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MICROMORPHIC THEORY OF MIXTURES APPLIED TO THE THEORY OF RODS

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1. Introduction

Studying heterogenous materials, the so-called mixtures, one comes across much bigger difficulties than in the study of single component materials. Due to the complexity of problems, arising from the study of mixture behaviour, different theories have appeared. All of them, to a lesser or a greater degree, use different simplifications, together with analogies of single component material mechanics continuum, in order to reach appropriate conclusions which are valid for the mixture as a whole. Because of that majority of approaches for the study of mixtures the following three physical principles, postulated by **C**. Truesdell [8], should be used:

a) All the properties of a mixture must be mathematical consequences of the properties of a constituent;

b) In order to describe the motion of a constituent, we may in imagination isolate it from the rest of the mixture, on condition that we allow properly for the actions of the other constituents upon it;

c) The motion of the mixture is governed by the same equations as the motion of a single body.

In further development of the theory of mixtures, papers which do not treat them as a classical continuum model, but as a micromorphic continuum [4], [5], have appeared. For such purpose **C**. Eringen [1] inferred the balance laws, using the general balance laws, and this was the main reason why in this paper we decided for such an approach.

In the current theory of mixtures there is only one paper [9] which refers to the study of onedimensional continuum. The paper appeared in 1975 and in it the general non-linear theory of mixtures of oriented curves was recommended. The authors mentioned the mathematical model for human spine as a possibility for the practical application of the studies.

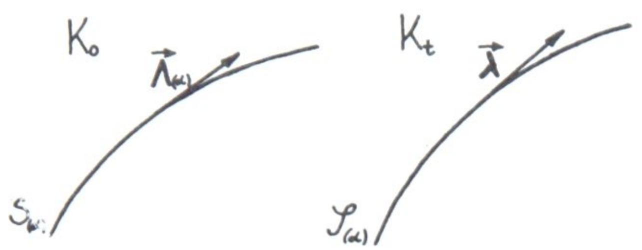
One can see from it that there are practical examples in nature to which one can apply the mixture theory of onedimensional continuum. Owing to it, we believe that in order to develop a theory of mixtures one should apply the micromorphic theory of mixtures and the corresponding balance laws and discontinuity conditions, applicable to threedimensional body [10], to rods as onedimensional bodies as well.

In paper [2] the micromorphic theory of three dimensional continuum was applied [1] and the general balance laws were derived, which were then applied to the theory of rods.

Using the results from paper [2], we deduced specific laws concerning the balance of the mass, momentum and energy of α-th constituent of the mixture and the sole mixture in the case of a rod.

2. **Kinematics**

We define the rod as onedimensional micromorphic continuum, which contains $\alpha = 1, 2, \ldots, n$ constituents of the mixture. To describe the movement of one constituent we shall apply, as in three dimensional case [10], the principle cited in the introduction under b) and in that way we shall assume the movement, micromovement and deformation only of the α-th constituent of the rod mixture (fig. 1).



(Fig. 1)

In the non deformed configuration K_0, which corresponds to the time t_0, the rod has a curve shape

$$X^K = X^K(S_{(\alpha)}), \tag{2.1}$$

where $S_{(\alpha)}$ is the arc of the curve of the α-th constituent. Unit vector of the curve tangent in the non deformed configuration is

$$\Lambda^K_{(\alpha)} = \frac{dX^K}{dS_{(\alpha)}}. \tag{2.2}$$

Considering that the microelements movement in the macroelement, relative to their centre, can adequately approximate it as a homogenous deformation, then the equations and deformations of the α-th constituent of the mixture are determined by

$$\begin{aligned} x^k_{(\alpha)} &= x^k_{(\alpha)}(\mathcal{S}_{(\alpha)}, t) \\ \chi^k_{(\alpha)K} &= \chi^k_{(\alpha)K}(\mathcal{S}_{(\alpha)}, t), \end{aligned} \tag{2.3}$$

where $\chi^k_{(\alpha)K}$ are microdeformation gradients of the α-th constituent of the mixture, which define the homogenous deformation.

Spacious curves arc of the α-th constituent of the mixture in the deformed configuration K_t is defined by

$$\mathcal{S}_{(\alpha)} = \mathcal{S}_{(\alpha)}(S_{(\alpha)}, t) = \int_0^{S_{(\alpha)}} \sqrt{g_{kl}\frac{\partial x^k}{\partial S}\frac{\partial x^l}{\partial S}}\, dS_{(\alpha)}. \tag{2.4}$$

The tangent's unit vector in the deformed configuration K_t is

$$\lambda^k_{(\alpha)} = \frac{\partial x^k}{\partial \mathcal{S}_{(\alpha)}} = \frac{\partial x^k}{\partial \mathcal{S}} = \lambda^k, \tag{2.5}$$

where for a partial derivative

$$\frac{\partial}{\partial \mathscr{S}_{(\alpha)}}=\frac{\partial}{\partial \mathscr{S}}. \tag{2.6}$$

applies. The average velocity of the rod's mixture $\dot{\mathscr{S}}$ is defined as

$$\rho \dot{\mathscr{S}}=\sum_{\alpha} \rho_{(\alpha)} \dot{\mathscr{S}}_{(\alpha)}, \tag{2.7}$$

whilst the average density is defined as

$$\rho=\sum_{\alpha} \rho_{(\alpha)}. \tag{2.8}$$

Diffusion velocity of the α-th constituent of the rod $U_{(\alpha)}$, relative to the average velocity $\dot{\mathscr{S}}$, is expressed by

$$U_{(\alpha)}=\dot{\mathscr{S}}_{(\alpha)}-\dot{\mathscr{S}}. \tag{2.9}$$

It follows from (2.7) and (2.9) that

$$\sum_{\alpha} \rho_{(\alpha)} U_{(\alpha)}=0. \tag{2.10}$$

For a magnitude $\psi=\psi(\mathscr{S}_{(\alpha)}, t)$, defined in all the points of the rod, the following two material derivatives will exist: material derivative of the constituent

$$\dot{\psi}=\frac{\partial \psi}{\partial t}+\frac{\partial \psi}{\partial \mathscr{S}} \dot{\mathscr{S}}_{(\alpha)} \tag{2.11}$$

and material ddrivative of the mixture

$$\dot{\psi}=\frac{\partial \psi}{\partial t}+\frac{\partial \psi}{\partial \mathscr{S}} \dot{\mathscr{S}} \tag{2.12}$$

It follows that the link between them

$$\dot{\psi}=\dot{\psi}+\frac{\partial \psi}{\partial \mathscr{S}} U_{(\alpha)}. \tag{2.13}$$

If the average value of the function for the mixture exists

$$\rho\psi\equiv\sum_{\alpha} \rho_{(\alpha)} \psi_{(\alpha)}, \tag{2.14}$$

then, using (2.9), (2.11) and (2.12), we obtain the so-called fundamental identity, which applies to the rod in the form

$$\begin{aligned}\sum_{\alpha} \rho_{(\alpha)} \dot{\psi}_{(\alpha)}=\rho \dot{\psi}+\psi\left[\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial \mathscr{S}}(\rho \dot{\mathscr{S}})\right]+\sum_{\alpha} \frac{\partial}{\partial \mathscr{S}}\left(\rho_{(\alpha)} \psi_{(\alpha)} U_{(\alpha)}\right)-\\ -\sum_{\alpha} \psi_{(\alpha)}\left[\frac{\partial \rho_{(\alpha)}}{\partial t}+\frac{\partial}{\partial \mathscr{S}}\left(\rho_{(\alpha)} \dot{\mathscr{S}}_{(\alpha)}\right)\right].\end{aligned} \tag{2.15}$$

3. General local balance laws of the micromorphic theory of rod

Applying the physical principle b) stated in the introduction and using the general form of the local balance laws of the micromorphic theory of rod [2], we can write the general form of the local balance laws and the main discontinuity conditions, which apply to each single constituent, i.e.

$$\frac{\partial \tau_{(\alpha)}}{\partial \mathcal{S}} + g_{(\alpha)} - \grave{\sigma}_{(\alpha)} = 0 \tag{3.1}$$

$$[\tau_{(\alpha)} - \psi_{(\alpha)} (\mathcal{S}'_{(\alpha)} - u)] = 0$$

where

$$\grave{\sigma}_{(\alpha)} = \grave{\psi}_{(\alpha)} + \psi_{(\alpha)} \frac{\partial \grave{\mathcal{S}}_{(\alpha)}}{\partial \mathcal{S}} . \tag{3.2}$$

4. Particular balance laws of constituents and mixture

Using the general form of the local balance laws of the α-th constituent of the rod given in (3.1), we can derive the appropriate balance laws of single physical values. If we summ the over all constituent of the mixture we would get the balance laws for those physical values, which will apply to the mixture as a whole.

a) Mass density. The mass density balance law is obtained when one assumes that

$$\psi'_{(\alpha)} = \rho'_{(\alpha)}, \quad \tau'^{k}_{(\alpha)} = 0, \quad g'_{(\alpha)} = \rho\hat{\beta}'_{(\alpha)}, \tag{4.1}$$

where these values are defined in relation to the microelement. $\rho'_{(\alpha)}$ is the mass density of the α-th constituent in microelement, and $\rho\hat{\beta}'_{(\alpha)}$ is the volume change of the mass of the α-th constituent in microelement due to a chemical reaction wiht some other or all constituents of the mixture.

The corresponding tensor fields are defined by

$$\tau^{k}_{(\alpha)} = 0$$

$$\psi_{(\alpha)} = \langle \rho_{(\alpha)} \rangle = \rho_{(\alpha)} \tag{4.2}$$

$$g_{(\alpha)} = \langle \rho\hat{\beta}_{(\alpha)} \rangle = \rho\hat{\beta}_{(\alpha)}$$

where $\rho_{(\alpha)}$ is the mass denisity of the α-th constituent of the mixture and $\langle \ \rangle$ is the mean value. Substituting (4.2) in (3.2), we get

$$\grave{\sigma}_{(\alpha)} = \grave{\rho}_{(\alpha)} + \rho_{(\alpha)} \frac{\partial \grave{\mathcal{S}}_{(\alpha)}}{\partial \mathcal{S}} , \tag{4.3}$$

thus, from $(3.1)_1$, the mass balance low of the α-th constituent of the mixture is

$$\grave{\rho}_{(\alpha)} + \rho_{(\alpha)} \frac{\partial \grave{\mathcal{S}}_{(\alpha)}}{\partial \mathcal{S}} = \rho\hat{\beta}_{(\alpha)} . \tag{4.4}$$

If $(4.2)_1$ and $(4.2)_2$ are substituted in $(3.1)_2$ one gets the discontinuity condition of the α-th constituent of the mixture

$$[\rho_{(\alpha)}(\dot{\mathscr{S}}_{(\alpha)}-u)]=0. \tag{4.5}$$

When in (4.4) we summ the over all constituent of the mixture and use (2.7), (2.8), (2.9) and (2.10), one gets the balance law of the mass density of the mixture

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial\mathscr{S}}(\rho\dot{\mathscr{S}})=0, \tag{4.6}$$

on condition that

$$\sum_{\alpha}\rho\hat{\beta}_{(\alpha)}=0, \tag{4.7}$$

which means that within the mixture the mass remains constant.

One gets the discontinuity condition for the mixture from (4.5), using (2.7)

$$[\rho(\dot{\mathscr{S}}-u)]=0. \tag{4.8}$$

Fundamental identity (2.15), using (4.4) and (4.6) becomes

$$\sum_{\alpha}\rho_{(\alpha)}\dot{\psi}_{(\alpha)}=\rho\dot{\psi}+\sum_{\alpha}\frac{\partial}{\partial\mathscr{S}}(\rho_{(\alpha)}\psi_{(\alpha)}U_{(\alpha)})-\sum_{\alpha}\rho\hat{\beta}_{(\alpha)}\psi_{(\alpha)}. \tag{4.9}$$

b) Momentum. In this case are

$$\begin{aligned}
&\psi'_{(\alpha)}=\rho'_{(\alpha)}\underset{\sim}{v}'_{(\alpha)}\\
&\tau'^{k}_{(\alpha)}=\underset{\sim}{t}'^{k}_{(\alpha)}\\
&\tau'_{(\alpha)}=\underset{\sim}{t}'^{k}_{(\alpha)}n'_{(\alpha)k}=\underset{\sim}{t}'_{(n'_{\omega})}=\underset{\sim}{t}'_{(\alpha)}\\
&g'_{(\alpha)}=\rho'_{(\alpha)}\underset{\sim}{f}'_{(\alpha)}
\end{aligned} \tag{4.10}$$

where $\underset{\sim}{v}'_{(\alpha)}$, $\underset{\sim}{t}'^{k}_{(\alpha)}$ and $\underset{\sim}{f}'_{(\alpha)}$ are the velocity vector, stress vector and body force of the α-th constituent respectively, defined in a microelement. The corresponding values, figuring in (3.1), are of the form

$$\begin{aligned}
&\psi_{(\alpha)}=\rho_{(\alpha)}\underset{\sim}{v}_{(\alpha)}+\rho_{(\alpha)}i^{k}_{(\alpha)}\underset{\sim}{\nu}_{(\alpha)k}\\
&\tau_{(\alpha)}=\underset{\sim}{t}_{(\alpha)}\\
&g_{(\alpha)}=\rho_{(\alpha)}\underset{\sim}{f}_{(\alpha)}
\end{aligned} \tag{4.11}$$

where $\underset{\sim}{t}_{(\alpha)}$ and $\underset{\sim}{f}_{(\alpha)}$ are the stress vector and body force of the α-th constituent of the mixture.

Substituting (4.11) in (3.2) and considering (4.4) one gets

$$\dot{\sigma}(\alpha)=\rho(\hat{\beta}_{(\alpha)}\underset{\sim}{v}_{(\alpha)}+\hat{\beta}^{k}_{(\alpha)}\underset{\sim}{\nu}_{(\alpha)k})+\rho_{(\alpha)}\dot{\underset{\sim}{v}}_{(\alpha)}+\rho_{(\alpha)}i^{k}_{(\alpha)}(\underset{\sim}{\nu}_{(\alpha)l}\nu^{l}_{(\alpha)k}+\dot{\underset{\sim}{\nu}}_{(\alpha)k}). \tag{4.12}$$

When $(4.11)_{2,3}$ and (4.12) is substituted in $(3.1)_1$ one gets

$$(4.13)\quad \frac{\partial \underset{\sim}{t}_{(\alpha)}}{\partial \varphi} + \rho_{(\alpha)} (\underset{\sim}{f}_{(\alpha)} - \dot{\underset{\sim}{v}}_{(\alpha)}) - \rho_{(\alpha)} \dot{i}^k_{(\alpha)} (\underset{\sim}{v}_{(\alpha) l} \, v^l_{(\alpha) k} + \dot{\underset{\sim}{v}}_{(\alpha) k}) = \rho (\hat{\beta}_{(\alpha)} \underset{\sim}{v}_{(\alpha)} + \hat{\beta}^k_{(\alpha)} \underset{\sim}{v}_{(\alpha) k}).$$

When we take into account that

$$(4.14)\quad \dot{i}^k_{(\alpha)} = 0,$$

which physically means that the mass centre of microelement is in the same point in the microelement, then it follows that

$$(4.15)\quad \overline{\dot{i}^k_{(\alpha)}} = 0, \qquad \hat{\beta}^k_{(\alpha)} = 0.$$

Then from (4.13) one gets the balance of the momentum of the α-th constituent in the form of

$$(4.16)\quad \frac{\partial \underset{\sim}{t}_{(\alpha)}}{\partial \mathscr{S}} + \rho_{(\alpha)} (\underset{\sim}{f}_{(\alpha)} - \dot{\underset{\sim}{v}}_{(\alpha)}) = \rho \, \hat{\beta}_{(\alpha)} \underset{\sim}{v}_{(\alpha)} \cdot$$

Discontinuity condition of the α-th constituent of the mixture will be obtained if one uses $(3.1)_2$, $(4.11)_{1,2}$ with (4.14),

$$(4.17)\quad [\underset{\sim}{t}_{(\alpha)} - \rho_{(\alpha)} \underset{\sim}{v}_{(\alpha)} (\mathscr{S}_{(\alpha)} - u)] = 0.$$

If we summ the over all constituents of the mixture (4.16), and use the fundamental identity (4.9), giving that $\psi_{(\alpha)} = \underset{\sim}{v}_{(\alpha)}$ and $\psi = v$ and the following definitions

$$(4.18)\quad \sum_\alpha \rho_{(\alpha)} \underset{\sim}{f}_{(\alpha)} = \rho \underset{\sim}{f},$$

$$(4.19)\quad \underset{\sim}{t}^k = \sum_\alpha [\underset{\sim}{t}^k_{(\alpha)} - \rho_{(\alpha)} \underset{\sim}{u}_{(\alpha)} U_{(\alpha)}],$$

we shall get the balance of the momentum for the mixture

$$(4.20)\quad \frac{\partial \underset{\sim}{t}}{\partial \mathscr{S}} + \rho (\underset{\sim}{f} - \dot{\underset{\sim}{v}}) = 0.$$

If we summ the over all constituents of the mixture (4.17) and use (4.19), we shall get the discontinuity condition of the mixture,

$$(4.21)\quad [\underset{\sim}{t} - \rho \underset{\sim}{v} (\dot{\mathscr{S}} - u)] = 0.$$

c) Energy. The appropriate values in this case are

$$(4.22)\quad \begin{aligned} \psi'_{(\alpha)} &= \rho'_{(\alpha)} \left(\varepsilon'_{(\alpha)} + \frac{1}{2} \underset{\sim}{v}_{(\alpha)} \underset{\sim}{v}_{(\alpha)} \right) \\ \tau'_{(\alpha)} &= \underset{\sim}{t}'_{(\alpha)} \underset{\sim}{v}'_{(\alpha)} + q'_{(\alpha)} \\ \tau'^k_{(\alpha)} &= \underset{\sim}{t}'^k_{(\alpha)} \underset{\sim}{v}'_{(\alpha)} + q'^k_{(\alpha)} \\ g'_{(\alpha)} &= \rho'_{(\alpha)} \underset{\sim}{f}'_{(\alpha)} \underset{\sim}{v}'_{(\alpha)} + \rho'_{(\alpha)} h'_{(\alpha)} \end{aligned}$$

where $\varepsilon'_{(\alpha)}$, $\underset{\sim}{t}'_{(\alpha)}$, $\underset{\sim}{t}'^{k}_{(\alpha)}$, $q'^{k}_{(\alpha)}$ and $h'_{(\alpha)}$ represent internal energy density, stress vectors, heat flux vector and body heat supply of the α-th constituent respectively, defined in a microelement. The appropriate values from (3.1) and (3.2) have the form

$$\psi_{(\alpha)} = \rho_{(\alpha)}\left(\varepsilon_{(\alpha)} + \frac{1}{2}\underset{\sim}{v}_{(\alpha)}\underset{\sim}{v}_{(\alpha)} + \frac{1}{2}\underset{\sim}{\nu}_{(\alpha)r}\underset{\sim}{\nu}_{(\alpha)s}\, i^{rs}_{(\alpha)}\right)$$

$$\tau_{(\alpha)} = \underset{\sim}{t}_{(\alpha)}\underset{\sim}{v}_{(\alpha)} + \underset{\sim}{t}^{r}_{(\alpha)}\underset{\sim}{\nu}_{(\alpha)r} + q_{(\alpha)} \tag{4.23}$$

$$g_{(\alpha)} = \rho_{(\alpha)}\underset{\sim}{f}_{(\alpha)}\underset{\sim}{v}_{(\alpha)} + \rho_{(\alpha)}\underset{\sim}{f}^{r}_{(\alpha)}\underset{\sim}{\nu}_{(\alpha)r} + \rho_{(\alpha)}h_{(\alpha)}$$

where $\varepsilon_{(\alpha)}$, $\underset{\sim}{t}_{(\alpha)}$, $\underset{\sim}{t}^{r}_{(\alpha)}$, $q_{(\alpha)}$ and $h_{(\alpha)}$ represent internal energy density, stress vectors, heat flux vector and body heat supply per unit volume of the α-th constituent respectively.

Substituting (4.23) in (3.2) and considering (4.4) we get

$$\dot{\sigma}_{(\alpha)} = \rho_{(\alpha)}\left[\varepsilon'_{(\alpha)} + \underset{\sim}{v}_{(\alpha)}\underset{\sim}{v}'_{(\alpha)} + \underset{\sim}{\nu}_{(\alpha)s}\, i^{rs}_{(\alpha)}\left(\dot{\underset{\sim}{\nu}}_{(\alpha)r} + \underset{\sim}{\nu}_{(\alpha)k}\nu^{k}_{(\alpha)r}\right)\right] + \tag{4.24}$$

$$+ \rho\,\hat{\beta}_{(\alpha)}\left(\varepsilon_{(\alpha)} + \frac{1}{2}\underset{\sim}{v}_{(\alpha)}\underset{\sim}{v}_{(\alpha)}\right) + \frac{1}{2}\rho\,\underset{\sim}{\nu}_{(\alpha)r}\underset{\sim}{\nu}_{(\alpha)s}\,\hat{\beta}^{rs}_{(\alpha)}.$$

If $(4.23)_{3,6}$ and (4.24) is substituted in $(3.1)_1$ one will obtain the energy balance law of the α-th constituent in the form of

$$\rho_{(\alpha)}\,\varepsilon_{(\alpha)} - \underset{\sim}{t}_{(\alpha)}\frac{\partial \underset{\sim}{v}_{(\alpha)}}{\partial \mathscr{S}} - \underset{\sim}{t}^{r}_{(\alpha)}\frac{\partial \underset{\sim}{\nu}_{(\alpha)r}}{\partial \mathscr{S}} - \left(\bar{\underset{\sim}{t}}^{r}_{(\alpha)} - \lambda^{r}\underset{\sim}{t}^{r}_{(\alpha)}\right)\underset{\sim}{\nu}_{(\alpha)r} - \frac{\partial q_{(\alpha)}}{\partial \mathscr{S}} - \rho_{(\alpha)}h_{(\alpha)} = \tag{4.25}$$

$$= \rho\,\hat{\beta}_{(\alpha)}\left(\frac{1}{2}\underset{\sim}{v}_{(\alpha)}\underset{\sim}{v}_{(\alpha)} - \varepsilon_{(\alpha)}\right) + \frac{1}{2}\rho\,\hat{\beta}^{rs}_{(\alpha)}\,\underset{\sim}{\nu}_{(\alpha)r}\underset{\sim}{\nu}_{(\alpha)s}.$$

Substituting $(4.23)_{1,3}$ in $(3.1)_2$ one will get the discontinuity condition of the α-th constituent in the form of

$$\left[\underset{\sim}{t}_{(\alpha)}\underset{\sim}{v}_{(\alpha)} + \underset{\sim}{t}^{r}_{(\alpha)}\underset{\sim}{\nu}_{(\alpha)r} + q_{(\alpha)} - \rho_{(\alpha)}\left(\varepsilon_{(\alpha)} + \frac{1}{2}\underset{\sim}{v}_{(\alpha)}\underset{\sim}{v}_{(\alpha)} + \right.\right. \tag{4.26}$$

$$\left.\left. + \frac{1}{2}\underset{\sim}{\nu}_{(\alpha)r}\underset{\sim}{\nu}_{(\alpha)s}\, i^{rs}_{(\alpha)}\right)(\dot{\varphi} - u)\right] = 0.$$

If equation (4.25) summs the over all constituents and if fundamental identity (4.9) is used, after a longer calculation and rearranging, one will get the energy balance law for the mixture,

$$-\rho\,\dot{\varepsilon} + \underset{\sim}{t}\frac{\partial \underset{\sim}{v}}{\partial \mathscr{S}} + \underset{\sim}{t}^{r}\frac{\partial \underset{\sim}{v}_{r}}{\partial \mathscr{S}} + \left(\bar{\underset{\sim}{t}}^{r} - \lambda^{r}\underset{\sim}{t}^{r}\right)\underset{\sim}{\nu}_{r} + \frac{\partial q}{\partial \mathscr{S}} + \rho\,h = 0, \tag{4.27}$$

where the following definitions are used

$$\rho\,\varepsilon=\sum_{\alpha}\rho_{(\alpha)}\left(\varepsilon_{(\alpha)}+\frac{1}{2}\,\underset{\sim}{u}_{(\alpha)}\,\underset{\sim}{u}_{(\alpha)}+\frac{1}{2}\,\underset{\sim}{\theta}_{(\alpha)\,r}\,\underset{\sim}{\theta}_{(\alpha)\,s}\,i^{rs}_{(\alpha)}\right)$$

$$\underset{\sim}{t}^{k}=\sum_{\alpha}\left[\underset{\sim}{t}^{k}_{(\alpha)}-\rho_{(\alpha)}\,\underset{\sim}{u}_{(\alpha)}\,U_{(\alpha)}\right]$$

$$\underset{\sim}{t}^{kr}=\sum_{\alpha}\left[\underset{\sim}{t}^{kr}_{(\alpha)}-\rho_{(\alpha)}\,\underset{\sim}{\theta}_{(\alpha)\,r}\,U_{(\alpha)}\,i^{rs}_{(\alpha)}\right]$$

(4.28)

$$\underset{\sim}{t}^{r}=\sum_{\alpha}\left[\underset{\sim}{t}^{r}_{(\alpha)}-\rho_{(\alpha)}\,\underset{\sim}{u}_{(\alpha)}\,U_{(\alpha)}-\rho_{(\alpha)}\,\underset{\sim}{\theta}_{(\alpha)\,s}\,\theta^{r}_{(\alpha)\,k}\,i^{ks}_{(\alpha)}\right]$$

$$q^{k}=\sum_{\alpha}\left[q^{k}_{(\alpha)}+\underset{\sim}{t}^{r}_{(\alpha)}\,\underset{\sim}{u}_{(\alpha)}+\underset{\sim}{t}^{kr}_{(\alpha)}\,\underset{\sim}{\theta}_{(\alpha)\,r}-\rho_{(\alpha)}\left(\varepsilon_{(\alpha)}+\frac{1}{2}\,\underset{\sim}{u}_{(\alpha)}\,\underset{\sim}{u}_{(\alpha)}+\frac{1}{2}\,\underset{\sim}{\theta}_{(\alpha)\,r}\,\underset{\sim}{\theta}_{(\alpha)\,s}\,i^{rs}_{(\alpha)}\right)U_{(\alpha)}\right]$$

$$\rho\,h=\sum_{\alpha}\rho_{(\alpha)}\left(h_{(\alpha)}+\underset{\sim}{f}_{(\alpha)}\,\underset{\sim}{u}_{(\alpha)}+\underset{\sim}{f}^{r}_{(\alpha)}\,\underset{\sim}{\theta}_{(\alpha)\,r}\right).$$

If one summs the over all constituents of the mixture (4.26) and uses the definitions (4.28) one will get the discontinuity condition of the mixture

$$\left[\underset{\sim}{t}\,\underset{\sim}{v}+\underset{\sim}{t}^{r}\,\underset{\sim}{\nu}_{r}+q-\rho\left(\varepsilon+\frac{1}{2}\,\underset{\sim}{v}\,\underset{\sim}{v}+\frac{1}{2}\,\underset{\sim}{\nu}_{r}\,\underset{\sim}{\nu}_{s}\,i^{rs}\right)(\dot{\mathscr{S}}-u)\right]=0. \tag{4.29}$$

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МИКРОМОРФНАЯ ТЕОРИЯ МАТЕРИЯЛА СМЕСИ ПРИМЕННЕНАЯ К ТЕОРИИ СТЕРЖНЕЙ

П. Цветкович

Резюме

Пользуясь общей формой локальных законов баланса микроморфной теории стержневых систем, написанна общая форма локальных законов баланса и условий перерыва для компоненты материяла смеси. Применяя общую форму балансного уравнения к частностям, выведенны законы баланса массы, количества движения и енергии, вместе с соответствующими условями перерыва для компоненты и для смеси в целом.

МИКРОМОРФНА ТЕОРИЈА МЕШАВИНЕ ПРИМЕЊЕНА НА ТЕОРИЈУ ШТАПОВА

Предраī Цвеīковић

Резиме

Користећи општи облик локалних закона баланса микроморфне теорије штапова написан је општи облик локалних закона баланса и услова дисконтинуитета штапа за сваки састојак мешавине појединачно. Примењујући те законе баланса изведени су закони баланса масе, количине кретања и енергије као и одговарајућих услова дисконтинуитета за случај једног састојка мешавине и мешавине у целини.

Предраг Цветковић
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Овај рад је део истраживачког пројекта Института за механик ПМФ у оквиру програма Заједнице наука СР Србије