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Estimation of frictional resistance to a non-Newtonian open-channel flow by application of Bingham and pseudoplastic rheological models

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Abstract

In this paper the equation of a laminar uniform one-dimensional free surface flow for a Herschel-Bulkley fluid is derived. On the basis of the equation, an expression for Darcy-Weisbach friction factor is defined and the extended Reynolds number is introduced. Using results of experiments carried out using water-kaolinite clay mixtures with different solid concentrations, the friction factor is estimated for the Bingham and pseudoplastic models. Results are compared with theoretical values. Accuracy of the friction factor estimation is also given.

1 Introduction

When the frictional resistance to a non-Newtonian fluid flow is considered, the analysis of the fluid rheological properties plays an important role. For the rheological model adopted, an expression for the friction factor can be derived, based on the laminar flow equation. However, it is not easy to decide what rheological model to apply, even for the same type of material. Mixtures of water and clay, that are a subject of investigation in this paper, were previously modelled most commonly by a Bingham model, [1] -[7]. At extremely low shear rates, the pseudoplastic model was suggested, [2], [8]. Recently, the more general Herschel-Bulkley model was applied, [9].

In order to obtain a relationship between Darcy-Weisbach friction factor Reynolds number, various concepts were performed for a Bingham fluid. When using the standard Reynolds number formulation, from the equation of laminar uniform one-dimensional free-surface flow, a non-linear dependence of the friction factor on Reynolds and Hedström numbers was defined, (see [7], [10] and [11]). Applying the coefficient of "effective viscosity", Hedström number is eliminated, so that the friction factor depends only on the Reynolds number. This approach is widely used by Chinese researches, (see [4], [10] and [12]). From the equation of laminar uniform one-dimensional free-surface flow of Bingham fluid, an extended Reynolds number was defined so that the standard Moody diagram, (valid for Newtonian fluids), can be applied without any modification for the friction factor estimation, [13].

Based on the concept given by Ogihara and Miyazawa, [14] and [15], for the pipe non-Newtonian flow, the extended Reynolds number formulation for a Herschel-Bulkley fluid is introduced in this paper, the definition being easily simplified to both Bingham and pseudoplastic models. Using results of experiments carried out using water-kaolinite clay mixtures with different solids concentrations, the friction factor is estimated for the Bingham and pseudoplastic models, and compared with theoretical values. Accuracy of the friction factor estimation, depending on the rheological model chosen, is also given.

The turbulent non-Newtonian flow is associated with much smaller difference in the friction factor (with respect to the water flow resistance) then in the laminar flow regime. In a turbulent flow over smooth boundary the friction factor follows Blasius formula, and for the fully turbulent flow a Colebrook-type flow resistance equation can be used. It is noted that a thick mixture easily enters the transitional zone, and is much more difficult to achieve the fully turbulence than a water flow, the transitional zone being much wider than in a Newtonian flow.

The turbulent flow resistance is not considered in this paper, but reader is referred to papers [4], [7], [10], [12], [16] and [17].

2 Rheological modelling

Rheological properties of a non-Newtonian fluid in the laminar flow regime can be described using Herschel-Bulkley model in a wide range of shear rates, [9]. For a point of the fluid analyzed, the model is described as

$$\begin{aligned} \frac{du}{dt} &= 0, \qquad \tau \leq \tau_c, \\ \tau &= \tau_c + \eta\left(\frac{du}{dt}\right)^m, \quad \tau > \tau_c, \end{aligned} \tag{1}$$

where u is the flow velocity (m/s), y - flow depth (m), du/dt - shear rate (s^{-1}), τ - shear stress (N/m^2), τ_c - yield stress (N/m^2), η - coefficient of rigidity (Ns/m^2), μ - coefficient of viscosity of a Newtonian fluid (Ns/m^2), and m - flow behavior index (dimensionless), see Fig. 1.

The above equation is somewhat modified when considering a finite volume of a fluid, so that a constant, depending on the flow geometry, is introduced. In the following text, the integral-type of the equation is considered.

Substituting $m = 1$ in equation (1), the Bingham model is obtained, while taking $\tau_c = 0$ and $m < 1$ gives the equation of a pseudoplastic. Rheological parameters τ_c, η and m are estimated experimentally by rotational or capillary tube viscometer. The illustration of the parameters estimation procedure follows.

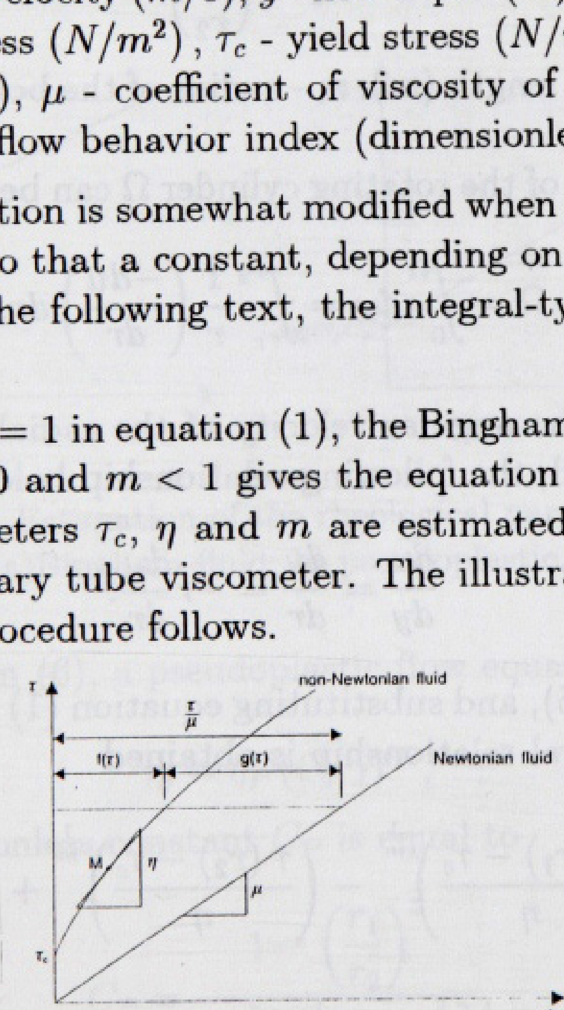


Fig. 1. - Shear diagram of a Newtonian and non-Newtonian fluid.

2.1 Rotational viscometer

Consider a fluid laminar flow in the annulus between two concentric cylinders of a rotational viscometer, the inner cylinder, (usually referred to as the "bob"), is rotating while the outer, (the "cup"), is held stationary ("Couette" type of device). The angular speed of the rotating cylinder Ω (s^{-1}), and the torque applied to stationary cylinder M (Nm), are measured, and then the shear stress τ_o (N/m^2), and the shear rate γ (s^{-1}) at the bob surface are calculated, (see [18] - [20])

$$\tau_o = \tau(r_1) = \frac{M}{2r_1^2 \pi h}, \tag{2}$$

$$\gamma = \frac{2\Omega}{1 - \left(\frac{r_1}{r_2}\right)^2}, \tag{3}$$

where h is the bob length (m), r_1 - radius of the bob (m), and r_2 radius of the cup (m).

The angular speed of the rotating cylinder Ω can be described as follows

$$\Omega = \int_0^{\Omega} d\omega = \int_{r_1}^{r_2} \frac{1}{r}\left(\frac{-du}{dr}\right) dr, \tag{4}$$

where ω (s^{-1}) is the angular velocity at the radial distance r. For the geometry described, the following relationship holds

$$\frac{du}{dy} = \frac{du}{dr} = -r\frac{d\omega}{dr}. \tag{5}$$

Using expression (5), and substituting equation (1) for $(-du/dr)$ in (4), the following general relationship is obtained

$$\Omega = \frac{m}{2}\left\{\left(\frac{\tau(r_1) - \tau_c}{\eta}\right)^{\frac{1}{m}} - \left(\frac{\tau(r_2) - \tau_c}{\eta}\right)^{\frac{1}{m}} + \sum_1^{\infty} (-1)^i \left(\frac{\tau_c}{\eta}\right)^i \right.$$

$$\left. \times \frac{1}{1 - mi}\left[\left(\frac{\tau(r_1) - \tau_c}{\eta}\right)^{\frac{1}{m} - i} - \left(\frac{\tau(r_2) - \tau_c}{\eta}\right)^{\frac{1}{m} - i}\right]\right\} \tag{6}$$

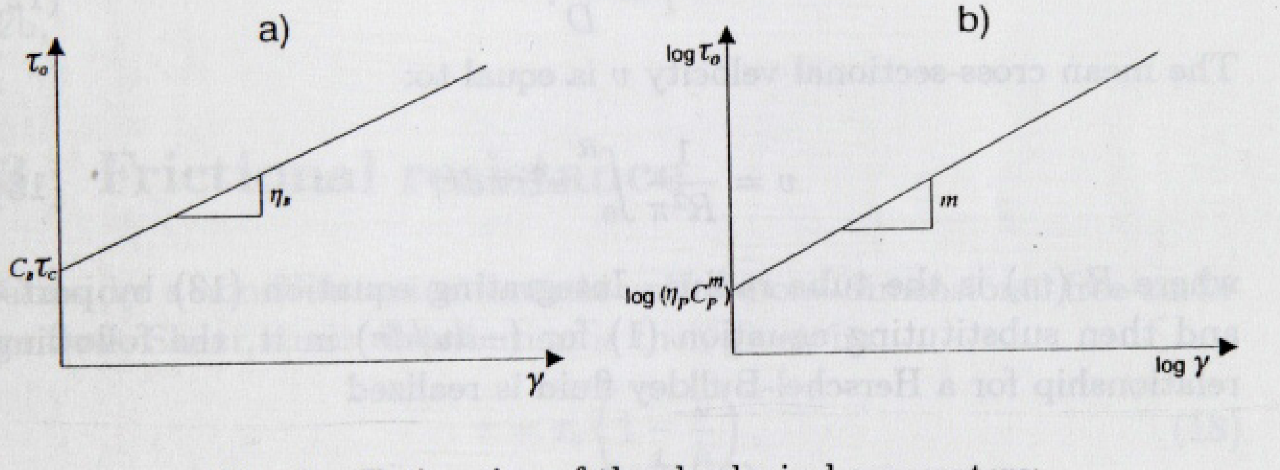
Taking $m = 1$ in (6), flow equation of a Bingham fluid is accomplished, (see [20] and [21])

$$\tau_o = C_B \tau_c + \eta_B \gamma, \tag{7}$$

where

$$C_B = \frac{2 \ln \dfrac{r_2}{r_1}}{1 - \left(\dfrac{r_1}{r_2}\right)^2}, \tag{8}$$

is a dimensionless constant depending on the radii of the bob and the cup, and η_B - coefficient of rigidity of a Bingham fluid (Ns/m^2). Flow data acquired by the viscometer can be fitted by linear relationship (7), the slope of the line being equal to η_B, and the intercept at the ordinate axis to $C_B \tau_c$, (which can give us the value of the yield stress τ_c), see Fig. 2a.



Fig, 2, - Estimation of the rheological parameters:
a)Bingham fluid; b) pseudoplastic.

Taking $\tau_c = 0$ in (6), a pseudoplastic flow equation is defined, (see [19] - [21])

$$\tau_o = \eta_P \left(C_P \gamma\right)^m, \tag{9}$$

where the dimensionless constant C_P is equal to

$$C_P = \frac{1 - \left(\dfrac{r_1}{r_2}\right)^2}{m\left[1 - \left(\dfrac{r_1}{r_2}\right)^{\frac{2}{m}}\right]}, \tag{10}$$

and η_P - consistency index (Ns^m/m^2). Now, the slope of the line on the $\log \tau_o - \log \gamma$ plot is equivalent to the flow behavior index m, while the intercept equals the value of $\log(\eta_P C_P^m)$, enabling us to calculate the value of the coefficient η_P, (see Fig. 2b).

2.2 Capillary tube or pipeline viscometer

The essential feature of this device is the measurement of the frictional pressure drop Δp (N/m^2) and the velocity v (m/s) of laminar flow through a cylindrical tube of known length L (m) and diameter D (m). The wall shear stress τ_o and the wall shear rate γ are then calculated, (see [18] and [19])

$$\tau_o = \frac{D\Delta p}{4L}, \tag{11}$$

$$\gamma = \frac{8v}{D}. \tag{12}$$

The mean cross-sectional velocity v is equal to:

$$v = \frac{1}{R^2\pi}\int_0^R u2r\pi dr, \tag{13}$$

where R (m) is the tube radius. Integrating equation (13) by parts, and then substituting equation (1) for $(-du/dr)$ in it, the following relationship for a Herschel-Bulkley fluid is realized

$$v = \left(\frac{\tau_o}{\eta}\right)^{\frac{1}{m}} \frac{D}{8}\Psi\left(m, \frac{\tau_c}{\tau_o}\right), \tag{14}$$

where a dimensionless function $\Psi = \Psi(m, \tau_c/\tau_o)$ is written as

$$\Psi = 4\frac{m}{m+1}\left(\frac{\tau_c}{\tau_o}\right)^2 + \frac{2m}{2m+1}\left(\frac{\tau_c}{\tau_o}\right)\left(1-\frac{\tau_c}{\tau_o}\right)$$

$$+\frac{m}{3m+1}\left(1-\frac{\tau_c}{\tau_o}\right)^2 \times \left(1-\frac{\tau_c}{\tau_o}\right)^{\frac{m+1}{m}}. \tag{15}$$

If a Bingham model is adopted, taking $m = 1$ in equations (14) and (15), the result is the well-known Buckingham-Reiner's equation, (see

[18], [20] and [22])

$$v = \frac{\tau_o D}{8\eta_B}\left(1 - \frac{4}{3}\frac{\tau_c}{\tau_o} + \frac{1}{3}\left(\frac{\tau_c}{\tau_o}\right)^4\right). \tag{16}$$

Omitting of the last term in brackets of equation (16), (which is reasonable if $\tau_c/\tau_o < 0.4$, because the error becomes smaller than 5%), leads to equation (7), giving a value of $C_B = 4/3$. The parameters τ_c and η_B are estimated the same way as described in Fig. 2a.

Flow equation of a pseudoplastic, obtained from equations (8) and (9) by taking $\tau_c = 0$, may be written as, (see [20] and [22])

$$v = \frac{m}{2(3m+1)} D \left(\frac{\tau_o}{\eta_P}\right)^{\frac{1}{m}} \tag{17}$$

It is the form of equation (9), with $C_P = (3m+1)/(4m)$. The parameters τ_o and η_P are estimated the same way as described in Fig. 2b.

3 Frictional resistance

Consider a non-Newtonian laminar uniform one-dimensional free-surface flow. Shear stress distribution is then given by

$$\tau = \tau_o\left(1 - \frac{y}{h}\right), \tag{18}$$

where τ_o is the bottom shear stress (N/m^2), and h - total flow depth (m). The bottom share rate is equal to

$$\gamma = \frac{3v}{h}. \tag{19}$$

Comparing the shear diagrams of a Newtonian and non-Newtonian fluid illustrated in Fig. 1, the following functions can be defined, in a similar way as did Ogihara and Miyazawa, [14], [15], for a pipe flow

$$f(\tau) = \begin{cases} 0, & \tau \le \tau_c, \\ \left(\dfrac{\tau - \tau_c}{\eta}\right)^{\frac{1}{m}}, & \tau > \tau_c, \end{cases} \tag{20}$$

and

$$g(\tau) = \frac{\tau}{\mu} - f(\tau) = \begin{cases} \dfrac{\tau}{\mu}, & \tau \le \tau_c, \\ \dfrac{\tau}{\mu} - \left(\dfrac{\tau - \tau_c}{\eta}\right)^{\frac{1}{m}}, & \tau > \tau_c. \end{cases} \tag{21}$$

The fluid velocity u at a depth y is given by

$$u = \int_0^y \frac{du}{dy} dy = \frac{h}{\tau_o} \int_\tau^{\tau_o} f(\tau)\, d\tau = \frac{h}{\tau_o} \int_\tau^{\tau_o} \left(\frac{\tau}{\mu} - g(\tau)\right) d\tau. \tag{22}$$

Substituting equation (20) in (22), the following expression is obtained after integration

$$u(\tau) = \frac{h}{2\tau_o \mu}\left(\tau_o^2 - \tau^2\right) + \frac{h}{\tau_o}\left(G(\tau) - G(\tau_o)\right), \tag{23}$$

where

$$G(\tau) = \int_0^\tau g(\tau)\, d\tau. \tag{24}$$

Integration of equation (24) gives the Herschel-Bulkley fluid flow velocity distribution over the flow depth

$$u(\tau) = \frac{m}{m+1} \frac{h}{\tau_o} \left[(\tau_o - \tau_c) \left(\frac{\tau_o - \tau_c}{\eta}\right)^{\frac{1}{m}} - (\tau - \tau_c)\left(\frac{\tau - \tau_c}{\eta}\right)^{\frac{1}{m}} \right], \tag{25}$$

or, using (18),

$$u(y) = \frac{m}{m+1} \left(\frac{\tau_o}{h\mu}\right)^{\frac{1}{m}} y_c^{\frac{1}{m}+1} \left[1 - \left(1 - \frac{y}{y_c}\right)^{\frac{1}{m}+1} \right], \tag{26}$$

where y_c is the flow depth for which $\tau = \tau_c$.
Velocity distribution for a Bingham fluid or a pseudoplastic can easily be derived from equations (25) or (26).

In the vicinity of the fluid surface, where $\tau < \tau_c$, (if the fluid possesses the yield stress), there is an unsheared portion of fluid which moves as a solid plug. Velocity of the plug equals

$$u\left(\tau \leq \tau_c\right) = u\left(\tau_c\right) = \frac{m}{m+1} h \frac{\left(\tau_o - \tau_c\right)}{\tau_o} \left(\frac{\tau_o - \tau_c}{\eta}\right)^{\frac{1}{m}}, \tag{27}$$

$$u\left(y \geq y_c\right) = \frac{m}{m+1} \left(\frac{\tau_o}{h\mu}\right)^{\frac{1}{m}} y_c^{\frac{1}{m}+1}. \tag{28}$$

For a Bingham fluid $(m = 1)$, from equation (27) the plug velocity equals (see [2], [7] and [21]),

$$u\left(\tau_c\right) = \frac{h}{2\tau_o \eta_B} \left(\tau_o - \tau_c\right)^2. \tag{29}$$

or, from (19)

$$u\left(y_c\right) = \frac{\tau_o}{2h\mu_B} y_c^2. \tag{30}$$

Flow velocity distribution in dependence on the value of the flow behavior index m can be written in the form

$$X\left(Y\right) = 1 - \left(1 - Y\right)^{\frac{1}{m}+1}, \tag{31}$$

if dimensionless numbers X and Y are defined as

$$X = \frac{u\left(y\right)}{\frac{m}{m+1} \left(\frac{\tau_o}{h\eta}\right)^{\frac{1}{m}} y_c^{\frac{1}{m}+1}}, \tag{32}$$

and

$$Y = \frac{y}{y_c}. \tag{33}$$

Relationship (31) is illustrated in Fig. 3.

The mean cross-sectional flow velocity v is equal to

$$v = \frac{1}{h} \int_0^h u\left(y\right) dy = \frac{1}{\tau_o} \int_0^{\tau_o} u\left(\tau\right) d\tau. \tag{34}$$

Substituting equations (23) and (24) in (25), and after integration:

$$v = \frac{\tau_o h}{3\mu}\left(1 - X(\tau_o)\right), \tag{35}$$

where the dimensionless function $X(\tau_o)$ equals

$$X(\tau_c) = \frac{3\mu}{\tau_o^3}\left[\tau_o G(\tau_o) - \int_0^{\tau_o} G(\tau)\,d\tau\right] \tag{36}$$

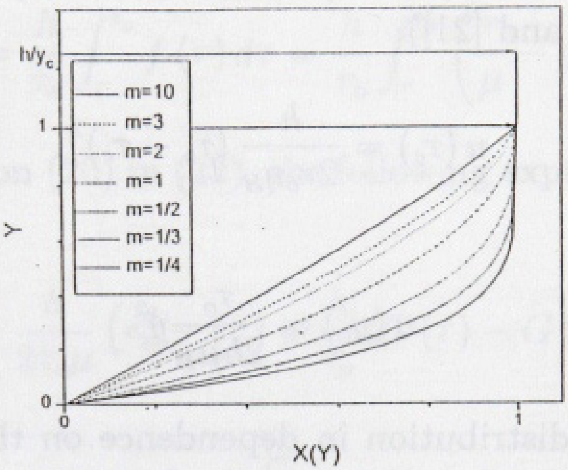


Fig. 3. - Velocity distribution over the flow depth for different m values [9].

By integration of equation (36), the expression for $X(\tau_o)$ is defined

$$X(\tau_o) = 1 - \frac{\mu}{\tau_o}\left(\frac{\tau_o}{\eta}\right)^{\frac{1}{m}}\Phi\left(m, \frac{\tau_c}{\tau_o}\right), \tag{37}$$

where a dimensionless function $\Phi = \Phi\left(m, \frac{\tau_c}{\tau_o}\right)$ is expressed by

$$\Phi = \frac{3m}{(m+1)(2m+1)}\left[1 + m\left(1 + \frac{\tau_c}{\tau_o}\right)\right]\left(1 - \frac{\tau_c}{\tau_o}\right)^{\frac{m+1}{m}}, \tag{38}$$

which, combining it with equation (35), gives the equation of laminar uniform one-dimensional free-surface flow of a Herschel-Bulkley fluid in the form

$$v = \left(\frac{\tau_o}{\eta}\right)^{\frac{1}{m}}\frac{h}{3}\Phi\left(m, \frac{\tau_c}{\tau_o}\right). \tag{39}$$

Taking into consideration a definition of the Darcy-Weisbach friction factor f

$$f = \frac{\tau_o}{\frac{1}{2}\rho_m v^2}, \tag{40}$$

where ρ_m (kg/m^3) denotes density of the given fluid, the following equation is obtained from (35)

$$f = \frac{6}{Re\,[1 - X(\tau_o)]} = \frac{6}{Re_1}, \tag{41}$$

where Reynolds number Re is equal to

$$Re = \frac{\rho_m v h}{\mu}. \tag{42}$$

Substituting the equation (37) for $X(\tau_o)$, the extended Reynolds number is then defined as

$$Re_1 = Re\,[1 - X(\tau_o)] = \frac{\rho_m v h}{\mu}\left(\frac{\tau_o}{\eta}\right)^{\frac{1}{m}} \Phi = \frac{\rho_m v^{2-m} h^m}{\mu 3^{m-1}} \Phi^m. \tag{43}$$

Equation (41) means that, even though the fluid has non-Newtonian properties, the friction factor can be determined by applying the standard Moody diagram, if the extended Reynolds number, according to formulation (43), is introduced.

For a Bingham fluid, the following expression for the function Φ is obtained from (38)

$$\Phi = 1 - \frac{3}{2}\frac{\tau_c}{\tau_o} + \frac{1}{2}\left(\frac{\tau_c}{\tau_o}\right)^3. \tag{44}$$

The flow equation is formed by simplifying equation (39), (see [2], [7], [11], [12], [16] and [24])

$$v = \frac{\tau_o h}{3\eta_B}\left[1 - \frac{3}{2}\frac{\tau_c}{\tau_o} + \frac{1}{2}\left(\frac{\tau_c}{\tau_o}\right)^3\right], \tag{45}$$

and the extended Reynolds number equals:

$$Re_1 = \frac{\rho_m v h}{\eta_B}\left[1 - \frac{3}{2}\frac{\tau_c}{\tau_o} + \frac{1}{2}\left(\frac{\tau_c}{\tau_o}\right)^3\right]. \tag{46}$$

For a pseudoplastic, the following relationships are derived, [22]:

$$\Phi = \frac{3m}{2m+1}, \tag{47}$$

$$v = h\frac{m}{2m+1}\left(\frac{\tau_o}{\eta_P}\right)^{\frac{1}{m}}, \tag{48}$$

$$Re_1 = \frac{\rho_m v^{2-m} h^m}{\eta_P 3^{m-1}}\left(\frac{3m}{2m+1}\right)^m \tag{49}$$

The value of $Re = 2100$ associated with transition from laminar to turbulent flow regime may be recommended, [7]. The transition velocity can be calculated using equations (46) and (49).

4 Experiments

4.1 Properties of investigated fluid

The fluids used in experiments are water-kaolinite clay mixtures with different solid particles concentrations. The mean diameter of the particles is 0.006 mm.The specific density of the clay material is 2.65, and its chemical composition: SiO_2 ($\approx 50\%$), Al_2O_3 (37%), CaO ($< 5\%$) and Fe_2O_3 (<3%). Nine solid-liquid mixtures were investigated, whose density and solids concentrations, (by volume C_V and by weight C_W), are presented in Table 1.

Table 1. Basic physical properties of investigated fluids [21].

C_v (%)	2.0	4.0	6.2	8.6	11.2	13.9	16.9	20.1	23.6
C_w (%)	5	10	15	20	25	30	35	40	45
ρ_m (kg/m³)	1032	1066	1103	1142	1184	1230	1279	1332	1389

4.2 Rheological measurements

Rheological parameters were determined on the basis of measurements by the coaxial cylinder rotational viscometer. The experiments and equipment were described in an earlier paper [21]. Only laminar flow data were used for rheological modelling. Mixtures with solids concentration up to 8.6 % were found to be Newtonian, whereas shear diagrams of the thicker mixtures point to a non-Newtonian character, the results being in agreement with findings by [23]. Treating the thicker mixtures as Bingham fluids, parameters were estimated by [11]. In this paper, parameters are determined assumig the pseudoplastic behavior of the mixtures, according to procedure described in Sect. 2, (see Fig. 4). Values of the parameters for both rheological models are shown in Table 2, (where n denotes the number of data points used for definition of the models, and r^2 is the coefficient of determination).

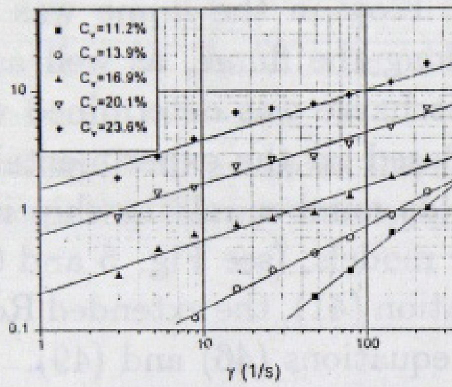


Fig. 4. - Estimation of the pseudoplastic parameters on the basis of measurements by rotational viscometer.

Table 2. Rheological parameters of the investigated mixtures.

	Bingham fluid					Pseudoplastic				
C_v (%)	C_B (/)	τ_c (Pa)	η_B (mPas)	n (/)	r^2 (/)	m (/)	C_P (/)	η_P (mPas^m)	n (/)	r^2 (/)
2.0	1.02	0.00	1.18	4	0.99	1.00	1.00	1.18	4	0.99
4.0	1.02	0.00	1.22	4	0.99	1.00	1.00	1.22	4	0.99
6.2	1.02	0.00	1.88	5	0.99	1.00	1.00	1.88	5	0.99
8.6	1.02	0.00	2.16	6	0.99	1.00	1.00	2.16	6	0.99
11.2	1.02	0.10	3.30	4	0.99	0.99	1.01	4.59	4	0.99
13.9	1.02	0.20	6.90	7	0.99	0.70	1.02	29.82	7	0.99
16.9	1.02	0.69	12.10	9	0.98	0.45	1.03	201.37	9	0.97
20.1	1.02	1.96	20.10	9	0.96	0.42	1.04	628.08	9	0.97
23.6	1.02	6.19	43.00	9	0.91	0.41	1.04	1499.73	9	0.95

4.3 Frictional resistance estimation

Experiments were carried out in a laboratory flume at Hydraulic Laboratory of the Faculty of Civil Engineering in Belgrade. The laboratory rig and the measurement equipment were described in detail by [21]. The steady circulation of the mixtures was ensured by a sludge pump. The concentration of any particular mixture was constant throughout experiments, a condition easily satisfied by the relatively small size of the experimental rig. Electrical probes, connected to a data acquisition and processing system, were used for continuous measurement of depths and velocities. Flow in the flume was nearly uniform. The levels were recorded along the flume, as well as point velocities over a cross-section. The discharge was determined volumetrically, and by velocity integration. Based on the experimental results, [21], friction factor extended Reynolds number relationship is illustrated for Bingham and pseudoplastic models, (see Fig. 5 and 6). Theoretical curves are obtained using equation (41), the extended Reynolds number values being calculated using equations (46) and (49).

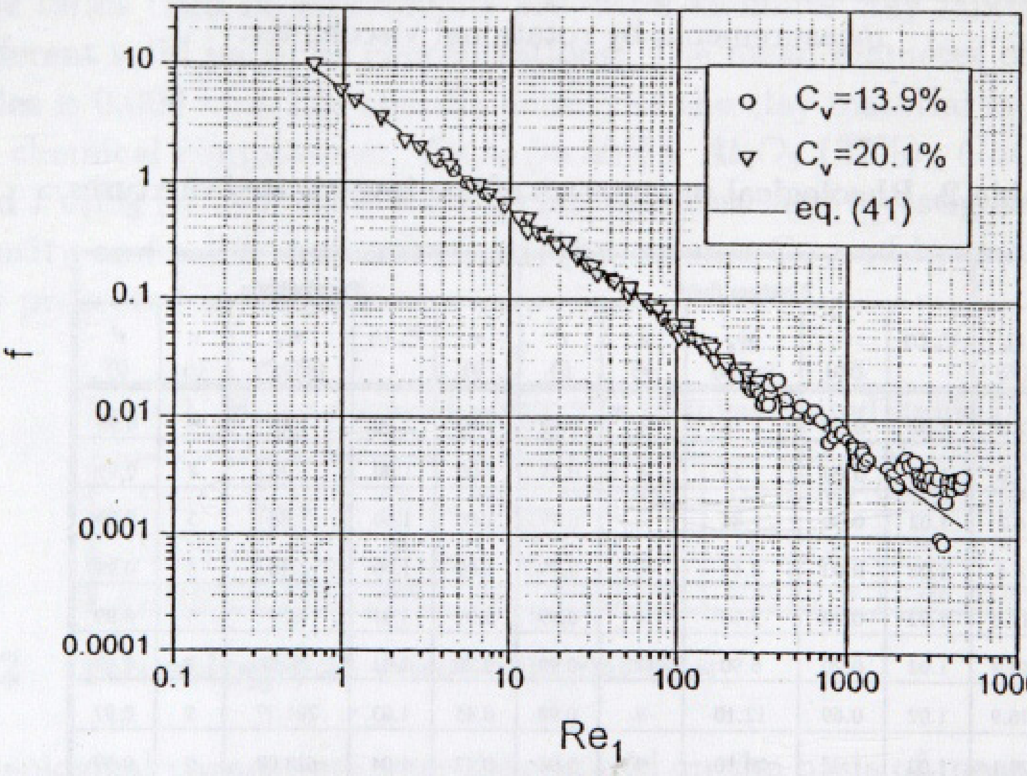


Fig. 5. - Friction factor-extended Reynolds number relationship for the Bingham model.

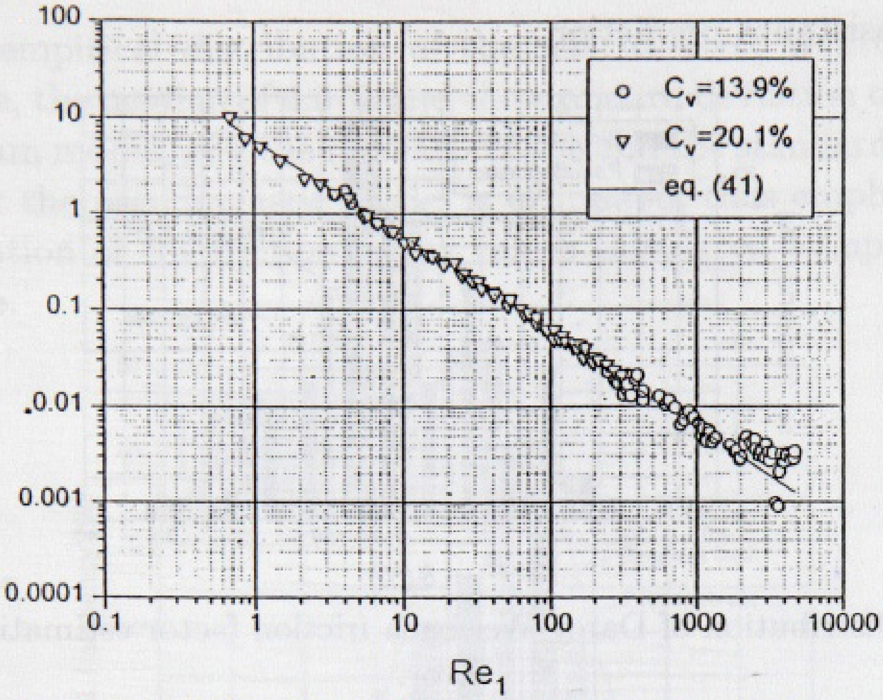


Fig. 6.- Friction factor-extended Reynolds number relationship for the pseudoplastic.

5 Accuracy of the flow resistance estimation

In order to analyze an accuracy of the flow resistance estimation, relative error is defined as

$$\delta = \frac{f_t - f}{f_t} 100\%, \tag{50}$$

where f_t denotes the theoretical value of the friction factor, calculated using the equation (41). In Fig. 7 the distribution of the relative error is shown for the whole range of shear rates. An approximation of the empirical distribution by the normal distribution gives the median of 2.5 % and the standard deviation of 12 % for the Bingham model, and the median - 0.5 % and the standard deviation - 11.3 %for the pseudoplastic model, which points to relatively equalized accuracy in the flow resistance prediction.

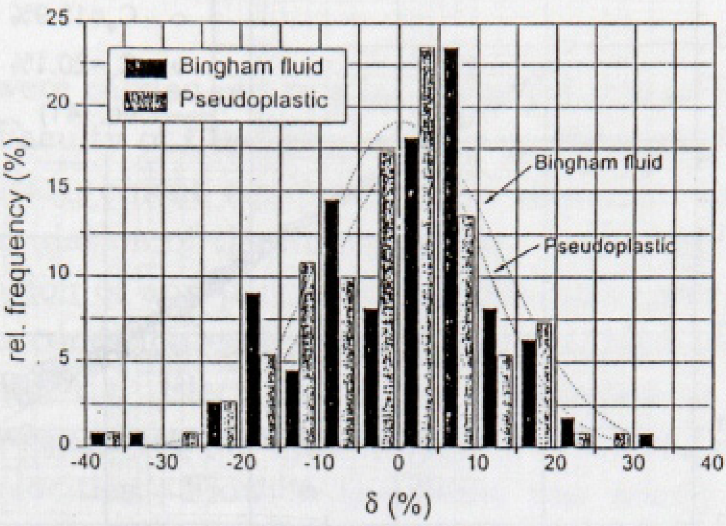


Fig. 7. - Distribution of Darcy-Weisbach friction factor estimation relative error.

However, analyzing the relative error dependence on the rate of shear, see Fig. 8, it can be concluded that the friction factor is often overestimated in a zone of the smallest shear rates, if the Bingham model is used, (which is logical, when considering the form of shear diagrams). Regarding the relative error for the pseudoplastic, practically no dependence on the rate of shear is observed.

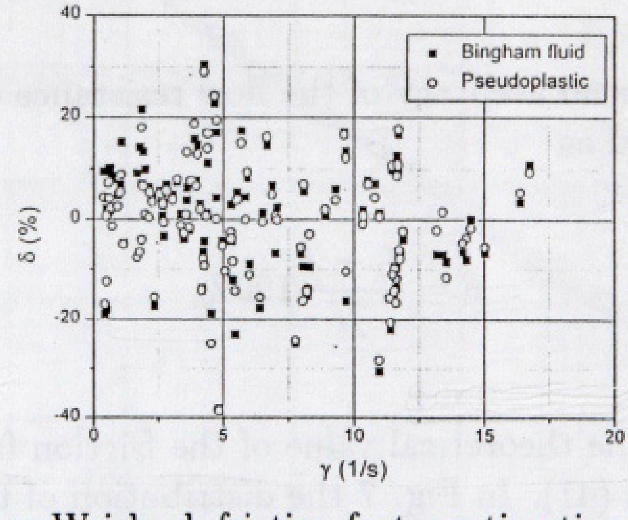


Fig. 8. - Darcy-Weisbach friction factor estimation relative error in dependence on the shear rate.

When the zone of the shear rates up to 10 s^{-1} is considered, a certain advantage of the pseudoplastic model application can be found, see Fig

9. If the empirical distribution of the relative error is fitted by the normal one, the median of 4.6 % and the standard deviation of 8.7 % for the Bingham model, and the median - 0.8 % and the standard deviation - 7.3 % for the pseudoplastic model is estimated, thus emphasizing an overestimation of the friction factor values calculated by applying the former one.

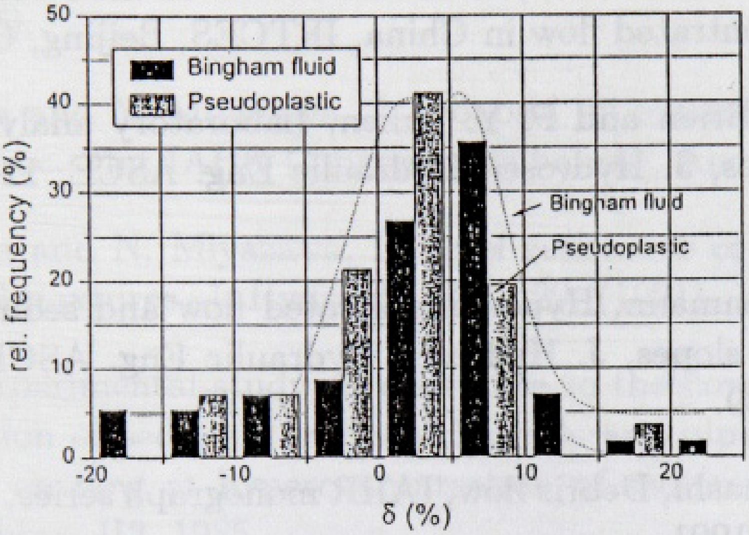


Fig. 9. - Distribution of Darcy-Weisbach friction factor estimation relative error for shear rates up to 10 s^{-1}.

6 Conclusion

Using the Herschel-Bulkley rheological model, the equation of laminar uniform one-dimensional flow can be derived, the relationship being easily simplified to widely used Bingham and pseudoplastic models. On the basis of the equation, an expression for Darcy-Weisbach friction factor is defined. The frictional resistance estimation can be simplified to the use of Moody diagram for Newtonian fluids, if the extended Reynolds number, including all rheological fluid parameters, is introduced. Accuracy level of the friction factor estimation, analyzed on the basis of experimental results using water-clay mixtures, appears to be nearly equal if Bingham model or pseudoplastic is applied, except for the small shear rates, where the friction factor is somewhat overestimated, (about 5 %) when Bingham model is applied.

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Proračun otpora trenja u otvorenom toku jednog nenjutnovskog fluida primenom Bingham-ovog i pseudoplastičnog reološkog modela

U radu je izvedena jednačina jednolikog, ravanskog toka Herschel-Bulkley-jevog fluida sa slobodnom površinom, u lamilarnom režimu tečenja. Prikazana je jednačina kojom se opisuje odgovarajući raspored brzine po dubini toka. Na osnovu koncepta, ranije primenjenog na slučaj tečenja pod pritiskom, definisan je Reynolds-ov broj u opštijem obliku, tako da obuhvati sva tri parametra modela Herschel-Bulkley-ja. Primenom "proširenog" Reynolds-ovog broja, proračun Darcy-Weisbach-ovog koeficijenta linijskih otpora se svodi na upotrebu Moody-jevog dijagrama, koji važi za "čistu" vodu. Uprošćavanjem napisanih jednačina dobijaju se relacije za, u praksi najčešće korišćene, Bingham-ov i pseudoplastični fluid. Teorijska razmatranja praćena su rezultatima eksperimenata, izvedenim sa mešavinama vode i gline. Reološki parametri mešavina odredjeni su na osnovu merenja rotacionim viskozimetrom. Prema dobijenim diagramima smicanja definsani su reološki modeli analiziranih mešavina, (Bingham-ov i pseudoplastični). Korišćenjem usvojenih modela, sračunat je koeficijent otpora i uporedjen sa teorijskom zavisnošću. Izvršena je analiza tačnosti odredjivanja koeficijenta trenja, u nameri da se zaključi koji je od dva reološka modela, (Bingham-ov ili pseudoplastični), pogodniji za primenu.