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MODELING TWO-PHASE FLOW OF GAS-SOLID PARTICLES MIXTURE DURING COMBUSTION

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1. Introduction

We meet with the need for modeling two-phase flow of gas-particles mixture during combustion when we consider processes of ignition and flamespreading through propellant charges. These processes have exceptional practical importance because their studing enables:

- determination influence of a ignition system and a propellant charge organisation on interior ballistics characteristics,
- more accurate prediction of interior ballistics characteristics,
- prediction conditions for creation of shock waves in a gun barrel which cause irregular function of projectile and damage of gun barrel,
- consideration conditions for occuring propellant grains fracture caused by their impact on a prajectile base or chamber parts or due to intergranular stress an contacts between propellant grains.

All these are the questions which need answers during design of propellant charges especially because requirements for better interior ballistics performances of modern guns. During investigations processes of ignition and flamespreading through propellant charges we meet three basic problems: theoretical modeling, numerical modeling of the developed theoretical model and experimental investigations which, besides determination of the basic parameters of processes, make the base for the model verificatian.

2. Theoretical model

During studing a problem of flamespreading through propellant charge we meet two-phase flow of a gas and solid particles composed of propellant grains and their gas cambustian products. Having on mind the great complexity of considered processes it was necessary to accept certain assumptions which enabled creatian of conservatian equatians and relations for constitutive laws.

2.1 Basic assumptions

The basic assumptions used during model creation are:

- the gas and solid phases occupy separate complementary regions and within each region the material may be treated as a homogeneous continuum,
- the flow of heterogeneous mixture, composed of two interacting continua, can be described by appropriatly defined averages of flow properties,
- if solid phase combustion occurs, the energy deposition is taken to be in the gas only,
- the solid phase is deformable and incompressible, however locally no relative motion between the solid particles is considered; thus the average stress in the solid phase is an isotropic normal stress,
- the influence of solid phase deformation on the particle surface area is neglected, and the interfacial average of the particle velocity is equal to the volume average in the absence of burning,
- the interphase drag and the interphase heat transfer are determined from steady state correlations,
- the Noble-Abel equatian of state will be employed; the specific heats are taken to be independent of temperature,
- the regression rate of the burning propellant surface is a function of the average gas properties and the propellant surface temperature,
- heat transfer to the solid phase is treated as a one-dimensional process in order to determine the propellant surface temperature,
- the pressure drop at the gas-solid interface is negligible.

2.2 Conservatin equations

Starting from the local-instant equations, using averaging, we create the equations of conservation of mass, momentum and energy for both phases (time average will be denoted by single overbar and phase average by double overbar)

The continuity equation for the gas phase:

$$\frac{\partial(\epsilon\bar{\bar{\rho}})}{\partial t}+\nabla(\epsilon\bar{\bar{\rho}}\vec{V}_g)=(1-\epsilon)\,\frac{S_b\rho_b}{W_b}\langle u\rangle^i \tag{1}$$

The continuity equation for the solid phase:

$$\frac{\partial(1-\epsilon)}{\partial t}+\nabla\left[(1-\epsilon)\vec{V}_b\right]=-(1-\epsilon)\,\frac{S_b}{W_b}\langle u\rangle^i \tag{2}$$

The momentum conservation equation for the gas phase:

$$\frac{\partial\left(\epsilon\bar{\bar{\rho}}\vec{V}_g\right)}{\partial t}+\nabla\left(\epsilon\bar{\bar{\rho}}\vec{V}_g\vec{V}_g\right)=-\epsilon\nabla\bar{\bar{p}}+\nabla\left[\epsilon\left(T_g+T^T\right)\right]-$$
$$-(1-\epsilon)\,\frac{S_b}{W_b}\langle\vec{F}\rangle^i+\vec{V}_b\,(1-\epsilon)\,\frac{S_b\rho_b}{W_b}\langle u\rangle^i \tag{3}$$

The momentum conservation equatian for the solid phase

$$\frac{\partial\left[\left((1-\epsilon)\rho_b\vec{V}_b\right)\right]}{\partial t}+\nabla\left[(1-\epsilon)\,\rho_b\vec{V}_b\vec{V}_b\right]=-(1-\epsilon)\,\nabla\bar{\bar{p}}+\nabla\left[(1-\epsilon)\,R\right]+ \\ +(1-\epsilon)\,\frac{S_b}{W_b}\langle\vec{F}\rangle^i-\vec{V}_b\,(1-\epsilon)\,\frac{S_b\rho_b}{W_b}\langle u\rangle^i \tag{4}$$

The energy conservation equation for the gas phase:

$$\frac{\partial\left(\epsilon\bar{\bar{\rho}}\bar{h}_g\right)}{\partial t}+\nabla\left(\epsilon\bar{\bar{\rho}}\vec{V}_g\bar{h}_g\right)=-\nabla\left[\epsilon\left(\bar{\bar{q}}+q^T\right)\right]+\frac{D}{dt}\left(\epsilon\bar{\bar{p}}\right)+\epsilon\Phi+ \\ +\epsilon\bar{\bar{\rho}}\bar{\epsilon}-\bar{\bar{p}}\vec{V}_g\nabla\epsilon+(1-\epsilon)\,\frac{S_b}{W_b}\vec{V}_b\langle\vec{F}\rangle^i+\bar{\bar{q}}\nabla\epsilon- \\ -(1-\epsilon)\,\frac{S_b}{W_b}\langle q\rangle^i+(1-\epsilon)\,\frac{S_b\rho_b}{W_b}\langle u\rangle^i\left(\bar{h}_b+h_s+\frac{\vec{V}_b\vec{V}_b}{2}\right) \tag{5}$$

The energy conservation equation for the solid phase:

$$\frac{\partial\left[(1-\epsilon)\,\rho_b\bar{h}_b\right]}{\partial t}+\nabla\left[(1-\epsilon)\,\rho_b\vec{V}_b\bar{h}_b\right]=\frac{D\left[(1-\epsilon)\left(\bar{\bar{p}}+R\right)\right]}{dt}+ \\ +\bar{\bar{p}}\vec{V}_b\nabla\,(1-\epsilon)-(1-\epsilon)\,\frac{S_b}{W_b}\vec{V}_g\langle\vec{F}\rangle^i-\bar{\bar{q}}\nabla\epsilon+ \\ +(1-\epsilon)\,\frac{S_b}{W_b}\langle q\rangle^i-(1-\epsilon)\,\frac{S_b\rho_b}{W_b}\langle u\rangle^i\left(\bar{h}_b+h_s+\frac{\vec{V}_b\vec{V}_b}{2}\right) \tag{6}$$

More detailed description of process for getting these equations is qiven in references [1, 2].

2.3 Constitutive laws

In the conservation equations for mass, momentum and energy it is necessary to define the interphase drag, the interphase heat transfer, the intergranular stress, the propelant combustion rate, the propellant grain size calculation, etc. For that purpose we use the constitutive laws.

2.3.1 Interphase drag

The equations used for determination the force of interphase drag are:

$$\langle\vec{F}\rangle^i=\frac{\bar{\bar{\rho}}\left(\vec{V}_g-\vec{V}_b\right)\left|\vec{V}_g-\vec{V}_b\right|}{6\epsilon^2}f \tag{7}$$

$$f = \begin{cases} 1.75 & \epsilon < \epsilon_0 \\ 1.75\left(\frac{1-\epsilon}{1-\epsilon_0}\frac{\epsilon_0}{\epsilon}\right)^{0.45} & \epsilon_0 < \epsilon \le \epsilon_1 \\ 0.3 & \epsilon_1 < \epsilon \le 1 \end{cases} \tag{8}$$

In the equation (8) ϵ_0 is the starting porosity, and ϵ_1 is the porosity given by the following expression:

$$\epsilon_1 = \left(1 + 0.02\frac{1-\epsilon_0}{\epsilon_0}\right)^{-1} \tag{9}$$

2.3.2 Interphase heat transfer

For modeling flamespreading through the propellant charge we use the Denton's relation for a nonfluidized charge, and Gelperin's relation for fluidized charge [3] (table 1).

Table 1

Researchers	$Nu_b =$	Re^* range	Porosity	Particles
Denton	$0.65Re_b^{0.7}Pr^{0.3}$	$50 \le Re \le 50000$	0.37	spheres
Gelperin and Einstein	$2 + 0.4Re_b^{2/3}Pr^{1/3}$	$Re_b > 200$	/	spheres

$$* \ Re_b = \frac{\rho\epsilon(V_g - V_b)d_b}{\mu_g}$$

2.3.3 Intergranular stress

The intergranular stress R represents the differenee between mean stresses in the gas and solid phase. Namely, the granular charge can be compressed to the one maximum value. The force with which the charge resist to compression is called the intergranular stress or the force particle-particle. The intergranular stress is given by the following correlation:

$$R_b = \begin{cases} -\rho_b a_r^2 \frac{\epsilon_0-\epsilon}{1-\epsilon}\frac{\epsilon_0}{\epsilon} & \epsilon \le \epsilon_0 \\ -\frac{\rho_b a_r^2}{2(1-\epsilon)k_s}\left[1 - e^{-2k_s(1-\epsilon)}\right] & \epsilon_0 < \epsilon \le \epsilon_k \\ 0 & \epsilon_k < \epsilon \end{cases} \tag{10}$$

2.3.4 Heat conduction through propellant grain

The propellant grain surface temperature is necessary for determination the heat transfer between the gas and solid phases and determination the conditions for ignition and flamespreading through propellant charge. The access to the propellant grain surface temperature determination is caused by the fact that the thermal wave penetration depth in particle is small, because the process of flamespreading lasts only a few milliseconds. The thermal wave penetration depth in particle δ and the heat transfer coefficient a_b are given by expressions:

$$\delta(t) = \frac{5}{2}R\left\{1 - \left[1 - \frac{8}{5}\left(\frac{\bar{\bar{T}}_b - T_{bo}}{\bar{\bar{T}}_{pb} - T_{po}}\right)^{1/2}\right]\right\} \tag{11}$$

$$a_b = \frac{3k_b}{\delta}\frac{\left(\bar{\bar{T}}_{pb} - T_{bo}\right)}{\bar{\bar{T}}_{pb} - \bar{\bar{T}}_b} \simeq \frac{3k_b}{\delta} \tag{12}$$

2.3.5 Particle-size calculation

For the particle-size calculation the Spalding's method [4] is used. This approach consists of calculation a "shadow" volume fraction of the solid phase ϵ_b^* that, is to say, the volume fraction which the solid phase would have possessed in the absence of combustion; the velocities, however, being the same as the phases actually possesses. Therefore, ϵ_b^* obeys equation (2), but for the absence of the mass-transfer rate. So we have:

$$\frac{\partial \epsilon_b^*}{\partial t} + \nabla\left(\epsilon_b^* \vec{V}_b\right) = 0 \tag{13}$$

Then the particle-size can be compute from the correlation:

$$\frac{D_b}{D_{bo}} = \left(\frac{\epsilon_b}{\epsilon_b^*}\right)^{1/3} \tag{14}$$

2.3.6 Some more important relations

As the ignition criterion in the paper the critical temperature of the propellant grain surface is used. This criterion has the following mathematical form:

$$\begin{aligned} (u)^i &= 0 \quad \bar{\bar{T}}_{pb} < T_{pri} \\ (u)^i &> 0 \quad \bar{\bar{T}}_{pb} \geq T_{pri} \end{aligned} \tag{15}$$

The gas state is determined by the Noble-Abel equation of state

$$\bar{\bar{p}}\left(\frac{1}{\bar{\bar{\rho}}} - \alpha\right) = R_g \bar{\bar{T}}_g \tag{16}$$

For the propellant burning law the following relation is used:

$$(u)^i = a\bar{\bar{p}}^n + b \tag{17}$$

3. Numerical method

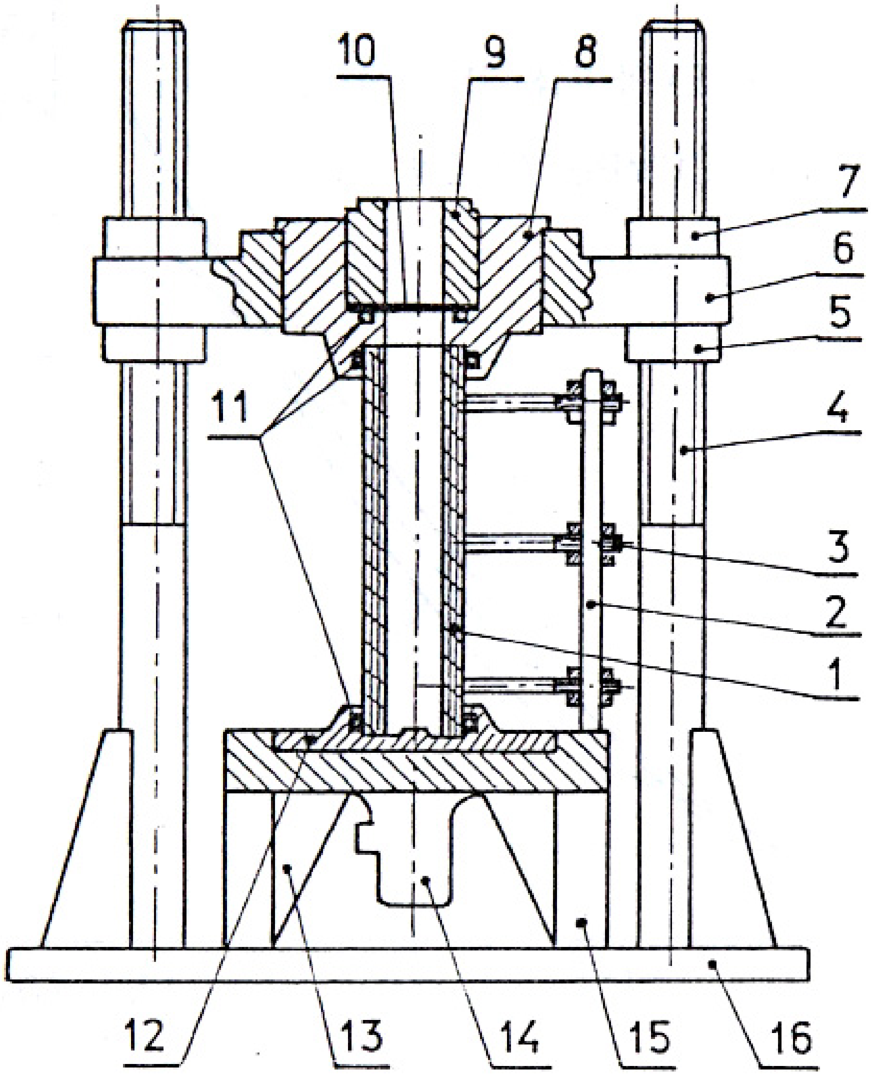
For the equations discretisation the staggered grid is used. The finite difference equations creation is carried out by the integration of the partial differential equations over the finite control volumes. Because of the high degree of nonlinearity of the equations, the calculation must be performed by iterative means. At each time interval, the solution procedure proceeds according to following steps:

1. The determination of the boundary conditions at the both boundaries for all variables.
2. The solution of the finite-domain equations for the enthalpies of the two phases individually, by the use of the partial-elimination algorithm (PEA) [5].
3. The solution of the finite-domain equation for the solid phase volume fractions at all grid points by way of the tridiagonal matrix algorithm (TDMA).
4. The determination of the gas phase volume fractions.
5. The determination of the pressure distribution which is appropriate to the momentum equations of the two phases added together. This is a simple forward-integration process.
6. Using the pressure distribution, the solution for the velocities of the two phases individually by the partial-elimination algorithm.
7. The computation the consequent errors in the "volumetric continuity" equation for the two phases added together, and formulation the "pressure-correction" equation.
8. The solution for the pressure corrections by TDMA and the coresponding velocities and densities corrections.
9. The returning to the step 2. and repeating the cycle of computations until the continuity errors are sufficiently small.
10. Proceed to the next time interval, and start the cycle of operations again from step 1.

4. Experimental investigations

The apparatus for the experimental investigations of flamespreading through propellant charges is designed and manufactured (figure 1). The apparatus basic component is the fiberglass tube. The fiberglass is used for manufacturing the tube (the propellant chamber imitation), because it is necessary to provide flamespreading tracking using the high-speed cinematography and the impulse radiography. On the tube wholes are drilled for acceptance of piezo-gauges for measuring pressures, and the tube ends are processed so as to provide good sealing. The fiberglass tube bursts when a pressure reaches the value of about 500 bars (the pressure greater than a projectile start pressure). The flamespreading through the propellant charge composed of granular seven-perforated nitrocellulose (about 96.5% nitrocelullose content) propellant is considered. The propellant mass was 1100 g which was enaugh to fill the fiberglasse tube. The propellant charge ignition was carried out by a black powder placed on the bottom of the propellant chamber.

In the table 2 the survey of used instrumentation and observed phenomena during experimental investigations is given.



1 - fiberglass tube　　2 - support of guage holder
3 - guage holder　　4 - column
5 - lower nut　　6 - upper plate
7 - upper nut　　8 - tube holder
9 - membrane tightener　　10 - membrane
11 - gasket　　12 - bottom
13 - rib　　14 - breech
15 - lateral plate　　16 - pedestal

Fig.1. The apparatus for experimental investigations of a propellant charge ignition

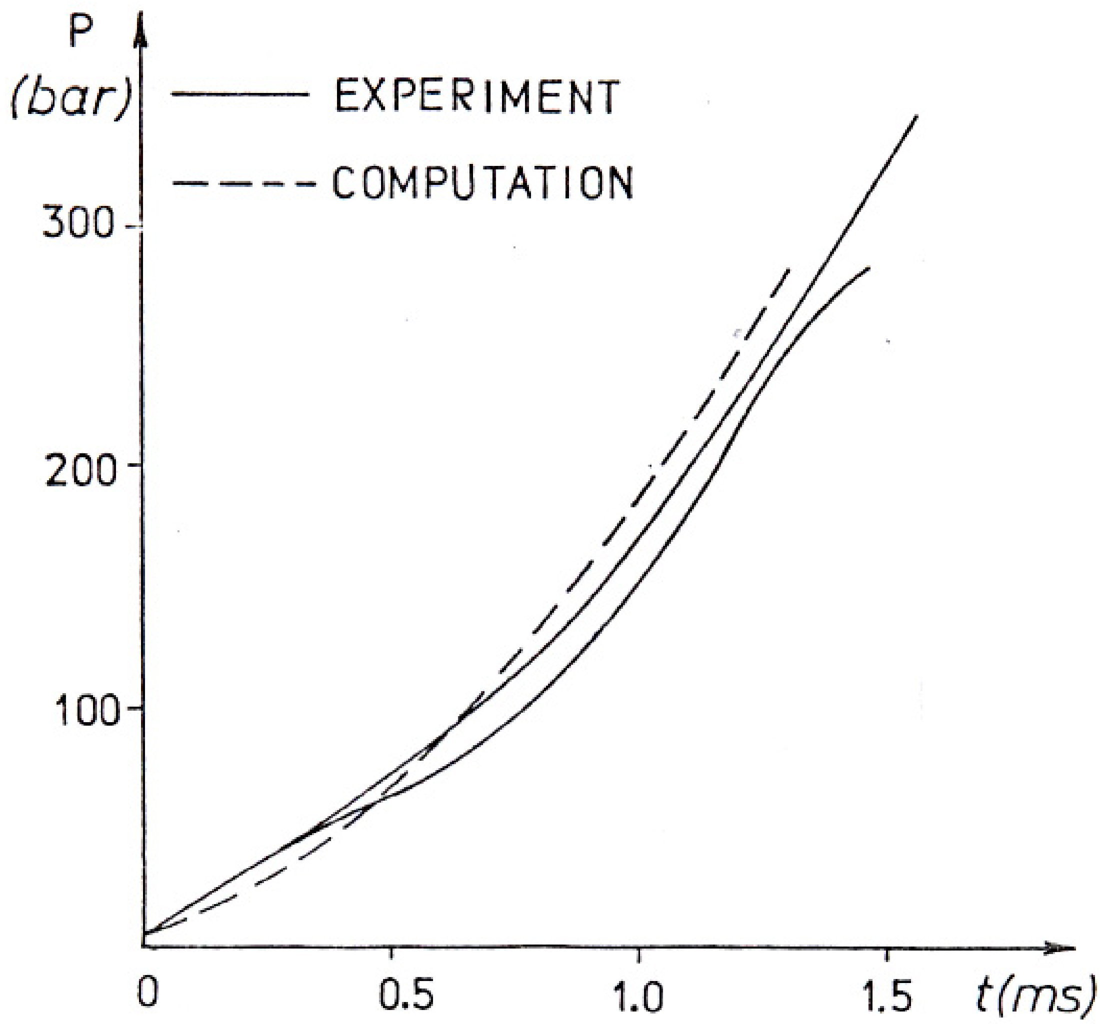


Fig.2. Measured and computational pressures on 83 mm from the chamber bottom when 8 g of a black powder of fine granulation is used for ignition

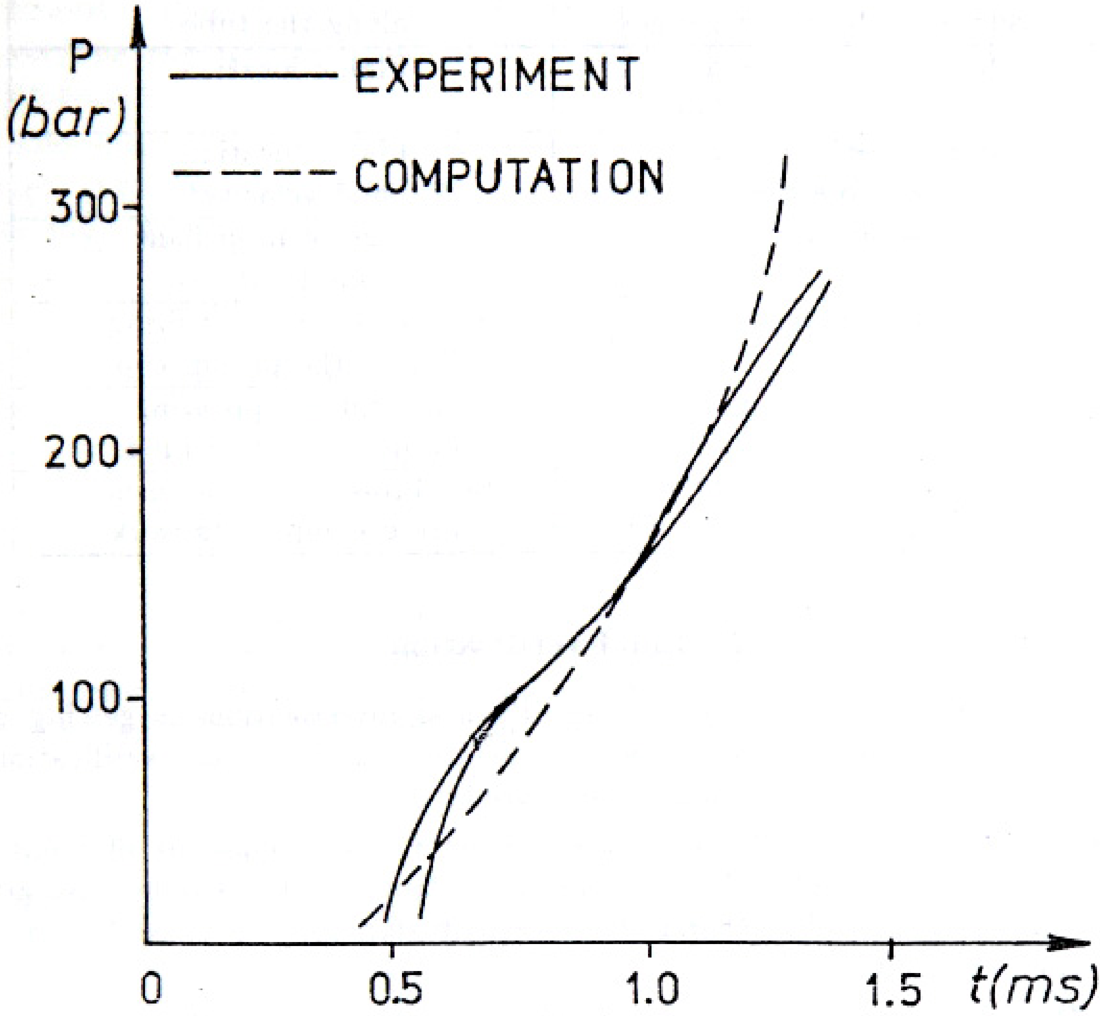


Fig.3. Measured and computational pressures on 220 mm from the chamber bottom when 8 g of a black powder of fine granulation is used for ignition

Table 2

INSTRUMENTATION	OBSERVED PHENOMENA
Equipment for pressure measurement by piezo gauges	Pressure vs. time along the tube
High-speed camera (frame-to-frame method)	Flame location
High-speed camera (streak method)	Flame location and velocity
X-ray apparatus	Change of propellant grains locations
Cantact detektor	Impact moment of firing pin into the primer cup
Optical detektor	Moment of appearing luminous light in tube
Optical equipment with generators	Synchronisation the measuring equipments work

5.Model verification

One of the basic tasks for the experimental investigations is getting data whose comparison with the computation results will enable the verification of the theoretical modeling and numerical procedure.

On the figures 2 and 3 experimentaly determined diapasons of measured pressures on 83 mm, and 220 mm from the bottom of the chamber are given. Also on these figures, the computational results for ignition using 8 g of fine granulation are given.

In the table 3 the computational and experimental characteristic moments of pressure wave spreading (moments when the projectile base pressure reaches defined values) are given. Also in this table the mean velocities of pressure spreading between the places for pressure measuring, until the projectile start pressure is reached, are given.

Table 3

Black powder Mass (g)	Data source	t_{200} (ms)	t_{300} (ms)	t_{350} (ms)	V_{1-2} (m/s)	V_{2-3} (m/s)	V_{3-4} (m/s)
fine granulation	experimental	1.300	1.358	1.395	251	435	449
8	computation	1.140	1.250	1.300	311	381	437

The detailed results analysis shows following:

- the good correspondence between the experimental data and computational results for pressures vs. time is obtained,

- the main reason for the detected differences in the experimental and computational results is the fact that the used constitutive laws for the interphase drag, the interphase heat transfer and the intergranular stress are from fluidisation field; they are only available experimental laws at the moment; however, these laws are formulated for stationary conditions, but not for very nanstationary conditions that govern during flamespreading through propellant charges.

The mean computational and experimental velocities of flamespreading through propellant charge between the places for pressure measuring are given in the table 4.

Table 4

Black powder Mass (g)	Data source	V_{1-2} (m/s)	V_{2-3} (m/s)
fine granulation 8	experimental	248	356
	computation	258	361

In the table 5 experimental data and computational results of the propellant grains motions are given. From the table it can be seen good agreement between experimental and computational results. This mean that the model adequatly describes parameters of propellant grains motions.

Table 5

Black powder Mass (g)	Grains starting location (mm)				Data source	Grains location change (mm)			
Fine granulation 8	89	115	208	221	experimental	22	27	24	29
					computation	24	24	21	25

6. Conclusions

The main conclusions that come from given considerations are:

- the developed model is one of the formal averaging models in which, starting from the conservation equations in the local-instant mode, averaging is performed and the microscopic balance equations for the gas and solid phase are given,
- the main constitutive laws are given,
- the numerical procedure for solution the discretisation equations of developed model is given,
- by comparison of experimental and computational results we found their good agreement for the case of the base ignition of the propellant charge;

on that way the verification of the developed theoretical-numerical model is performed.

- the developed theoretical-numerical model will serve as the basis for consideration all interior ballistic problems in the new light because it will enable computation the distribution of the individual process parameters without assuming their uniformity over the whole volume behind the projectile, which is the case for the classical access,
- the model enables determination influence of an ignition system and a propellant charge organisation on ignition of the propellant charge; the model also enables considerations conditions for occuring the propellant grains fracture due to impact on a projectile base or chamber parts or due to an intergranular stress on propellant grains contacts,
- using the developed theoretical-numerical-experimental access it is possible to predict conditions for shock waves creation in a gun barrel and irregular fuction of a projectile-gun system.

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MODELISATION D'ECOULEMENT BIPHASIQUE LORS DE LA COMBUSTION D'UN MELANGE GAS-PARTICULES SOLIDES

Dans l'article on donne la modélisation théorique et numérique d'écoulement biphasique lors de la propagation de la flamme dans un milieux granulaire. Le modèl théorique comprend les équations de balance pour les deux phases, ainsi que les lois constitutives nécessaires. Les équations discrétisées du modèl sont incorporées dans le procédé de calcul, assurant une solution convergeante du système d'équati ons. La vérification de l'accès théorique-numérique est effectuée par la comparaison avec les résultats experimentals. L'acceès général permet meilleur résolution de nombreux problèmes de la balistique intérieure.

MODELIRANJE DVOFAZNOG STRUJANJA SMEŠE GAS-ČVRSTE ČESTICE PRI SAGOREVANJU

U radu je dato teorijsko i numeričko modeliranje dvofaznog strujanja pri prostiranju plamena kroz granularnu sredinu. Teorijski model obuhvata jednačine održanja za obe faze, kao i neophodne konstitutivne zakone. Razvijene

diskretizovane jednačine formiranog modela uključene su u postupak proračuna koji obezbedjuje konvergentno rešenje sistema jednačina. Izvršena je verifikacija teorijsko-numeričkog pristupa kroz poredjenje sa eksperimentalnim rezultatima. Celokupni pristup omogućuje uspešnije rešavanje mnogih unutrašnjebalističkih problema.

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