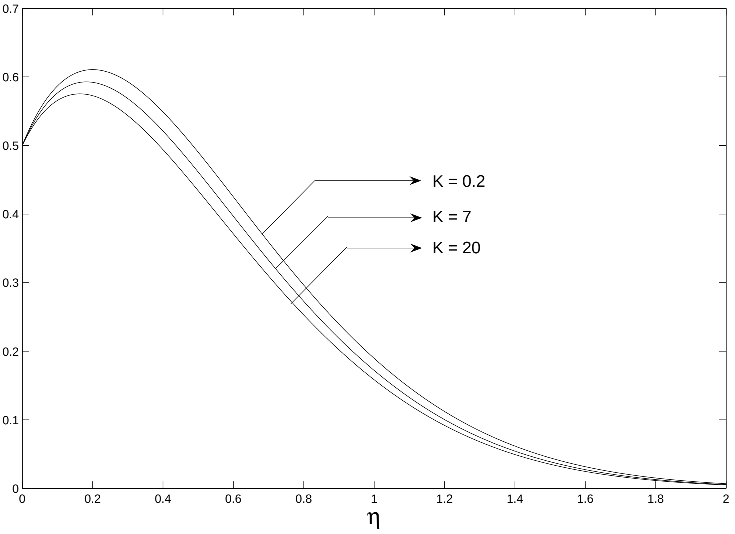

Figure 2: Velocity profiles for different K

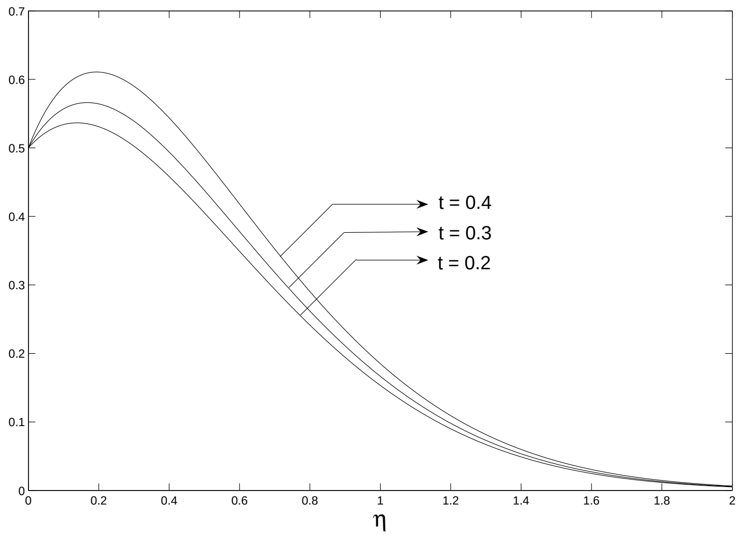

Figure 3: Velocity profiles for different t

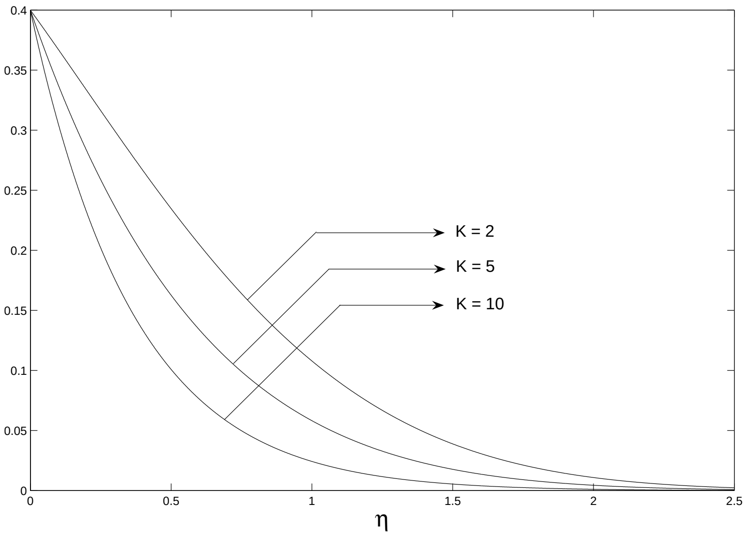

Figure 4: Concentration profiles for different K

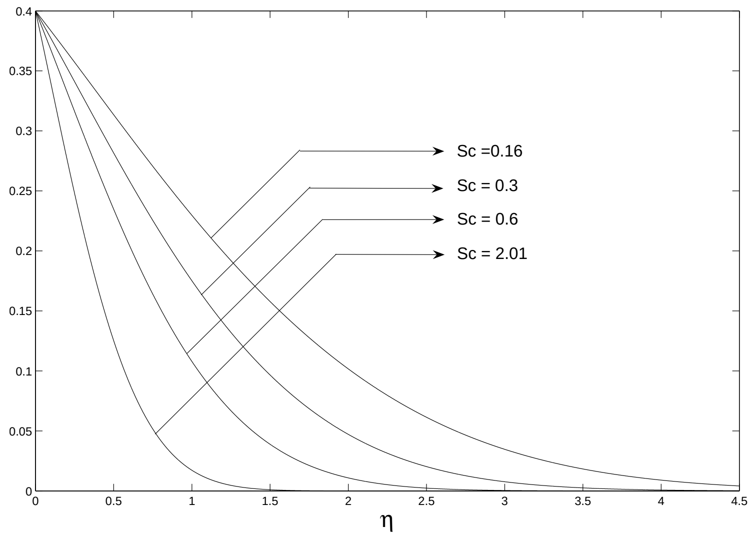

Figure 5: Concentration profiles for different Sc

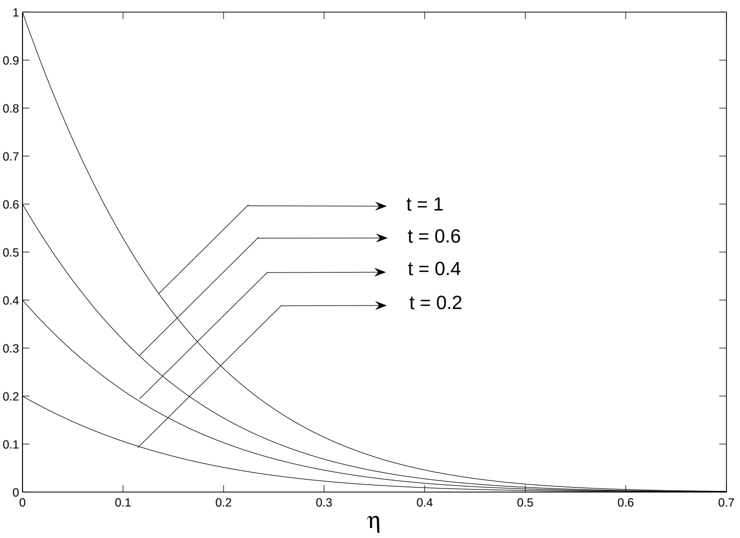

Figure 6: Temperature profiles for different t

number, Schmidt number and t. It is observed that the velocity increases with decreasing phase angle ωt or chemical reaction parameter K. As time increases, there is also a rise in velocity. The approximate method of Laplace transform technique should be applied only locally about each point of linearization. The solutions are valid only for small vales of time . It is also observed that the concentration increases with decreasing Schmidt number or chemical reaction parameter.

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Teorijska studija o efektu hemijske reakcije na vertikalno osculujuću ploču sa promenljivom temperaturom

UDK 536.7 534.142

Daje se tačno rešenje viskoznog nestišljivog tečenja preko beskonačne vertikalno oscilujuće ploče sa promenljivom temperaturom i difuzijom mase, uzimajući u obzir homogenu reakciju prvog reda. Temperatura ploče i nivo koncentracije rastu linearno sa vremenom. Bezdimenzione jednačine problema se formulišu metodom inverzne Laplasove transformacije kada ploča harmonijski osciluje u sopstvenoj ravni. Proučavaju se efekti brzine i koncentracije za razne vrednosti sledećih parametara: faznog ugla, parametra hemijske reakcije, termičkog Grashofovog broja, masenog Grashofovog broja, Šmitovog broja i vremena.

Dobijena rešenja važe samo za male vrednosti vremena. Uočava se da brzina raste sa opadanjem faznog ugla ili parametra hemijske reakcije.