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HEAT TRANSFER IN A SECOND-ORDER FLUID FLOW BETWEEN TWO PARALLEL PLATES WHEN ONE OF THEM MOVES WITH VELOCITY U(t)

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1. Introduction

In recent years, whether it is electrical, mechanical, chemical or atomic the study of heat transfer by laminar flow of Newtonian or Non-Newtonian fluids has gained considerable importance. Schlichting (1955), Goldstein (1938), Pai (1956) have studied several cases of heat transfer problems in viscous liquids. The problem of heat transfer by the lamineer flow of an elastico-viscous liquid between two parallel plates of uniform temperature has been studied by Jain (1963), Mishra (1964) and Rath and Satpathy (1974). Mishra (1965) extended this problem to the case when the plates are having linearly varying wall temperature.

Following Bhatnagar (1966) method of solution Tikeker (1967) has given a close form of solution of the differential equation for the velocity field. Sparrow and Gregg (1960) have studied the time of establishment of the flow of a viscous incompressible fluid past an unsteady rotating disk by considering the deviation of the instantaneous value of the skin-friction from the quasi-steady value. Panigrahi and Bhunya (1988) studied the flow of viscous second-order fluid flow between two parallel plates maintained at constant temperature and fluctuating temperature. Panigrahi and Bhunya (1989) have extended the above problem to the case when one of the plates moves with velocity $U(t)$. The present analysis is aimed to study the Heat Transfer phenomenon as an extension.

2. Formulation of the problem

In the present investigation a viscous incompressible second-order fluid occupying the space between two parallel infinite plates lying in the plane $y = \pm d$ is considered. The upper plate is assumed to move with a time dependent velocity $U(t)$ in its own plane and in x-direction while the flow of the fluid is entirely due to the motion of the plate and a pressure gradient in the same direction. Taking $u = u(y,t)$ and $p = p(x,t)$ the equation of continuity

$$\frac{\partial u}{\partial x} = 0, \tag{2.1}$$

the momentum equation

$$\frac{\partial u}{\partial t} - \frac{\mu_1}{\rho}\frac{\partial^2 u}{\partial y^2} - \frac{\mu_2}{\rho}\frac{\partial^3 u}{\partial y^2 \partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} = A(t) \quad \text{(say)} \tag{2.2}$$

and the energy equation

$$\rho c \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial y^2} + \mu_1 \left(\frac{\partial u}{\partial y}\right)^2 \tag{2.3}$$

are satisfied where ρ, c, k, μ_1 and μ_2 are density, specific heat, coefficient of thermal conductivity and material constants respectively.

The boundary conditions are

$$\begin{aligned} y &= -d, \qquad u = 0, \qquad T = T_0, \\ y &= d, \qquad u = U(t), \qquad T = T_1. \end{aligned} \tag{2.4}$$

3. Solution of the equations

Solution of equation (2.3) subject to the boundary condition (2.4) may be obtained by virtue of non-dimensionalisation through the following transformations:

$$\begin{aligned} \eta &= \frac{yU}{\nu}, \qquad \eta_d = \frac{dU}{\nu} \\ \frac{u}{U} &= f(\eta, \beta_r) = f_0 + \beta_1 f_1 + \beta_2 f_2 + \dots, \\ \frac{d^2 A(t)}{\nu U} &= A_0 + \beta_1 A_1 + \beta_2 A_2 + \dots, \\ \frac{T - T_0}{T_1 - T_0} &= \theta(\eta, \beta_r) = \theta_0(\eta) + \beta_1\theta_1(\eta) + \beta_2\theta_2(\eta) + \dots, \\ \beta_r &= \frac{d^r D^r U}{U^{r+1}}, \qquad D^r U = \frac{d^r U}{dt^r}, \qquad \beta_1 = \frac{d\dot{U}}{U^2}, \\ u &= U(f_0 + \beta_1 f_1 + \beta_2 f_2 + \dots) \end{aligned} \tag{2.5}$$

where

$\nu = \frac{\mu}{\rho}$	: kinematic viscouty
η	: Reynolds number
η_d	: Reynolds number with channel width d
f_0	: steady parabolic velocity profile
$f_1, f_2, \dots$	: unsteady velocity profile
$A_0, A_1, \dots$	: pressure gradient parameter
β_r	: expansion parameter
$\theta_0, \theta_1, \dots$	: gradients of temperature.

Making use of (2.4) and (2.5) equation (2.3) takes the form

$$\begin{aligned}&\beta_1 \frac{p}{\eta_d}\eta(\theta_0' + \beta_1\theta_1' + \beta_2\theta_2' + \dots) + \theta_1 \frac{p}{\eta_d}(\beta_2 - 2\beta_1^2) + \theta_2 \frac{p}{\eta_d}(\beta_3 - 3\beta_1\beta_2) \\ &= (\theta_0'' + \beta_1\theta_1'' + \beta_2\theta_2'' + \dots) + pE(f_0' + \beta_1 f_1' + \beta_2 f_2' + \dots)^2\end{aligned} , \qquad (2.6)$$

where $pE = p \times E$, $p =$ Prandtl number and $E =$ Ekert number. Equating the like powers of $\beta_1, \beta_2, \dots,$ and the independent terms

$$\theta_0'' = -pE{f_0'}^2,$$
$$\theta_1'' = \frac{p}{\eta_d}\eta\theta_0' - 2pEf_0'f_1'$$

and

$$\theta_2'' = \frac{p}{\eta_d}\theta_1 - 2pEf_0'f_2' \qquad (2.7)$$

the corresponding boundary conditions of (2.4) reduces to

$$\eta = -\eta_d, \qquad \theta_0 = \theta_1 = \theta_2 = 0$$

and

$$\eta = \eta_d, \qquad \theta_0 = 1, \quad \theta_1 = \theta_2 = 0. \qquad (2.8)$$

Solving (2.7), with the boundary conditions (2.8) we get

$$\begin{aligned}\theta_0 = -\,pE\Bigg\{&\frac{A_0^2\eta_d^2}{12}\left(\frac{\eta}{\eta_d}\right)^4 - \frac{A_0\eta_d}{6}\left(\frac{\eta}{\eta_d}\right)^3 + \frac{1}{8}\left(\frac{\eta}{\eta_d}\right)^2 + \frac{\eta}{\eta_d}\left(\frac{A_0\eta_d}{6} - \frac{1}{2pE}\right) \\ &-\frac{1}{2}\left[\frac{(A_0\eta_d)^2}{6} + \frac{1}{pE} + \frac{1}{4}\right]\Bigg\},\end{aligned} \qquad (2.9)$$

$$\theta_1 = \sum_{i=0}^{6} C_i\eta^i \qquad (2.10)$$

and

$$\theta_2 = \sum_{i=0}^{8} D_i\eta^i, \qquad (2.11)$$

where

$$\phi = (3\Lambda + \frac{1}{2} - \frac{1}{\eta_d})A_0 + \frac{1}{2\eta_d},$$
$$m = pE,$$
$$n = \frac{\Lambda_\phi}{2} - \frac{A_0}{16} + \frac{\phi}{4} + \frac{A_2}{2\eta_d^2},$$

$$r = \frac{A_0 A}{8} + \frac{\phi}{24},$$

$$s = \frac{7}{720} + \frac{\Lambda}{12},$$

$$\sigma = \frac{\Lambda}{12} + \frac{1}{72},$$

$$C_0 = \frac{m\eta_d}{6}\left\{\left[A_0\eta_d^2\left(\frac{pA_0}{15} + \frac{A_0}{5} - \phi\right) - \frac{p+1}{8}\right] - \frac{1}{4}\right\},$$

$$C_1 = m\left\{\left[\frac{pA_0}{4}\left(\frac{1}{90}\right) - \frac{A_0}{4}\left(\frac{14}{45}\right) + \frac{\phi}{6}\right]\eta_d - \frac{p}{12m}\right\},$$

$$C_2 = \frac{m}{2\eta_d},$$

$$C_3 = \frac{m}{6\eta_d}\left(\frac{A_0}{6} - \frac{pA_0}{6} + \frac{p}{2m\eta_d} - \phi\right),$$

$$C_4 = \frac{m}{6\eta_d}\left(A_0\phi - \frac{p+1}{8\eta_d^2}\right),$$

$$C_5 = \frac{mA_0}{20\eta_d^3}\left(\frac{p+2}{2}\right),$$

$$C_6 = \frac{mA_0}{30\eta_d^3}\left[-\frac{(p+3)}{3}\right],$$

$$D_0 = \frac{m\eta_d^4}{2}\left\{\frac{A_0^2}{280}\left(\frac{2p^2+6p+9}{18}\right) - \frac{1}{15}\left(\frac{pA_0\phi}{6} + 8A_0 r - \frac{p^2+p}{48\eta_d^2}\right) - \right.$$
$$- \frac{1}{6}\left(\frac{p}{24\eta_d^2} + \frac{A_0}{60\eta_d} + \frac{3\sigma}{\eta_d^2} - 4A_0 n\right) - \frac{p}{\eta_d^2}\left[A_0^2\eta_d^2\left(\frac{p+3}{90}\right) - \right.$$
$$\left.\left. - \frac{A_0\eta_d^2\phi}{6} + \frac{p+3}{48} - \frac{s}{p}\right]\right\},$$

$$D_1 = m\eta_d^2\left\{-\frac{A_0}{840}\left(\frac{p^2+2p+1}{2}\right) - \frac{1}{20}\left[\frac{pA_0(1-p)}{36} + \frac{p^2}{12m\eta_d} - \frac{p\phi}{6} - 4r - \right.\right.$$
$$\left. - 6A_0\sigma\right] + \frac{pA_0}{6}\left(\frac{p}{40} - \frac{p}{12mA_0\eta_d} + \frac{p}{36} + \frac{7}{90} + \frac{\phi}{6A_0} - \frac{1}{120pA_0\eta_d} + \right.$$
$$\left.\left. + \frac{2n}{pA_0} + \frac{2s}{p}\right)\right\},$$

$$D_2 = \frac{pm}{12}\left[\frac{A_0^2\eta_d^2}{5}\left(\frac{p+3}{3}\right) - A_0\eta_d^2\phi + \frac{p+3}{8} - \frac{6s}{p}\right],$$

$$D_3 = pmA_0\left(\frac{p}{72mA_0\eta_d} - \frac{19p}{2160} - \frac{7}{540} + \frac{1}{720pA_0\eta_d} - \frac{\phi}{36A_0} - \frac{n}{3pA} - \frac{s}{3p}\right),$$

$$D_4 = \frac{m}{3}\left(\frac{p}{96\eta_d^2} + \frac{A_0}{240\eta_d} + \frac{\sigma}{12\eta_d^2} - A_0 n\right),$$

$$D_5 = \frac{pm}{5\eta_d^2}\left(\frac{A_0}{144} - \frac{pA_0}{144} + \frac{p}{48m\eta_d} - \frac{\phi}{24} - \frac{r}{p} - \frac{6A_0\sigma}{4p}\right),$$

$$D_6 = \frac{pm}{10\eta_d^2}\left(\frac{A_0\phi}{18} - \frac{p}{164\eta_d^2} - \frac{1}{164\eta_d^2} + \frac{8A_0r}{3p\eta_d^2}\right),$$

$$D_7 = \frac{mA_0}{840\eta_d^4}\left(\frac{p^2+2p+1}{2}\right)$$

and

$$D_8 = \frac{mA_0^2}{560\eta_d^4}\left[-\frac{(2p^2+6p+9)}{18}\right].$$

4. Nusselt Number

The Nusselt number at the lower plate is

$$N_0 = \left(\frac{d\theta_2}{d\eta}\right)_{\eta=-\eta_d} = \sum_{i=1}^{8} iD_i(-\eta_d)^{i-1} \tag{2.12}$$

and at the upper plate is

$$N_1 = \left(\frac{d\theta_2}{d\eta}\right)_{\eta=\eta_d} = \sum_{i=1}^{8} iD_i\eta_d^{i-1} \tag{2.13}$$

5. Conclusion

From numerical solutions interesting conclusions have been drawn depicting graphs and tables **Fig.1** shows the temperature profile for different values of Reynolds numbers η. Temperature θ increases with the increase of Reynolds number η. The value of θ at 0 coincides with the value of the Reynolds number at $\eta = -1$ and at $\eta = 1$. For $\eta = 0.4$ the temperature attains a maximum value. **Fig.2** and **Fig.3** indicate the effect of elastic parameter Λ on Nusselt number at the lower and upper plates respectively. At the lower plate the value of Nusselt number N_0 increases whereas at the upper plate the value of Nusselt number N_1 decreases with the increase of elastic parameter Λ.

TABLE - 1

value of η	value of v_2
-1.0	0.00000
-0.8	0.00025
-0.6	0.00044
-0.4	0.00054
-0.2	0.00062
0.0	0.00070
0.2	0.00078
0.4	0.00083
0.6	0.00076
0.8	0.00050
1.0	0.00000
-1.0	0.00000
-0.8	0.00144
-0.6	0.00238
-0.4	0.00291
-0.2	0.00331
0.0	0.00372
0.2	0.00412
0.4	0.00433
0.6	0.00400
0.8	0.00267
1.0	0.00000

TABLE - 2

value of Λ	value of N_0
5.0	0.00015
10.0	0.00240
15.0	0.01221
20.0	0.03858
25.0	0.09416

TABLE - 3

value of Λ	value of N_1
5.0	-0.00023
10.0	-0.00399
15.0	-0.02039
20.0	-0.06456
25.0	-0.15761

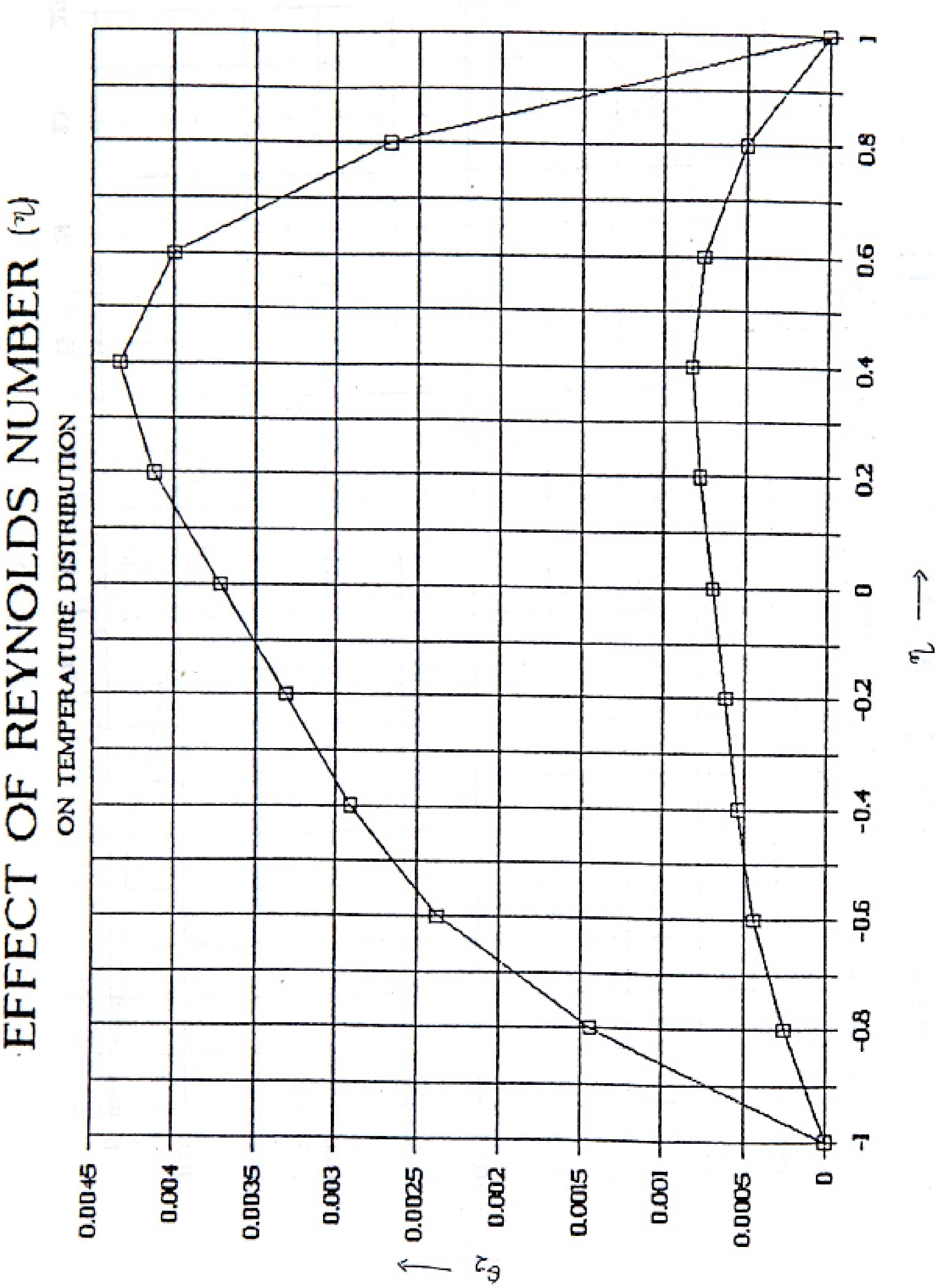

Figure 1. EFFECT OF REYNOLDS NUMBER (η) ON TEMPERATURE DISTRIBUTION

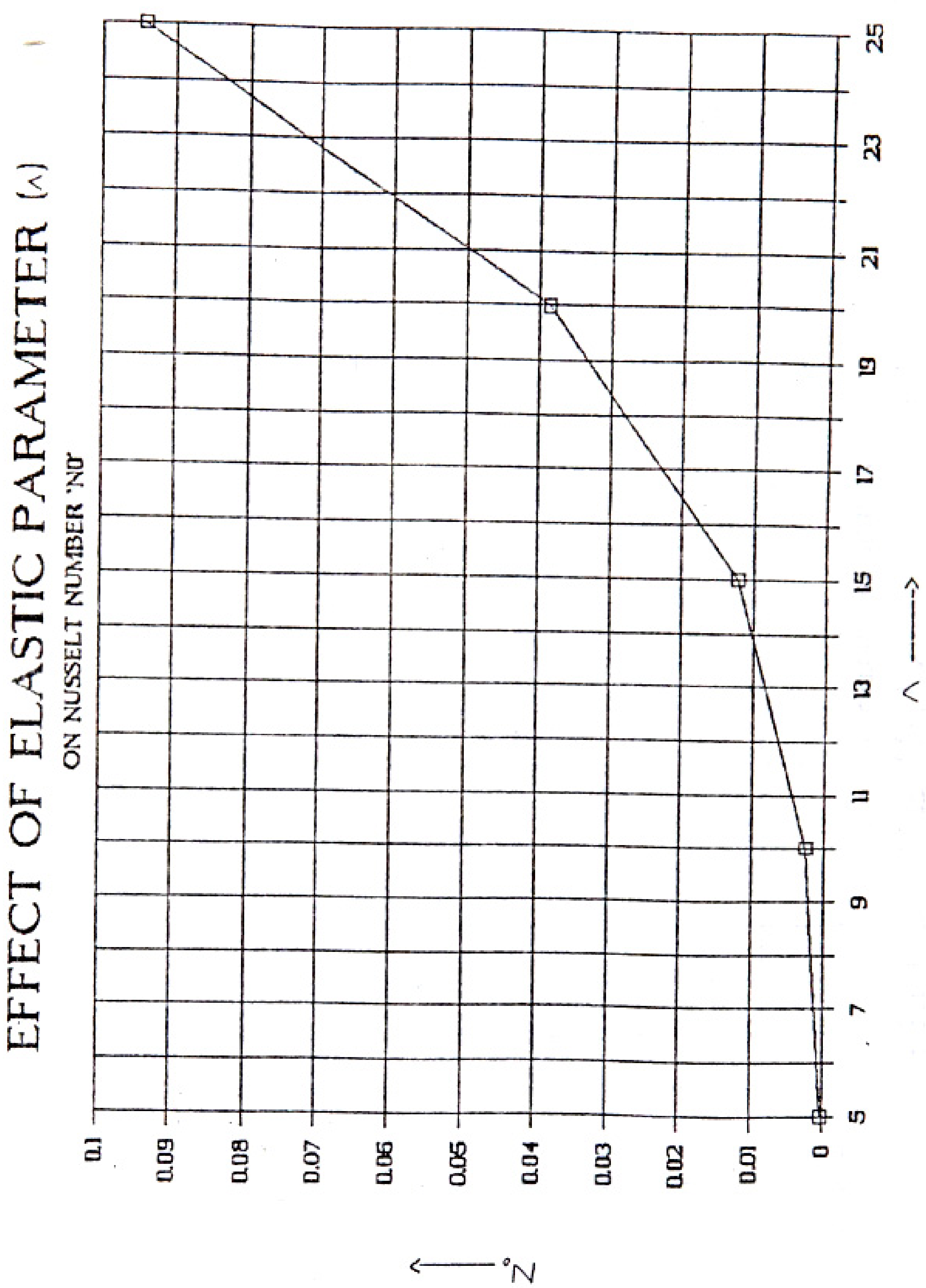

Figure 2. EFFECT OF ELASTIC PARAMETER (Λ) ON NUSSELT NUMBER N_0

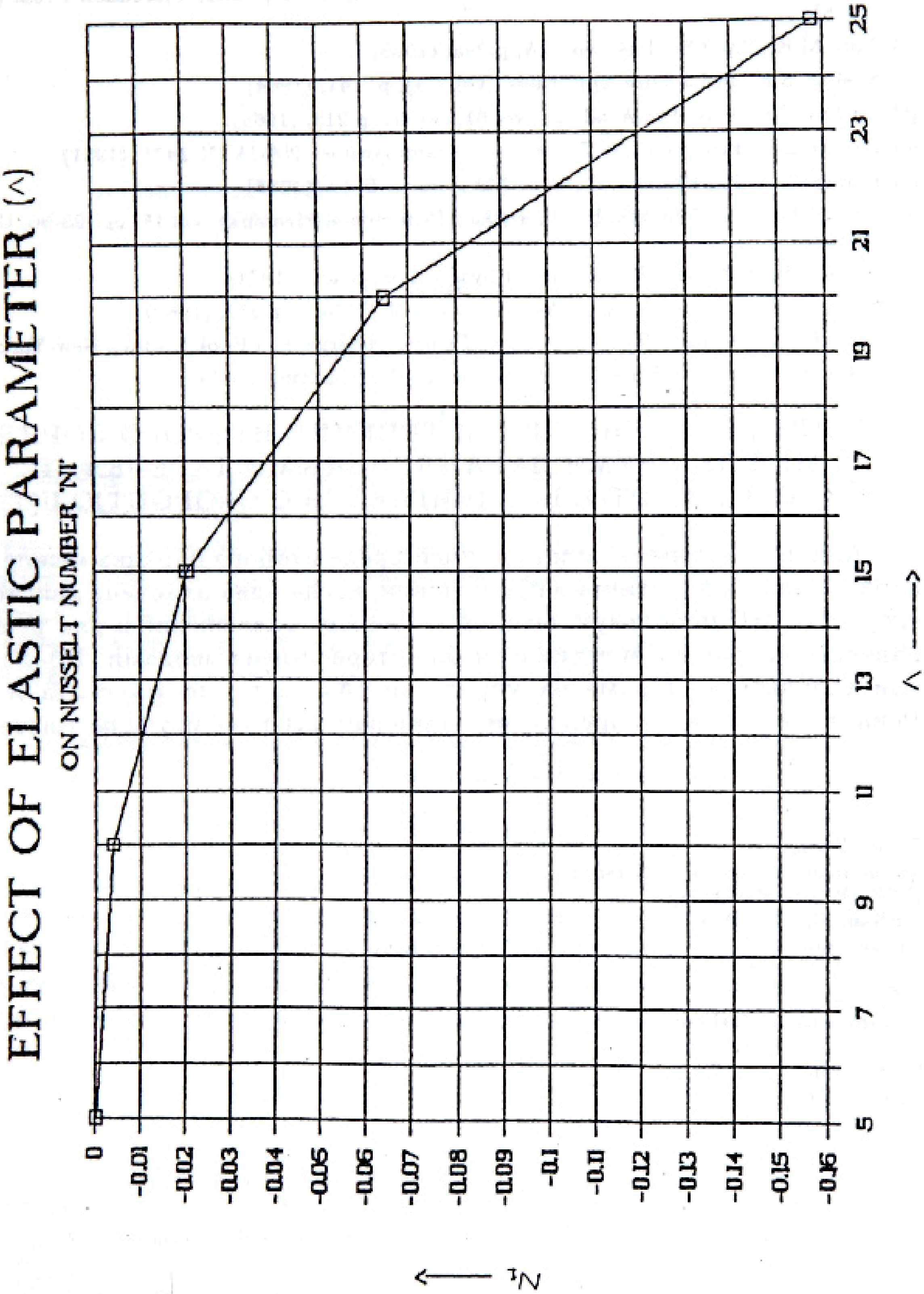

Figure 3. EFFECT OF ELASTIC PARAMETER (Λ) ON NUSSELT NUMBER N_1

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ТЕПЛОТА ПЕРЕНОСИТЬ В ТЕЧЕНИИ ВТОРОГО ПОРЯДКА МЕЖДУ ДВУМЯ ПАРАЛЛЕЛЬНЫМИ ТАРЕЛКАМИ, ОДНА ИЗ КОТОРЫХ ДВИГАЕТСЯ С СКОРОСТЮ U(T)

Анализ теплопередачи в текучей среде второго порядка между двумя паралледными пластинами при условии, когда одна пластина двигается со скоростю $U(t)$ используя методом серйоного разложения в ряд решено уравнение сохраненя энергии с очень интересными выводами. Последствия или результаты параметра упругости (Λ) на число Нусселта и число Рейнолдса (η) на распределение температуры тоже изучены тщательно.

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