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ALGEBRAIC EXPRESSIONS FOR STRESSES IN COMPOSITE AND PRESTRESSED STRUCTURES

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1. Basic equations

The equations determining the stresses in composite and prestressed structures with uncracked sections are:

— stress-strain relations for concrete and for all types of steel,

— Navier's assumption at the same time which represents the compatibility conditions of the unhomogeneous section,

— equilibrium equations of the section.

1.1 The Integral Form

The integral stress-strain relation for concrete (c) is symbolically written in the form:

$$\sigma_c = E_{co} \overset{\frown}{R} \varepsilon, \tag{1.1}$$

(see Appendix, item 1). The shrinkage is omitted, because it is unimportant for a further analysis. Prestressed steel (p), steel member (n) and reinforced steel (m) follow Hooke's law:

$$\sigma_k = E_k \varepsilon, \quad k = p, n, m. \tag{1.2}$$

In special cases the unhomogeneous cross-section does not contain all steel parts. Navier's assumption reads:

$$\varepsilon = \eta + \varkappa y, \tag{1.3}$$

$\eta = \eta\,(x, t, t_0)$ is a normal strain and $\varkappa = \varkappa\,(x, t, t_0)$ the change in the curvature of the bar axis (see Appendix, item 2).

When common strain ε and stresses are eliminated from the above mentioned equations, the basic equations are obtained. They are relevant to an arbitrary unhomogeneous section:

$$(1.4)\qquad \begin{aligned} E_u A_i \,\widehat{R}_{11}\,\eta + E_u S_i\,\widehat{R}_{12}\,\varkappa &= N, \\ E_u S_i\,\widehat{R}_{21}\,\eta + E_u J_i\,\widehat{R}_{22}\,\varkappa &= M, \end{aligned}$$

and represent a system of unhomogeneous integral equations. $N = N(x, t, t_0)$ is the axial force and $M = M(x, t, t_0)$ the bending moment.
The principal values of the operator matrix of system (1.4) are:

$$(1.5)\qquad \widehat{R}_h = \gamma'_h \widehat{1} + \gamma_h \widehat{R}, \qquad h = 1,2.$$

Their inverse operators are $\widehat{F}_h$ $(h = 1, 2)$ and the following holds for them:

$$(1.6)\qquad \widehat{R}_h\,\widehat{F}_h = \widehat{1}, \qquad \widehat{R}_h\,\widehat{F}_h = \widehat{F}_h\,\widehat{R}_h, \qquad h = 1,2.$$

The solution of basic equations (1.4) is expressed through inverse operators $\widehat{F}_h$:

$$(1.7)\qquad \begin{aligned} E_u\eta &= \frac{1}{\Delta\gamma}\,(\delta\gamma_2\,\widehat{F}_1 + \delta\gamma_1\,\widehat{F}_2)\,\frac{N}{A_i} + \frac{1}{\Delta\gamma}\,\gamma_{12}\,\frac{S_i}{A_i}\,(\widehat{F}_1 - \widehat{F}_2)\,\frac{M}{J_i}, \\ E_u\varkappa &= \frac{1}{\Delta\gamma}\,\gamma_{21}\,\frac{S_i}{J_i}\,(\widehat{F}_1 - \widehat{F}_2)\,\frac{N}{A_i} + \frac{1}{\Delta\gamma}\,(\delta\gamma_1\,\widehat{F}_1 + \delta\gamma_2\,\widehat{F}_2)\,\frac{M}{J_i}, \end{aligned}$$

Functions $F^*_h = F^*_h(\gamma_h, t, t_0) = \widehat{F}_h 1^*$ and:

$$(1.8)\qquad R^*_h = R^*_h(\gamma_h, t, t_0) = \widehat{R}_h 1^* = \gamma'_h 1^* + \gamma_h R^*, \quad h = 1,2,$$

will be used. The following is also introduced:

$$(1.9)\qquad \widehat{B}_h = \widehat{R}\,\widehat{F}_h, \quad B^*_h = B_h(\gamma_h, t, t_0) = \widehat{B}_h 1^*, \quad h = 1,2.$$

Functions B^*_h are called the basic functions. They simultaneously linearly depend on F^*_h:

$$(1.10)\qquad B^*_h = \frac{1}{\gamma_h}\,1^* - \frac{\gamma_h'}{\gamma_h}\,F^*_h, \quad h = 1,2.$$

The relation between inverse operators $\widehat{R}_h$ and $\widehat{F}_h$ (1.6) can be presented in the form:

$$(1.11)\qquad \widehat{B}_h\,\widehat{K}_h = \widehat{1}, \qquad \widehat{B}_h\,\widehat{K}_h = \widehat{K}_h\,\widehat{B}_h, \qquad h = 1,2,$$

containing the operators:

$$(1.12)\qquad \widehat{K}_h = \widehat{R}_h\,\widehat{F}. \qquad h = 1,2.$$

It is shown that functions $K^*_h = K^*_h(\gamma_h, t, t_0)$ linearly depend on the creep function:

$$(1.13)\qquad K^*_h = \widehat{K}_h 1^* = \gamma_h 1^* + \gamma_h' F^* = 1^* + \gamma_h' \varphi_r, \quad h = 1,2.$$

Integration of expression (1.11) yields:

$$\hat{B}_h K_h^* = 1^*, \qquad h = 1,2. \tag{1.14}$$

These relations give the unhomogeneous integral equations whose solutions are basic functions B_h^*. It means that they are directly determined by creep function F^*. Obviously, the basic functions are relevant to a determined section because, through quantities γ_h, they depend on their geometrical properties.

1.2 The Algebraic Form

In the system of equations mentioned, the integral equation (1.1) for concrete is substituted by the algebraic one:

$$\sigma_c = E_{co} \zeta'_c \varepsilon - \rho_c \sigma_{co}, \tag{1.15}$$

where:

$$\zeta'_c = \frac{1}{1 + \chi_F \varphi_r}, \qquad \rho_c = \frac{\varphi_r - \chi_F \varphi_r}{1 + \chi_F \varphi_r}. \tag{1.16}$$

χ_F is a free parameter, the values of which will be later on adopted. In a special case where this parameter takes the value of aging coefficient χ [1], (1.15) represents th e known AAEM Method.

The basic equations in the algebraic form are:

$$\begin{aligned} E_u A_{i\zeta} \eta + E_u S_{i\zeta} \varkappa &= N + \rho_c \left(N_0 \frac{A_{cr}}{A_i} + M_0 \frac{S_{cr}}{J_i} \right), \\ E_u S_{i\zeta} \eta + E_u J_{i\zeta} \varkappa &= M + \rho_c \left(N_0 \frac{S_{cr}}{A_i} + M_0 \frac{J_{cr}}{J_i} \right), \end{aligned} \tag{1.17}$$

where:

$$A_{i\zeta} = A_i - (1 - \zeta'_c) A_{cr}, \quad S_{i\zeta} = -(1 - \zeta'_c) S_{cr}, \quad J_{i\zeta} = J_i - (1 - \zeta'_c) J_{cr},$$

$N_0 = N(x, t_0, t_0)$, $M_0 = M(x, t_0, t_0)$, (see Appendix, item 2).

The algebraic expressions for stresses can be obtained by incorporating solutions of basic equations into (1.15) and (1.2). Therein free parameter χ_F appearb.

1.3 Special case: Homogeneous Concrete Structure

Equations determining the stresses are:

— stress-strain relation for concrete (1.1),
— Navier's assumption (1.3),
— equilifirium equations of the section.

For the system of principal centroidal axis of inertia, the basic equations reduce to two independent unhomogeneous integral equations:

(1.18) $$E_{co}\,A_c\,\hat{R}\,\eta = N, \qquad E_{co}\,J_c\,\hat{R}\,\varkappa = M,$$

with the solutions:

(1.19) $$E_{co}\,\eta = \hat{F}\,\frac{N}{A_c}, \quad E_{co}\,\varkappa = \hat{F}\,\frac{M}{J_c},$$

where $A_c = A_c(x)$ and $J_c = J_c(x)$ are the cross-section area and the moment of inertia of this area about the principal axis, respectively.

The basic equations in the algebraic form are:

(1.20) $$E_{co}\,A_c\,\zeta'_c\,\eta = N + \rho_c\,N_o, \quad E_{co}\,J_c\,\zeta'_c\,\varkappa = M + \rho_c\,M_o.$$

They are obtained when integral relation (1.1) is substituted by algebraic one (1.15) in the above system.

2. The Generalized AAEM Method

Theorem I. If in an unhomogeneous section the axial force and the bending moment linearly depend on functions R_1^* and R_2^*:

(2.1) $$N = N_1\,R_1^* + N_2\,R_2^* + N_3\,1^*, \quad M = M_1\,R_1^* + M_2\,R_2^* + M_3\,1^*,$$

$N_1, \ldots, M_3$ being arbitrary time-independent quantities, then functions η and $\varkappa$ determining the deformation of the unhomogeneous section, linearly depend on functions F_1^* and F_2^*:

(2.2) $$\eta = \eta_1\,F_1^* + \eta_2\,F_2^* + \eta_3\,1^*, \quad \varkappa = \varkappa_1\,F_1^* + \varkappa_2\,F_2^* + \varkappa_3\,1^*;$$

the corresponding operators $\hat{R}_h$ and $\hat{F}_h$ $(h = 1, 2)$ are inverse. Quantities $\eta_1, \ldots \varkappa_3$ are not time-dependent. The proof of this theorem has been omitted.

Expressions (2.2) could be written even through the basic functions:

(2.3) $$\eta = \bar{\eta}_1\,B_1^* + \bar{\eta}_2\,B_2^* + \bar{\eta}_3\,1^*, \quad \varkappa = \bar{\varkappa}_1\,B_1^* + \bar{\varkappa}_2\,B_2^* + \bar{\varkappa}_3\,1^*.$$

Theorem I represents the Generalized AAEM Method and is relevant to the unhomogeneous section of arbitrary geometrical properties.

2.1 Special Case: The AAEM Method

Theorem II. If the normal force and the bending moment in the homogeneous section linearly depend on the relaxation function:

(2.4) $$N = N_1\,R^* + N_2\,1^*, \quad M = M_1\,R^* + M_2\,1^*,$$

$N_1, \ldots, M_2$ are arbitrary time-independent quantities, functions η and $\varkappa$ determining the section deformation linearly depend on the creep function:

(2.5) $$E_{co}\,\eta = \frac{N_1}{A_c}\,1^* + \frac{N_2}{A_c}\,F^*, \qquad E_{co}\,\varkappa = \frac{M_1}{J_c}\,1^* + \frac{M_2}{J_c}\,F^*;$$

the operators $\hat{R}$ and $\hat{F}$ are inverse. The proof of this theorem has also been omitted.

In algebraic equations (1.20) free parameter χ_F appears through coeficients $\zeta_c^{'}$ and ρ_c. Its value will be determined from the condition that for the determined t and t_0 and for N and M which change according to law (2.4), value η i.e. $\varkappa$ should be the same as the one stemming from accurate expression (2.5). This condition yields the relation:

$$\chi_F = \chi = \frac{1}{1-R^*} - \frac{1}{\varphi_r}. \tag{2.6}$$

This shows that Theorem II represents another formulation of the known *AAEM* Method [1].

Quantities γ_h and γ'_h represent the discrete values of interval (0,1) (see Appendix, item 2), so that index h can be omitted in some considerations. From (1.8) it is shown that for $\gamma' = 0$, $R_1^* \equiv R_2^* \equiv R^*$, wherefrom it follows: $F_1^* \equiv F_2^* \equiv F^*$. Under this condition Theorem II becomes the special case of Theorem I which means that the *AAEM* Method represents the special case of the Generalized *AAEM* Method.

2.2 The Corrected Aging Coefficient

The relationship between inverse operators $\widehat{R}_h$ and $\widehat{F}_h$ is shown in form (1.11) and after integration, in form (1.14) i.e.

$$\widehat{B}\, K^* = \widehat{B}\,(1^* + \gamma'\,\varphi_r) = 1^*, \qquad 0 < \gamma' \leqslant 1. \tag{2.7}$$

Analogously with:

$$\widehat{R}\, F^* = \widehat{R}\,(1^* + \varphi_r) = 1^* \tag{2.8}$$

and with relation (2.6) coefficients:

$$\chi_\gamma = \frac{1}{1-B^*} - \frac{1}{\gamma'\,\varphi_r}, \quad 0 < \gamma' \leqslant 1; \quad \chi_{\gamma h} = \frac{1}{1-\overset{*}{B}_h} - \frac{1}{\gamma_h'\varphi_r}, \quad h = 1,2 \tag{2.9}$$

are defined.

The comparison between (2.7) and (2.8) displays that $K^* \equiv F^*$ and $B^* \equiv R^*$ for $\gamma' = 1$. Comparing (2.6) with (2.9) we infer that aging coeficient χ is the special case of coeficient χ_γ for $\gamma' = 1$.

To the homogeneous section corresponds aging coeficient χ. To the unhomogeneous section, associated, in a general case, with two integral equations (1.6), correspond two coefficients $\chi_{\gamma h}$ $(h = 1, 2)$. For the accepted creep function and determined t and t_0, coeficients $\chi_{\gamma h}$ depend on the section geometrical properties because they contain the compatibility conditions of the unhomogeneous section. They will be called corrected aging coeficients.

In order to get an insight into the influence of section geometrical properties on these coefficients, Figure 1 contains comparative values χ_γ for $\gamma' = 0.1$, 0.3, 0.5 and 1.0 corresponding to the creep function CEB 1978 Model [2]. For the same t and t_0 a lower value of χ_γ corresponds to the lower value of γ'. For $\gamma' = 1$, its highest value is obtained, which is, in fact, aging coefficient χ.

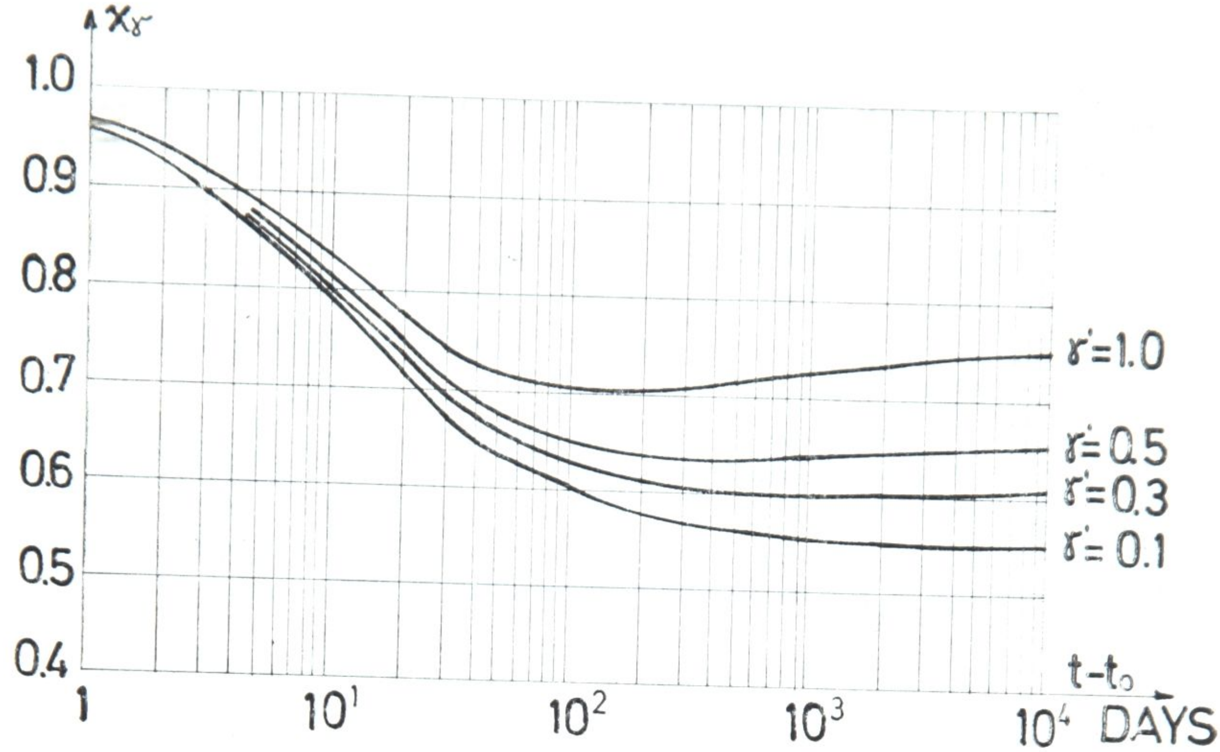

Fig. 1. Coefficients $\chi_\gamma(\gamma', t - t_0, t_0)$ for $t_0 = 3$ days $\varphi_{f1} = 3$ and $h_0 = 40$ cm.

3. Discussion of the algebraic stress expressions

The following theorem [3] is valid for stresses in unhomogeneous sections: if the normal force and the fiending moment change according to the law:

$$(3.1) \qquad N = \mathcal{N}_1 R^* + \mathcal{N}_2 1^*, \qquad M = \mathcal{M}_1 R^* + \mathcal{M}_2 1^*,$$

$\mathcal{N}_1, \ldots, \mathcal{M}_2$ being arbitrary time-independent quantities then, in the general case of the section geometrical properties, stresses depend linearly on the basic functions:

$$(3.2) \qquad \sigma_{KH} = \nu_K \left[U_{KH} 1^* + \sum_{h=1}^{2} W_{hkH} B_h^* \right], \; k = c, p, n, m; \quad H = G, P, S.$$

In the above quoted paper this theorem was given in a more general form, with the relaxation properties of prestressing steel being involved as well. Equations (3.2) represent accurate expressions for stresses in statically determinate and indeterminate structures, due to the influences of self-weight ($H = G$), prestressing by forces ($H = P$) and shrinkage ($H = S$), provided the conditions, established in [3], take place.

This theorem is a direct consequence of the generalized *AAEM* Method. The law of change of N and M (3.1) is obtained from (2.1) substituting function R^* for R_h^* according to (1.8).

The calculation of exact values of stresses at every point of the section and for every t and t_0 is the simplest from expressions (3.2). By means of the known numerical method [1] it is necessary to determine, once for ever, the values of function B^* for a series of discrete values t_0, $t - t_0$ and γ' as solutions of unhomogeneous integral equations (2.7). Another method consists of the determination of an approximate function B^*, utilizing a procedure analogous to the procedure which was used for obtaining the approximate function R^* [4].

Independently of this, we shall consider the algebraic expressions for stresses, obtained from (3.2), when basic functions B_h^* are replaced by corrected aging coefficients $\chi_{\gamma h}$ (2.9), respectively.

Two special cases of section geometrical properties appearing often in practice are given: sections where $I_m = I_p = 0$ and $y_m = y_p = y_a$ can be adopted (Fig. 2*a*) and sections where $I_p = I_c = 0$ and $y_p = y_c = y_v$ can be adopted (Fig. 2*b*) [5].

Except for stresses σ_{cH} ($H = G, S$) for sections in Figure 2*b*, all stresses in these special cases depend either on the creep function and one basic function or on the latter only. In those cases, the algebraic expressions contain only one corrected aging coefficient χ_γ.

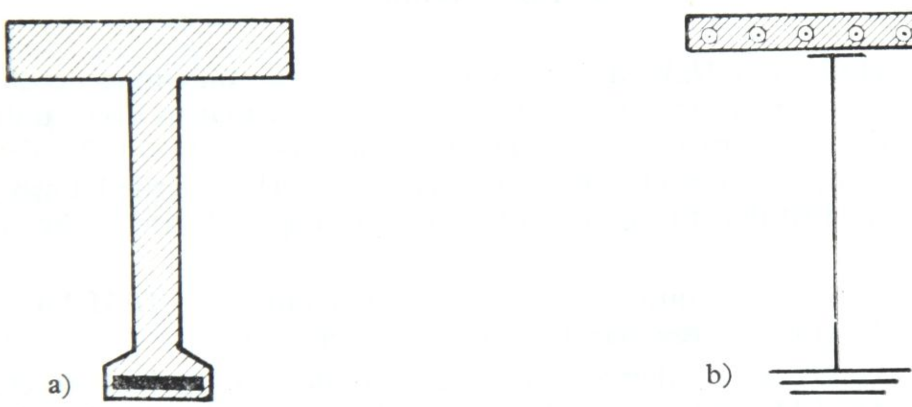

Fig. 2. Sections with special geometrical properties.

It must be emphasized that the same algebraic expressions can be derived by the procedure presented in Sub-chapter 1.2, the difference being that the free parameter χ_F appears now. If N and M change according to law (3.1), these expresions yield accurate values for stresses at every point of the section and for every t and t_0, provided the corrected aging coefficient $\chi_F = \chi_\gamma$, obtained from (2.9) is adopted. If another value for χ_F is accepted, the same expressions yield the approximate stress values. The aging coefficient once adopted, $\chi_F = \chi$ (the *AAEM* Method) [6], the approximate values predict a smaller change in stresses in the interval (t_0, t) than the accurate one. This is due to the fact that $\chi > \chi_\gamma$ for every value of γ', $0 > \gamma' > 1$. Thereby the effects of the concrete creep on the stress redistribution are underestimated. If $\chi_F = 1$ is adopted (the *EM* Method), these influuences are even further underestimated.

When stresses depend on two basic functions, two corrected aging coefficients $\chi_{\gamma h}$ ($h = 1, 2$) appear in algebraic expressions obtained from (3.2). If N and M follow law (3.1) these expressions provide the accurate values of stresses at every point of the section and for every t and t_0, when values $\chi_{\gamma h}$ are accepted according to (2.9). Such algebraic expressions cannot be derived by means of the procedure presented in Sub-chapter 1.2.

For practical applicability the significance of the algebraic stress expressions derived by procedure presented in Sub-chapter 1.2 is beyond any doubt; they are, in this case, approximate expressions. When using them, it is necessary to determine the value χ_F for every section so that the stress changes in the interval from t_0 to t at most points of the section are equal to, or greater than, those yielded by the accurate expressions. According to certain authors' exprriences this is achieved when:

— the greater value of the corrected aging ceofficient for the free parameter, $\chi_F = \chi_{Y2}$ is accepted, for sections without steel member (Fig. 2*a*);

— the lower value of the corrected aging coefficient for the free parameter $\chi_F = \chi_{Y1}$ is accepted, for sections with a steel member (Fig. 2b).

It is understood that by accepting $\chi_F = \chi$ (the *AAEM* Method), the effects of concrete creep are underestimated, because for every section the aging coefficient is greater than the greater value of the corrected aging coefficient.

4. Conclusion

The Generalized *AAEM* Method is relevant to the unhomogeneous section with arbitrary geometrical properties. It stems from that that to every unhomogeneous section correspond two corrected aging coefficients χ_{Yh} $(h = 1, 2)$, depending on the geometrical properties of that section. As its special case the known *AAEM* Method is obtained so that the aging coefficient χ is a special case of the corrected aging coefficient χ_Y.

On the basis of the consequences of the Generalized *AAEM* Method, the discussion of algebraic expressions for stresses is presented.

If these expressions are derived with the algebraic relation for concrete according to the *AAEM* Method, where the aging coefficient χ is introduced [6], the approximate values of stresses are obtained, predicting smaller stress changes in the interval from t_0 to t, than the correct one, which means that the concrete creep effects are underestimated.

If instead of a predetermined value of aging coefficient χ free parameter χ_F is introduced in the same algebraic expressions, then, by a suitable selection of its value for every section separately, the following are obtained:

— the exact values of stresses, in the above mentioned special cases which are often encountered, when the corrected aging coefficient value for χ_F is accepted according to expression (2.9);

— the approximate values of stresses in other cases; these values, in a large number of points of the section, predict the same or greater stress changes in the interval from t_0 to t, than the exact one, if χ_F is accepted for every section, dependent on its geometrical properties and according to a certain established criterion.

Appendix

1. The integral form of the stress-strain relation:

$$\varepsilon(t, t_0) = \frac{1}{E_{co}} \left[\frac{1}{e(t)} \sigma_c(t, t_0) + \int_{t_0}^{t} \frac{\partial f(t, \tau)}{\partial \tau} \sigma_c(\tau, t_0)\, d\tau \right],$$

is presented by means of the linear integral operator:

$$\hat{F} = \frac{1}{e}\hat{1} + \hat{f}, \quad e = e(t) = \frac{E_c(t)}{E_{co}}, \qquad E_{co} = E_c(t_0),$$

n the form:

$$\varepsilon = \frac{1}{E_{co}} \hat{F} \sigma_c.$$

$\hat{1}$ is the unity operator associated to the Dirac function.

Operators $\hat{F}$ and:

$$\hat{R} = e\,\hat{1} - \hat{\psi}$$

are inverse and the following is valid for them:

$$\hat{R}\,\hat{F} = \hat{1}, \qquad \hat{R}\,\hat{F} = \hat{F}\,\hat{R}.$$

The integral of function $\dot{g} = \dot{g}(t, \tau) = \frac{\partial g(t, \tau)}{\partial \tau}$ is defined:

$$g^* = g^*(t, t_0) = \hat{\dot{g}}\, 1^* = \int_{t_0}^{t} \frac{\partial}{\partial \tau} g(t, \tau)\, H(\tau - t_0)\, d\tau,$$

$1^* = H(t - t_0)$ is the Heaviside function.

$$F^* = \hat{F}\, 1^* = \frac{1}{e}\, 1^* + f^* = 1^* + \varphi_r, \quad \varphi_r = \frac{E_{co}}{E_{c28}}\, \varphi.$$

F^* is the dimensionless creep function and R^* the dimensionless concrete relaxation function.

The laws of algebra for ordinary numbers, including the commutativity law, are valid for the linear integral operators used in the present paper.

2. The unhomogeneous section is presented in Figure 3.

Its position at the bar axis is determined by coordinate $x \cdot A_c = A_c(x)$ is the area of the concrete part of the section, $S_c = S_c(x)$ and $J_c = J_c(x)$ are the static moment and the moment of inertia of this area about the z-axis, respectively. E_u is the relative modulus of elasticity.

$$\nu_c = \frac{E_{co}}{E_u}, \quad \nu_k = \frac{E_k}{E_u}, \quad k = p, m, m; \quad A_{cr} = \nu_c A_c, \quad S_{cr} = \nu_c S_c, \quad J_{cr} = \nu_c J_c.$$

$I_k = I_k(x)$ is the moment of inertia of the section part k ($k = c, p, n, m$) about the axis through the centroid C_k, parallel to the z-axis. $A_i = A_i(x)$ and $J_i = J_i(x)$ are the transformed section area and the moment of inertia of this area about the z-axis, respectively; $S_i = S_i(x) = \sqrt{A_i J_i}$.

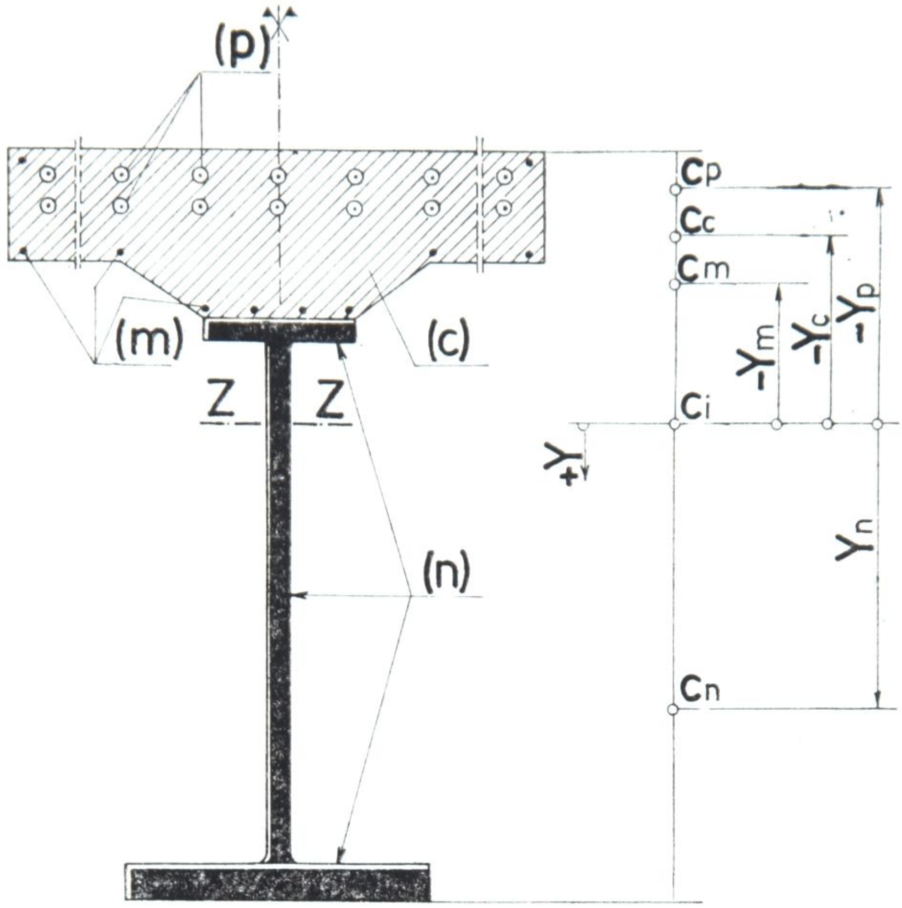

Fig. 3. The unhomogeneous section.

The following dimensionless values will be used:

$$\gamma_{11} = \frac{A_{cr}}{A_i}, \quad \gamma_{12} = \gamma_{21} = \frac{S_{cr}}{S_i}, \quad \gamma_{22} = \frac{J_{cr}}{J_i}.$$

The principal values of matrix $||\gamma_{hl}||$ are γ_1 and γ_2. The following notation will be also used:

$$\delta\gamma_1 = \gamma_1 - \gamma_{11}, \quad \delta\gamma_2 = \gamma_{11} - \gamma_2, \quad \Delta\gamma = \gamma_1 - \gamma_2, \quad \gamma_h' = 1 - \gamma_h, \quad h = 1,2.$$

The following holds:

$$0 < \gamma_2 < \gamma_1 < 1, \quad 0 < \gamma_1' < \gamma_2' < 1.$$

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LES EXPRESSIONS ALGÉBRIQUES POUR LES CONTRAINTES DANS LES STRUCTURES MIXTES ET PRÉCONTRAINTES

R é s u m é

La méthode *AAEM* généralisée a été dérivée pour les structures mixtes et précontraintes et on a demontré que deux coefficients de vieillissement corrigés, dépendant des caractéristiques géométriques de la section, correspondent a la section unhomogene dans un cas général. Sur cette base on a conclu que les expressions algébriques pour les contraintes, déterminées par la méthode *AAEM*, sousestiment les effets du fluage du béton dans la structure.

ALGEBARSKI IZRAZI ZA NAPONE U SPREGNUTIM I PRETHODNO NAPREGNUTIM KONSTRUKCIJAMA

I z v o d

Izvedena je Generalisana *AAEM* metoda za spregnute i prethodno napregnute konstrukcije i pokazano je da, u opštem slučaju geometrijskih karakteristika, nehomogenom preseku odgovaraju dva korigovana koeficijenta starenja. Na osnovu ovoga zaključeno je da algebarski izrazi za napone, određeni *AAEM* metodom, podcenjuju efekte puzanja betona u ovim konstrukcijama.

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